Using Deadband in Packet-Based Networked Control Systems

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Abstract—A packet-based deadband control approach is proposed for Networked Control Systems (NCSs). Within the packet-based control framework for NCSs, the proposed deadband control strategy takes full advantage of the packet-based data transmission in NCSs, and thus considerably reduces the use of the communication resources in NCSs whilst maintaining the system performance at a satisfactory level. The stability conditions of the closed-loop system are obtained and a numerical example illustrating the effectiveness of the proposed approach is presented.

Index Terms—Packet-based control, Deadband, Networked control systems, Communication constraints.

I. INTRODUCTION

Networked Control Systems (NCSs) has been a growing research area in the last decade, where the control loop is closed via some form of communication network, which thus enables its potential application to a vast range of remote and distributed control areas, such as remote surgery, automated highway systems, etc. [1]. However, the introduction of the communication channels can also mean imperfect data transmission in NCSs, which significantly degrades the system performance or even destabilizes the system at certain conditions, thus presenting a great challenge for conventional control and communication theory [2]–[5].

The early work on NCSs has been done mainly from the perspective of control theory [6]–[11]. In these studies, the dynamics of the communication network in NCSs has typically been modeled as some uncontrollable parameters within the control system and then a conventional control system rather than an NCS is actually considered. However, the reality is that the system performance of NCSs are affected by the dynamics of both the control system and the communication network, and the introduction of the communication network brings to the control system some extraordinary phenomena distinct from conventional control systems. Therefore the absence of investigating the properties of the communication network invariably introduces considerable conservativeness to both the analysis and design of NCSs. To deal with this issue, researchers have sought to integrate control and communication in their analysis and design of NCSs, which is the so-called “co-design” approach to NCSs. In this kind of studies, the imperfect packet-based data transmission has been extensively explored and as a result, a better system performance is expected than those using conventional control approaches [12]–[19].

Within the co-design framework, a packet-based control approach is recently proposed for NCSs where the packet-based data transmission structure in NCSs is more efficiently used to send a sequence of forward control signals together, instead of one at a time, which as a result can simultaneously effectively deal with network-induced delay, data packet dropout and data packet disorder in NCSs simultaneously [20]–[22]. Using this approach, a better system performance is achieved without requiring additional communication resource. It is noticed, however, the length of the Forward Control Sequence (FCS) used in the previously reported packet-based control approach in [20]–[22] is determined by the upper bound of the communication constraints rather than the capacity of the network. This can mean that the packet structure of the network has not been fully taken advantage of and thus can result in considerable conservativeness. Based on this observation, in this paper a packet-based deadband control approach is proposed by using more efficiently the packet structure in NCSs. This is done by extending the length of FCS to the maximum of what the network can contain, and then setting a deadband on FCS which allows transmission only in the presence of a sufficiently large change between the current FCS and the one last sent. As a result, this strategy can significantly reduce the data transmissions in NCSs and in the meanwhile retain the system performance at a satisfactory level. The stability of the closed-loop system is considered using delay-dependent analysis and the effectiveness of this approach is also illustrated by a numerical example.

The remainder of the paper is organized as follows. Section II presents the packet-based deadband control strategy for NCSs, the stability of the corresponding closed-loop system is then considered in Section III. A numerical example is presented in Section IV to illustrate the effectiveness of the proposed approach and Section V concludes the paper.

II. PACKET-BASED DEADBAND CONTROL FOR NCSs

In this paper, the following linear plant in discrete time is considered, which is controlled over the network by a remote controller as shown in Fig. 1:

\[ x(k+1) = Ax(k) + Bu(k) \] (1)

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where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$.

For the implementation of the packet-based deadband control approach to NCSs, the following assumptions with respect to the dynamics of the control system and the communication network, are required [20]–[22]. The rationality and implication of the assumptions are further discussed in Remark 1.

