RESOURCE LETTER

Roger H. Stuewer, *Editor*
School of Physics and Astronomy, 116 Church Street SE,
University of Minnesota, Minneapolis, Minnesota 55455

This is one of a series of Resource Letters on different topics intended to guide college physicists, astronomers, and other scientists to some of the literature and other teaching aids that may help improve course content in specified fields. [The letter E after an item indicates elementary level or material of general interest to persons becoming informed in the field. The letter I, for intermediate level, indicates material of somewhat more specialized nature; and the letter A indicates rather specialized or advanced material.] No Resource Letter is meant to be exhaustive and complete; in time there may be more than one letter on some of the main subjects of interest. Comments on these materials as well as suggestions for future topics will be welcomed. Please send such communications to Professor Roger H. Stuewer, Editor, AAPT Resource Letters, School of Physics and Astronomy, University of Minnesota, 116 Church Street SE, Minneapolis, MN 55455; e-mail: rstuewer@physics.umn.edu.

Resource Letter RPS-1: Research in problem solving

Leonardo Hsu
General College, University of Minnesota, Minneapolis, Minnesota 55455

Eric Brewe
Department of Physics, Hawaii Pacific University, Honolulu, Hawaii 96813

Thomas M. Foster
Department of Physics, Southern Illinois University, Edwardsville, Edwardsville, Illinois 62026

Kathleen A. Harper
Faculty and TA Development, The Ohio State University, Columbus, Ohio 43201

(Received 14 October 2003; accepted 23 April 2004)

This Resource Letter provides a guide to the literature on research in problem solving, especially in physics. The references were compiled with two audiences in mind: physicists who are (or might become) engaged in research on problem solving, and physics instructors who are interested in using research results to improve their students' learning of problem solving. In addition to general references, journal articles and books are cited for the following topics: cognitive aspects of problem solving, expert-novice problem-solver characteristics, problem solving in mathematics, alternative problem types, curricular interventions, and the use of computers in problem solving.

[DOI: 10.1119/1.1763175]

I. INTRODUCTION

Problem solving is an integral part of most physics courses. Many instructors would like their students to learn to use physics principles and concepts to solve problems. However, experienced instructors know that this is a difficult task. After even the most lucid lectures and textbook presentations, typically only a fraction of the students in a class is able to solve problems with the desired facility. To help more students become competent problem solvers, it can be useful to study how people solve and learn to solve problems. Such research can be used to guide improvements in instruction.

Just as superconductivity and polymer physics are subfields of condensed matter physics, the study of how students learn to solve problems can be considered a subfield of physics education research (PER). Other PER subfields include the study of how students gain conceptual understanding, how laboratory activities and demonstrations contribute to the learning of physics, and how the attitudes, expectations, and beliefs of students affect their learning of physics. Each of these subfields is concerned with how students learn physics, but studies a different aspect of the subject or limits itself to some domain such as laboratories or lectures. Research in problem solving also extends beyond physics education research, with links to cognitive science, psychology, and education. Besides physics and mathematics, researchers in those fields have performed studies using chess problems, logic puzzles, and a variety of other contexts (Refs. 6 and 8).

Resource Letter PER-1 (Ref. 16) contained references from all aspects of physics education research, including some basic ones from the problem-solving literature. The purpose of this Resource Letter is to provide for physicists a more thorough introduction to the literature, exploring in greater detail the multiple aspects of human problem solving that researchers have studied. What may at first seem like a narrow topic is actually quite a rich field. The references cited here focus primarily on problem solving in the context of introductory physics. Citations from other disciplines have been included where they may be of interest to physicists.

Publications about problem-solving range from basic research on the cognitive processes involved in problem solving, to studies of the effect of curricular materials on students' learning, to the development of computer programs designed to help students learn to solve problems. The references are organized into sections corresponding to major threads in the problem-solving literature. Section II contains general references on problem solving, giving a more detailed overview of the research than can be done here, while Sec. III features articles in which various definitions of what qualifies as a problem are considered. Although the question of what constitutes a problem is not itself a subject of research, the viewpoints expressed in the articles in Sec. III help to frame a context for the research studies that follow. Sections IV and V include citations from two major areas of research, the study of the cognitive aspects of problem solving (the thought processes required to solve problems) and the study of the problem-solving characteristics of novices and experts. Because of the close relationship between physics and mathematics and the similarities in problem solving in both of these fields, references to problem-solving research in mathematics are given in Sec. VI. Finally, Secs. VII–IX describe various types of applied research in problem solving, including the design of new types of problems, curricular development efforts based on problem-solving research, and the creation of computer systems to help students become better problem solvers. Within each section or subsection, the works are sorted by date, from earliest to most recent. Finally, we include a short glossary of terms that may not be familiar to physicists, but which are commonly found in the problem-solving literature.

