2017 2nd International Conference on Computational Modeling, Simulation and Applied Mathematics (CMSAM 2017)
ISBN: 978-1-60595-499-8

The Simulation Analysis of Optimal Execution Based on Almgren-Chriss Framework

Tian-min ZHOU, Can JIA and Han-dong LI*

School of Systems Science, Beijing Normal University, Beijing 100875, P.R. China

*Corresponding author

Keywords: Optimal execution, Monte Carlo simulation, Brown Bridge, Realized volatility.

Abstract. First, we compute the optimal execution strategy under the Almgren-Chriss framework, based on the assumption that the asset price follows the arithmetic Brownian motion with the linear impact function. On this basis, we discuss the price impact under the execution strategy with the parameter κ, and find that the trajectory becomes steeper as the parameter κ increasing. In the simulation analysis, we use the Brown Bridge to construct three asset price generation processes, and we compare the execution strategy between the Almgren-Chriss framework and the TWAP (The Time Weighted Average Price) strategy. The result shows that the TWAP strategy is superior to the AC strategy under an upward price trend, the two strategies have the same performance without price trend, and the AC strategy is better than the TWAP strategy under a downward price trend.

Introduction

The algorithmic trading has been greatly developed with the increasing availability of high-frequency data and the widespread use of electronic computers in modern financial markets. It refers to the general name of trading methods such as using computers to develop transaction decisions, submitting orders, managing orders and so on. In occident developed capital markets, the trading volume by using the algorithm trading has exceeded 70% of the total volume recently. The algorithmic trading has had a significant impact on the daily trading volume, execution speed and average transaction size of the stock market. One of the core issues of algorithmic trading is the liquidation of institutional investors. Due to constraints of market liquidity, the trading large number of shares will inevitably bring impacts on the stock price, which increases transaction costs. Therefore, the investors need to split the order into small ones to execute, however this will increase the transaction time. Thus, a good execution strategy is to consider the balance between the transaction costs and the time risks, that is, we need to minimize the cost of liquidation or maximize the benefits.

A common approach is to use the price impact model, such as the Almgren-Chriss model [1, 2], to obtain the optimal execution strategy. The model assumes that the execution strategy is time-discrete and the security price process follows the random walk. The impact of the transaction process on the price can be decomposed into permanent market impact and temporary market impact.

The optimal criterion based on this model is to find the balance between minimizing the price impact and minimizing the time risks. It mainly includes: (1) risk minimization. This criterion uses the variance, the quadratic variation or the VaR [3,4] as risk measures and maximizes its expected revenue of trading at the same time; (2) utility function criterion. This criterion uses the power function or exponential function [5] as a utility function and maximizes it; (3) minimizing expected execution cost criteria. Such as minimizing the expected execution costs by modeling the dynamic character of the distribution of bids and ask in the limit order book [6].

Based on the Almgren-Chriss model framework, we analyze the optimal trading path under the mean-variance criterion. Then we discuss the influence of the impact coefficient on the transaction cost by simulation. On this basis, we compare the differences between AC strategy and the TWAP strategy under different price trend through Monte Carlo simulation.

Models and Methods

Consider the problem of liquidating a large number of shares in a stock before time T to maximize the expected terminal wealth. We assume that the dynamics of the stock is characterized by

$$dS(t) = \sigma dB(t) - \gamma v(t)dt, \quad (1)$$

where $S(t)$ denotes the price of one share of the stock at time t, and $dB(t)$ is the increment of Brownian motion, σ represents the volatility of the stock and γ is the permanent impact. In Eq. 1, $v(t)$ is the trading rate that is characterized by

$$v(t) = -\frac{dx(t)}{dt}, \quad (2)$$

where $x(t)$ denotes the number of holdings in the stock and $x(t)$ satisfies $x(0) = X$, $x(T) = 0$. According to the implement shortfall [7], our execution cost is expressed by

$$C = XS(0) - \int_0^T \big(S(t) - \eta v(t)\big)\, dx(t), \quad (3)$$

where the η represents the temporary impact. According to the Almgren-Chriss mean-variance discretization model [2], we obtain the value function under the given period

$$V\big(x(t)\big) = \min_{v\in A}\{E[C] + \lambda Var[C]\}$$
$$= \min_{v\in A} \int_t^T \big(\gamma v(u)x(u) + \eta v^2(u) + \lambda\sigma^2 x^2(u)\big)\, du, \quad (4)$$