**Assumption 1:** The sum of the network-induced delay and consecutive data packet dropout in both channels (denoted by $\tau_{sc}$ and $\tau_{ca}$ respectively) are upper bounded, i.e.,

$$\bar{\tau}_{sc} \triangleq \max_{k \geq 1} \{ \bar{x}_{sc} + \tau_{sc,k} \} < \infty$$  \hspace{1cm} (2a)

$$\bar{\tau}_{ca} \triangleq \max_{k \geq 1} \{ \bar{x}_{ca} + \tau_{ca,k} \} < \infty$$  \hspace{1cm} (2b)

where $\tau_{sc,k}$, $\bar{x}_{sc}$ and $\tau_{ca,k}$, $\bar{x}_{ca}$ represent the network-induced delay and the upper bound of consecutive data packet dropout in the sensor-to-controller and the controller-to-actuator channels respectively.

**Assumption 2:** The control components in NCSs including the sensor, the controller and the actuator are time synchronized and data packets are sent with time stamps to notify when they were sent.

**Remark 1:** From Assumption 1 it is seen that, the controller (or the actuator) is always able to receive the sampled data (or the control signal) within a finite time period, which is reasonable in practice as well as necessary in theory; Assumption 2 implies that the delays in the sensor-to-controller channel and in the round trip are bounded by the controller and the actuator respectively. Similar assumptions have also been made for the implementation of the packet-based control approach in [20]–[22].

In order to present the packet-based deadband control approach to the system in (1) clearly and for completeness, the basic ideas of the packet-based control approach for NCSs will first be presented in the following subsection. We then point out its deficiency and propose the deadband control strategy for the packet-based control approach, by more efficiently using the packet structure in NCSs.

**A. Exploring the packet structure–packet-based control for NCSs**

Denote the effective load of the data packet being used in the NCS illustrated in Fig. 1 by $B_p$ and the data size required for encoding a single step of the control signal by $B_c$. The number of control signals that one data packet can contain can then be obtained as

$$N = \frac{B_p}{B_c}$$

(3)

where $\lfloor \frac{B_p}{B_c} \rfloor = \max\{ \varsigma | \varsigma \in \mathbb{N}, \varsigma \leq \frac{B_p}{B_c} \}$.

The key point of the previously reported packet-based control approach is to realize that $N$ in (3) is usually larger than $\tau_{ca}$. This observation thus enables us to send a sequence of forward control signals simultaneously over the network instead of one at a time as typically done in conventional control systems, with the length of the sequence being $\tau_{ca} + 1$. That is, at time $k$, instead of calculating and sending only current control signal $u(k)$, the following FCS $U_p(k|k - \tau_{sc,k})$ is packed into one data packet and sent to the actuator,

$$U_p(k|k - \tau_{sc,k}) = [u(k|k - \tau_{sc,k}) \ldots u(k + \tau_{ca}|k - \tau_{sc,k})]$$

(4)

On receiving $U_p(k|k - \tau_{sc,k})$, the actuator is then able to select from it the appropriate control signal to actively compensate for current communication constraints in NCSs. For example, if the delay in the control-to-actuator channel for $U_p(k|k - \tau_{sc,k})$ is $\tau_{ca}$, the actuator may thus choose $u(k + \tau_{ca}|k - \tau_{sc,k})$ at time $k + \tau_{ca}$ to apply it to the plant to compensate for the communication constraints (Notice here that all the time instants mentioned are based on the controller side). This packet-based control approach, as shown in [20]–[22], generally leads to a better performance of NCSs than that using conventional control approaches, since more of the properties of the control system and the communication network have been considered. The reader is referred to [20]–[22] for further details of this approach.