II. GENERAL REFERENCES

A. Journals

Because problem solving has been studied in a wide range of fields, many nonphysics journals contain research on problem solving. We list relevant journals in three categories: (1) those with research specific to physics contexts; (2) those with research specific to science contexts; and (3) those with research in a wide range of science and nonscience domains. These lists are not exhaustive, but the bulk of the articles related to problem solving in physics appear in the following journals.

Problem solving in physics
American Journal of Physics (including the *Physics Education Research Supplement*)
Physics Education
The Physics Teacher

Problem solving in the sciences
International Journal of Science Education (formerly *European Journal of Science Education*)
Journal of College Science Teaching
Journal of Research in Science Teaching
Science Education

Problem solving in general
Cognition and Instruction
Cognitive Science
Educational Psychology
Journal of Educational Psychology
Journal of the Learning Sciences

B. Conference proceedings

1. **Problem Solving and Education: Issues in Teaching and Research**, edited by D. T. Tuma and F. Reif (Erlbaum, Hillsdale, NJ, 1980). These are proceedings from a 1978 conference on problem solving, and chapter authors include many recognized names in the field. This is an excellent background resource, as the issues described are all relevant to problem solving today, although some of these areas have seen significant progress in the past 25 years. The summary chapter provides an overview. (I)
2. **Teaching and Learning Mathematical Problem Solving**, edited by E. A. Silver (Erlbaum, Hillsdale, NJ, 1985). The proceedings from a 1983 conference on mathematical problem solving, this book contains chapters describing mathematics education research, cross-disciplinary perspectives (including physics), and directions for research. While the book is slightly outdated, the opening chapter by J. Kilpatrick outlining the issues with mathematical problem solving is relevant to physics instruction. The chapter by J. I. Heller and H. N. Hungate provides a glimpse into the early days of physics problem-solving research. (I)
3. **The Changing Role of Physics Departments in Modern Universities: Proceedings of the International Conference on Undergraduate Physics Education**, edited by E. F. Redish and J. S. Rigden (American Institute of Physics, Woodbury, NY, 1996). These proceedings highlight curriculum-reform efforts by the physics-education-research community, including several aimed at improving student problem-solving skills. (E)
4. **Proceedings of the 2003 Physics Education Research Conference**, edited by S. Franklin, K. Cummings, and J. Marx (PERC, New York, 2004). This conference included a targeted poster session focusing on different methods used in problem-solving research. Some of these posters are described in papers in the proceedings. (E)

C. Books and book chapters

Some of the works listed below address multiple aspects of problem solving. Where appropriate, these works are also included in subsequent sections, along with annotations targeted towards particular topics.

5. ‘‘Solving problems,’’ R. Hyman and B. Anderson, in **The R&D Game: Technical Men, Technical Managers, and Research Productivity**, edited by D. Allison (MIT Press, Cambridge, MA, 1969), pp. 90–105. This work provides a broad view of problem solving from an engineering perspective and includes examples to illustrate the main points. The authors draw upon previous research to present a series of precepts for teaching general problem solving. (E)
6. **Human Problem Solving**, A. Newell and H. A. Simon (Prentice Hall, Englewood Cliffs, NJ, 1972). Newell and Simon describe their information-processing theory for describing problem-solving behavior. They consider problems from the domains of chess, logic, and cryptoarithmetic puzzles. Their analysis focuses on how subjects perform on problem-solving tasks. (A)
7. ‘‘Problem solving ability,’’ M. T. H. Chi and R. Glaser, in **Human Abilities: An Information Processing Approach**, edited by R. J. Sternberg (Freeman, New York, 1985), pp. 227–250. The authors give an overview of the cognitive issues involved in problem solving and problem-solving research. They describe types of problems, explain the roles of representations and knowledge, and briefly discuss ill-defined problems. (I)
8. **Thinking, Problem Solving, Cognition**, R. E. Mayer (Freeman, New York, 1992). This is a general textbook on problem solving from the point of view of cognitive psychology that includes a historical overview of the research. (E)
9. ‘‘Research on problem solving: Physics,’’ D. P. Maloney, in **Handbook of Research on Science Teaching and Learning**, edited by D. L. Gabel (Macmillan, New York, 1993), pp. 327–356. This comprehensive overview addresses many of the major issues in problem-solving research and instruction. The final section raises several important issues for physics instructors, including the definition of a problem and the goals of a physics course with respect to problem solving. Other chapters provide overviews of problem-solving research in other fields (such as chemistry and earth science) and for other audiences (such as elementary and middle schools). (I)
10. **Word Problems: Research and Curriculum Reform**, S. K. Reed (Erlbaum, Mahwah, NJ, 1999). This book reports on research from cognitive psychology and mathematics aimed at developing a ‘‘big picture’’ of how students approach and solve word problems. Reed discusses many of the issues relevant to this Resource Letter, including several curricular reforms. The mathematical lens of this work can help to broaden a physicist’s perspective on problem solving. (I)