where v is permissible control rate and A is permissible control set, then we get HJB equation

$$-\frac{\partial V}{\partial t} = \min_{v\in A}\left\{\gamma v(t)x(t) + \eta v^2(t) - v(t)\frac{\partial V}{\partial x}\right\} + \lambda\sigma^2 x^2(t) \quad (5)$$

and the optimal trading trajectory is given by

$$x(t) = X\left(\frac{e^{\kappa(T-t)} - e^{\kappa(t-T)}}{e^{\kappa T} - e^{-\kappa T}}\right) = \frac{sinh\big(\kappa(T-t)\big)}{sinh\,\kappa T}X, \quad (6)$$

where $\kappa = \sqrt{\lambda\sigma^2/\eta}$. That means when our trading time interval is very small, we can ignore the influence of permanent impact on the strategy. Then the trading rate can be given by

$$v_\kappa = -\frac{cosh\big(\kappa(T-t)\big)}{sinh\,\kappa T}X. \quad (7)$$

As long as $X > 0$, the number of transactions n at each time t must be positive. That is, for the trading of large positions, the solution we will get must be a monotonically decreasing function, the rate of decline determined by the parameter κ.

The Impact on the Optimal Liquidation by Parameter κ

In order to see more clearly the changes in the trajectory with the change of κ, let the time and the number of liquidation stocks change within the range of [0,1]. We rescale the original conditions, let $\hat{X} = 1$, $\hat{T} = 1$ and we assume that transaction times is $N = 240$ during $\hat{T}$ and $\Delta t = 1/N$ the Then we use the above parameters to represent the trading trajectory

$$\hat{x}(t) = \frac{sinh\big(\hat{\kappa}(\hat{T}-t)\big)}{sinh\,\hat{\kappa}\hat{T}}, \quad (8)$$

where $2/\Delta t^2(cosh(\hat{\kappa}\,\Delta t) - 1) = \lambda\sigma^2\big(\eta/\hat{T} - \gamma/2N^2\big)$. The parameter κ of the optimal execution strategy not only reflects the liquidation's influence on the price process (relate to impact coefficient), and also determine the shift rate of the optimal trading trajectory. So, we let κ change from small to large by simulation, and get a set of optimal trading trajectory curves shown as Fig. 1.

Figure 1 shows that as κ continues to increase, the steeper the trading trajectory curves. The steep execution strategy curve represents that at the beginning of the trading, the investors prefer to avoid risks and choose to execute a large number of positions to avoid the impact of subsequent price fluctuations. The extreme cases are to execute all positions at first time, however we know that the market cannot absorb such a huge order resulting in large stock impact.

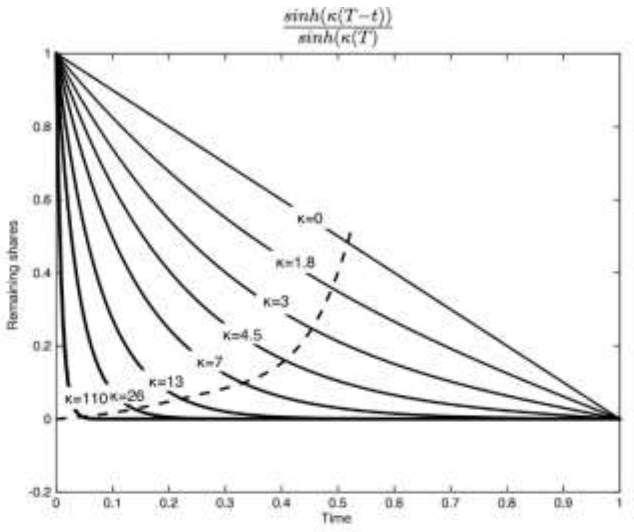

Figure 1. The impact on optimal trajectory by parameter κ.

Data Generation Mechanism

Assume that the stock price process follows the arithmetic Brownian motion, the temporary impact and permanent impact are linear function for each time $g(v) = \gamma v,\ h(v) = \eta v$. Then we establish three sample path for stock states by Brown Bridge: (1) the price process 1: $\{S(t), 0 \le t \le 1 | S(0) = 5, S(1) = 5\}$, (2) the price process 2: $\{S(t), 0 \le t \le 1 | S(0) = 5, S(1) = 4\}$, (3) the price process 3: $\{S(t), 0 \le t \le 1 | S(0) = 5, S(1) = 6\}$, and the three price processes respectively represent the day's stock movements unchanged, declining, or rising.