**B. Making full use of the packet structure–packet-based deadband control for NCSs**

Conventional control networks, such as ControlNet, DeviceNet, etc., which have been specially optimized for control applications and thus can meet the real time requirement of control systems to a certain extent, have been widely used in industrial processes for several decades [23]. However, it is seen that more and more network-based control applications are now using Internet rather than conventional control networks, due to the low cost, easy maintenance, remote control capability, etc. brought by the Internet, and thus Internet-based NCSs constitutes the main topic of this paper. Unlike conventional control networks, the Internet is a data network instead of a real time network, which means, it is optimized for data transmission and thus difficult to meet the critical real time requirement of control systems. Therefore, for Internet-based NCSs, we have to deal with worse communication conditions such as larger delay, more data packet dropout and disorder, etc.

In the meanwhile, it is noticed that as a data network, Internet uses data packets with a much larger size than that in conventional control networks. Take Ethernet as an example, which is widely used as the Local Area Network (LAN) in

![Fig. 1. The block diagram of a networked control system.](image-url)
but not determined by the upper bound of the communication is extended to the maximum of what a data packet can contain to design the following modified FCS where the length of FCS can then contain at least $2^3$ most control applications. In the Ethernet case, one data packet $65536$ different control signals is often used and ample for transmission rather than the data packet size [23], [24].

On the other hand, a $16$-bit data which can encode $2^{16} = 65536$ different control signals is often used and ample for most control applications. In the Ethernet case, one data packet can then contain at least $23$ such control signals (it can contain much more since the typical size of the data packet used in Ethernet is around several hundreds bytes and the maximum is $1500$ bytes) while in a typical Internet-based control application, where for example the plant and the controller are located respectively in the University of Glamorgan, Pontypridd, UK and the Chinese Academy of Sciences, Beijing, China, the network-induced delay (data packet dropout as well) in the controller-to-actuator channel is upper bounded by $4$ sampling periods with the sampling period being $0.041s$ [22]. From this analysis, it is readily seen that $N$ in (3) is normally much larger than $\bar{\tau}_{ca}$ in practice. This observation thus motivates us to design the following modified FCS where the length of FCS is extended to the maximum of what a data packet can contain but not determined by the upper bound of the communication constrain in the controller-to-actuator channel as in [20]–[22],

$$U(k|k - \tau_{sc,k}) = [u(k|k - \tau_{sc,k}) \ldots u(k+N-1|k - \tau_{sc,k})]$$ (5)

It is seen that using such a modified FCS, we have hardly increase the delay of transmitting it and what is more, by using the following deadband control strategy, we can significantly reduce the use of the communication resources while retain the system performance at a satisfactory level.

The motivation of proposing the deadband control strategy is due to the fact that the communication constraints play a dominant role in the system performance of NCSs and for a better system performance, we have to decrease possible congestion in the network by reducing the use of the communication resources. On the other hand, much more redundant forward control signals are packed into one data packet using FCS in (5). This enables us to set a deadband for FCSs and send only those that have a sufficiently large change compared with the last sent FCS. In this way, the use of the communication resources can be significantly reduced and the system performance can still be retained at a satisfactory level if the deadband is carefully chosen.

The block diagram of packet-based deadband control for networked control systems is illustrated in Fig. 2, where it is seen that this structure is different from conventional control approaches mainly in two aspects: the packet-based deadband controller and the so-called Control Action Selector (CAS) at the actuator side. For the latter, which consists of a register to store only the latest data packet and a logic comparator to determine which data packet contains the latest information and thus can be used to deal with data packet disorder and to actively compensate for network-induced delay, the reader is referred to [20]–[22] for details.

The packet-based deadband controller is used to produce FCS in (5) and, different from previously reported packet-based control approaches, also to determine whether a newly produced FCS should be sent or not. For this purpose, a register is present at the controller side to store the last sent FCS which is denoted by $U(k - \bar{\tau}_k|k - \bar{\tau}_k - \tau_{sc,k} - \bar{\tau}_{ca})$ at time $k$ at the controller side, where $k - \bar{\tau}_k$ is the time when the last FCS was sent. The newly produced FCS $U(k|k - \tau_{sc,k})$ at time $k$ will be sent to the actuator if the following inequality satisfies,

$$\max_{0 \leq i \leq N - \bar{\tau}_k - 1} \left| \frac{\delta u_{ki}}{u(k+i|k - \tau_{sc,k})} \right| > \Delta$$ (6)

where $\Delta$ is the deadband set for FCSs and $\delta u_{ki} = u(k+i|k - \tau_{sc,k}) - u(k+i|k - \bar{\tau}_k - \tau_{sc,k} - \bar{\tau}_{ca})$. On the other hand, in order that there is always a control signal available at the actuator side, FCS has to be sent at least once within $N - \bar{\tau}_{ca}$ time steps, which also implies that $\bar{\tau}_k \leq N - \bar{\tau}_{ca} - 1, \forall k$.

The algorithm of the packet-based deadband control approach to NCSs can be organized as follows.

Algorithm 1 (Packet-based deadband control):
S1. Initiation. Set $k = 0$, $g_k = 0$.
S2. The sensor samples the plant and sends the sampled data packet to the controller.
S3. At time $k$ at the controller side, if either 1) (6) satisfies; or 2) $g_k = N - \tau_{ca} - 1$, then send the current FCS to the actuator, update the register of the controller to be this FCS, and let $g_{k+1} = 1$, $k = k + 1$; otherwise let $g_{k+1} = g_k + 1$, $k = k + 1$ and wait for the next time instant.
S4. On receiving a new FCS, CAS compares its time stamp with that of $\bar{\tau}$. If the time stamp of the new FCS is less than $\bar{\tau}$, then send the current FCS to the controller side; update the actuator side of the controller to be this FCS; and wait for the next time instant.

The control signal taken at time $k$ at the actuator side is determined by

$$u^p(k) = u(k|k - \tau_k^p)$$. \hspace{1cm} (7a)

where $\tau_k^p$ denotes the round trip delay of the FCS being used at time $k$ (see Fig. 2 and references [20]–[22]). According to the structure of the FCS in (4), it is readily concluded that $\tau_k^p \leq \bar{\tau}$, where $\bar{\tau} = \tau_{sc} + \tau_{ca}$ is the upper bound of the delay and consecutive data packet dropout for the round trip.

With the use of FCS in (5) and the corresponding deadband control strategy in (6), the control signal used may be based on older sampled data information with the control action taken at time $k$ being

$$u(k) = u(k|k - \tau_k^s)$$. \hspace{1cm} (7b)

where $\tau_k^s$ denotes the round trip delay of the FCS being used in the packet-based deadband control case and it is seen that $\tau_k^s \geq \tau_k^p, \forall k$. Though the control signal in (7b) may be based on older sampled data information, with the deadband control strategy, the difference between $u(k)$ and $u^p(k)$ is however bounded, which also helps to maintain the system performance with the packet-based deadband control approach at a satisfactory level,

$$|u(k) - u^p(k)| \leq \Delta|u^p(k)|$$ \hspace{1cm} (7c)

For simplicity, in this paper state feedback is used and thus the control law in (7a) and (7b) can be explicitly represented by

$$u^p(k) = k_{rs}^{p} x(k - \tau_k^p)$$ \hspace{1cm} (7d)

and

$$u(k) = k_{rs}^{s} x(k - \tau_k^s)$$ \hspace{1cm} (7e)

where $\tau_k^p$ and $\tau_k^s$ are the network-induced delays in the sensor-to-controller and the controller-to-actuator channels with respect to $\tau_k^p$ and $\tau_k^s$, respectively, that is, $\tau_k^p = \tau_{sc}^p + \tau_{ca}^p$, $\tau_k^s = \tau_{sc}^s + \tau_{ca}^s$, and $k_{rs}^{p}, k_{rs}^{s}$ are the corresponding feedback gains. With the control law in (7e), the stability of the closed-loop system is then analyzed in the following subsection.