Then we use the stock index futures of China Securities Index 300 stock index futures from February 2, 2015 to June 18, 2015, including 90 trading days and 1-min interval period of the closing price, to calculate the realized volatility. According to the explanation from Anderson and Bollerslev [8], if the logarithmic returns satisfies the Itô process, $RV = \sum ln^2(S_i/S_{i+1})$. Then we calculate the realized 90 days volatilities of the futures and compare the realized volatility of simulated data with that of actual data. We find that when $\sigma = 1.5$, the mean value of realized volatility of generated data and the actual data have no significant difference at the 5% significance level (two independent simple t tests). That is, our generated data under the certain conditions to meet the real stock price operation process.

Simulation Analysis

Under the given simulation mechanism above, we compare the TWAP strategy with AC Strategy. TWAP strategy is to uniformly distribute the trading position into a trading period [9]. Then we specify that the execution strategy is the simple sell behavior, the number of the shares is one, the execution time is one, and the number of equal time intervals transaction is 240. Then we simulate the Brownian motion of different volatilities, and set risk preference, temporary impact and permanent impact separately be as follow: $\lambda = 1e-5,\ \eta = 2.5e-6,\ \gamma = 2.5e-6$. We carried out 5000 times of Monte Carlo simulation, and compare the number of the two strategies which transaction cost was lower than the opponent in 5000 experiments. The results are as follow:

Table 1. Compare the two strategies with the price trend unchanged.

σ	The type of the strategy	The number of wins	probability
0.1	TWAP	2535	0.5070
0.1	AC	2465	0.4930
0.475	TWAP	2540	0.5080
0.475	AC	2460	0.4920
0.95	TWAP	2482	0.4964
0.95	AC	2518	0.5036
1.5	TWAP	2519	0.5038
1.5	AC	2481	0.4962
2	TWAP	2513	0.5026
2	AC	2487	0.4974

Table 1 shows that in the price process 1, there is no rise or fall trend in the price process. No matter how σ value changes, the TWAP strategy and AC strategy are similar, and we cannot distinguish which one is better.

Table 2. Compare the two strategies with downward price trend.

σ	The type of the strategy	The winning times	probability
0.1	TWAP	0	0
0.1	AC	5000	1
0.475	TWAP	0	0
0.475	AC	5000	1
0.95	TWAP	67	0.0134
0.95	AC	4933	0.9866
1.5	TWAP	396	0.0792
1.5	AC	4604	0.9208
2	TWAP	747	0.1494
2	AC	4253	0.8506

Table 2 shows that in the price process 2, the price process has a downward trend. The transaction cost of AC strategy is significantly less than the TWAP strategy when σ changes from small to large, and as σ increases, the number of wins of TWAP strategy will increase, however σ will not increase without limitation in real situation.

Table 3. Compare the two strategies with increasing price trend.

σ	The type of the strategy	The winning times	probability
0.1	TWAP	5000	1
0.1	AC	0	0
0.475	TWAP	5000	1
0.475	AC	0	0
0.95	TWAP	4934	0.9868
0.95	AC	66	0.0132
1.5	TWAP	4319	0.8638
1.5	AC	681	0.1362
2	TWAP	4282	0.8564
2	AC	718	0.1436

Table 3 shows that in the price process 3, the price process has a upward trend. The cost of TWAP strategy is significantly less than the AC strategy when σ changes from small to large. As σ increases, the number of wins AC strategy will increase however σ will not increase without limitation in real situation.

Summary

In this paper, we analyze the optimal strategy of selling a large number of stock in financial market. Based on the assumption that the asset price follows the arithmetic Brownian motion, the mean-variance criterion and the linear impact function, we obtain the optimal trading strategy expression under the Almgren-Chriss framework.

On this basis, we first discuss the influence of the parameter κ changes on the trading strategy. It is found that as the parameter κ increases, the trading trajectories become steeper, which represents that at the beginning of the transaction, the investors prefer to avoid risks and choose to execute a large number of positions, in order to avoid the transaction by the follow-up price fluctuations.

Secondly, we use the Brown Bridge to construct three asset price generation processes, and then compare this strategy with the TWAP strategy. The result shows that the TWAP strategy is superior to the AC strategy under the upward price trend, the two strategies have the same performance without price trend, and AC strategy is significantly better than the TWAP strategy under downward price trend.

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