B. Stability analysis

With the control law in (7e), the closed-loop system for the plant in (1) can be obtained as

$$x(k + 1) = Ax(k) + Bk_{rs}^{s} x(k - \tau_k^s)$$ \hspace{1cm} (8)

Denote the set of $k_{rs}^{s} \tau_{sc}^{s} \tau_{ca}^{s} \chi$, $1 \leq \tau_{sc}^{s} \leq \tau_{ca}^{s}, 1 \leq \tau_{ca}^{s} \leq N - 1$ by $\Gamma$ and let $\bar{k} = \max \{||k_{rs}^{s} \tau_{sc}^{s} \tau_{ca}^{s} \chi || | k_{rs}^{s} \tau_{sc}^{s} \tau_{ca}^{s} \chi \in \Gamma \}$, where $|| \cdot ||$ denotes the Euclidean norm. Then we have the following

$$Bk_{rs}^{s} \tau_{sc}^{s} \tau_{ca}^{s} = B_m \cdot D_{rs}^{s}$$ \hspace{1cm} (9)

where $B_m = \frac{1}{\bar{k}}B$ and $D_{rs}^{s} \tau_{sc}^{s} \tau_{ca}^{s} = k_{rs}^{s} \tau_{sc}^{s} \tau_{ca}^{s} \chi / \bar{k}$. It is readily seen from the definition that $||D_{rs}^{s} \tau_{sc}^{s} \tau_{ca}^{s} \chi || \leq 1, \forall 1 \leq \tau_{sc}^{s} \leq \tau_{ca}^{s}, 1 \leq \tau_{ca}^{s} \leq N - 1$.

For the closed-loop system in (8), we obtain the following stability theorem based on the delay-dependent analysis technique.

**Theorem 1:** The system in (1) using the control law in (7e) and the packet-based deadband control approach presented in Section II is stable if there exist $P_i = P_i^T > 0, i = 1, 2, 3, X = (X_{11}^T \quad X_{12}^T \quad X_{21} \quad X_{22}) \geq 0, Q_i = 1, 2$ with appropriate dimensions and $\lambda > 0$ satisfying the following two LMI s,

$$X_{11} X_{12} + Q_1 X_{21} \geq 0 \hspace{1cm} (10)$$

$$\begin{bmatrix}
    \Phi_{11} & \Phi_{12} (A - I)^T H P_i B_m \\
    * & \Phi_{22} + \lambda I
  \end{bmatrix}
  \begin{bmatrix}
    0 & 0 \\
    0 & -H H B_m
  \end{bmatrix}
  \begin{bmatrix}
    \Phi_{11} & \Phi_{12} (A - I)^T H P_i B_m \\
    * & \Phi_{22} + \lambda I
  \end{bmatrix}^T
  < 0 \hspace{1cm} (11)$$

where

$$\Phi_{11} = (A - I)^T P_i + Q_1 + Q_1^T + (\tau_{sc} + N - 1) X_{11} + (\tau_{sc} + N - 2) P_2 + P_1 (A - I)$$
\[
\Phi_{12} = Q_2^T - Q_1 + (\tau_{ac} + N - 1)X_{12},
\]
\[
\Phi_{22} = -P_2 - Q_2 - Q_2^T + (\tau_{ac} + N - 1)X_{22},
\]
\[
H = P_1 + (\tau_{ac} + N - 1)P_3.
\]

**Proof:** Let \( d_1 = 2, d_2 = \tau_{ac} + N - 1, \Delta A(k) = A_f = B = \Delta B(k) = 0, \Delta A_p(k) = BK_{P, \Delta} \tau_{ac,k} \) in Theorem 7.3 in [27], and notice (9). The above theorem can then be obtained using the same techniques as in [27] and thus we omit the details.

IV. A NUMERICAL EXAMPLE

Consider the system in (1) with the following system matrices borrowed from [22], which is seen to be open-loop unstable,

\[
A = \begin{pmatrix}
0.98 & 0.1 \\
0 & 1
\end{pmatrix}, B = \begin{pmatrix}
0.04 \\
0.1
\end{pmatrix}.
\]

In the simulation, the initial state for the system in (1) is set as \( x_0 = [-1 \ 1]^T \) and the upper bounds of the delay and consecutive dropout are \( \tau_{ac} = 2, \tau_{sc} = 2 \) respectively.

The effectiveness of the proposed packet-based deadband control approach is illustrated by comparing with the previous packet-based control approach in [22]. For this purpose, the packet-based controller in [22] and the packet-based deadband controller in this paper are both designed using the same receding horizon approach as proposed in [22], which yields the following feedback gain for the packet-based control approach,

\[
K^p = [(K^p_1)^T \ (K^p_2)^T \ (K^p_3)^T]^T
\]

and the feedback gain for the packet-based deadband control approach with \( N = 6 \) and \( \Delta = 0.1, \)

\[
K = [K_{0.2}^T \ K_{0.1}^T \ K_{0.3}^T]
\]

\[
K_0 = \begin{pmatrix}
k_{0.0} \\
k_{0.1} \\
k_{0.2} \\
k_{0.3} \\
k_{0.4} \\
k_{0.5}
\end{pmatrix} = \begin{pmatrix}
-0.6438 \\
-0.5242 \\
-0.4198 \\
-0.2736 \\
-0.1845 \\
-0.1335
\end{pmatrix}
\]

\[
K_1 = \begin{pmatrix}
k_{1.0} \\
k_{1.1} \\
k_{1.2} \\
k_{1.3} \\
k_{1.4} \\
k_{1.5}
\end{pmatrix} = \begin{pmatrix}
-0.5432 \\
-0.4198 \\
-0.3419 \\
-0.2736 \\
-0.1845 \\
-0.1335
\end{pmatrix}
\]

\[
K_2 = \begin{pmatrix}
k_{2.0} \\
k_{2.1} \\
k_{2.2} \\
k_{2.3} \\
k_{2.4} \\
k_{2.5}
\end{pmatrix} = \begin{pmatrix}
-0.4542 \\
-0.3573 \\
-0.3419 \\
-0.2736 \\
-0.1845 \\
-0.1335
\end{pmatrix}
\]

It is seen from the comparison of the state responses in Fig. 3 that the system performance with the deadband control strategy is still maintained at a satisfactory level. This can also be verified by looking into the comparison of the control inputs for both cases in Fig. 4 where it is seen that the control inputs to both systems are very close. It is worth mentioning, however, only around 40% of FCSs are sent to the actuator with the deadband control strategy. The effectiveness of the packet-based deadband control approach can also been seen from Fig. 5, where with the same data packet transmission percentage (with heavy transmission load), the packet-based deadband control approach yields a far better system performance than...
the packet-based control approach.

V. CONCLUSIONS

By noticing that the packet structure has not been made fully use in the previously reported packet-based control approach, a deadband control strategy is thus proposed within the packet-based control framework. The proposed packet-based deadband control approach can effectively reduce the use of the communication resource in NCSs whilst retain the system performance at a satisfactory level. Stability conditions are obtained and the effectiveness of the proposed approach is verified by a numerical example. Future developments will focus on further theoretical analysis of the effects on the packet-based control approach of using the deadband control strategy, stabilized controller design in the presence of the deadband control approach can effectively reduce the use of the packet-based control framework. The proposed packet-based approach, a deadband control strategy is thus proposed within the packet-based control approach of using the deadband control approach can effectively reduce the use of the communication resource in NCSs whilst retain the system performance at a satisfactory level. Stability conditions are obtained and the effectiveness of the proposed approach is verified by a numerical example. Future developments will focus on further theoretical analysis of the effects on the packet-based control approach of using the deadband control strategy, stabilized controller design in the presence of the deadband control approach and corresponding experimental verification using Internet-based test rig for networked control systems.

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