11. The Psychology of Problem Solving, edited by J. E. Davidson and R. J. Sternberg (Cambridge U. P., Cambridge, UK, 2003). This reference does not address problem solving in physics specifically, but provides a contemporary summary of the psychological aspects of problem solving, including such topics as problem recognition, transfer, working memory, and other topics discussed in this Resource Letter. (A)

D. Other overviews

12. "Teaching problem solving—a scientific approach," F. Reif, Phys. Teach. **19**, 310–316 (1981). Describing the groundwork for much of the problem-solving research during the past 25 years, Reif presents what he feels to be issues central to problem solving: the results of cognitive-science research, a strategy of successive refinements in solving problems, the need for a structured knowledge base (including problem schemata), and the importance of qualitative descriptions of problem situations. (E)

13. "Learning to think like a physicist: A review of research-based strategies," A. Van Heuvelen, Am. J. Phys. **59**, 891–897 (1991). This article focuses on two of the major themes throughout the problem-solving literature: qualitative representations and hierarchical knowledge structures, and identifies these as important aspects of physics reasoning. Instructional strategies based upon these problem-solving components are described. (E)

14. "Millikan Lecture 1994: Understanding and teaching important scientific thought processes," F. Reif, Am. J. Phys. **63**, 17–32 (1995). This paper summarizes many of Reif's contributions to the current state of problem-solving research in physics. Reif discusses key aspects of expert problem solving, suggests a method for teaching problem-solving skills, and describes an implementation study of this method. (E)

15. "Cognitive Skill Acquisition," K. VanLehn, Annu. Rev. Psychol. **47**, 513–539 (1996). This article provides a review of the research on problem solving from a cognitive-science perspective. (E)

16. "Resource Letter PER-1: Physics Education Research," L. C. McDermott and E. F. Redish, Am. J. Phys. **67**, 755–767 (1999). This Resource Letter gives a broad overview of the literature in physics-education research. In particular, Sec. IV B includes several problem-solving references. (E)

III. DEFINITIONS OF "PROBLEM"

What exactly is a problem? Are the questions at the end of a chapter in a physics textbook problems? Is a problem for one person necessarily a problem for another person? "Problem" and "problem solving" can have different meanings in different situations. The references below discuss the definitions of these terms in different contexts, which can provide insight into the meaning and key elements of problem solving.

17. Mathematical Discovery, G. Polya (Wiley, New York, 1962). Polya defines problem solving as the search "for some action appropriate to attain a clearly conceived, but not immediately attainable, aim" and adds that "where there is no difficulty, there is no problem" (p. 117). This definition is thus a subjective one, varying from person to person. This book summarizes much of Polya's work, including his four-step approach to solving problems. (E)

18. Problem Solving in High School and College Students, H. J. A. Rimoldi, H. M. Fogliatto, J. B. Erdmann, and M. B. Donnelly (Loyola U.P., Chicago, 1964). Although this work is primarily a report on two studies on problem-solving training, the sections on Introduction and Review of Related Research in Phase I of the study present several views of what a problem is. One view, for example, states that a problem includes "a goal, obstacles to reaching the goal, and no clear perception of the means of obtaining it" (p. 3). The authors indicate that there is a difference between reasoning and conditioning and that this difference is relevant to problem solving. (A)

19. Human Problem Solving, A. Newell and H. A. Simon (Prentice Hall, Englewood Cliffs, NJ, 1972). Newell and Simon write that "A person is confronted with a *problem* when he wants something and does not know immediately what series of actions he can perform to get it" (p. 72). This book details the authors' information-processing theory of problem solving and presents a theoretical view involving manipulation of symbolic structures and representations. (A)

20. "Turning exercises into problems: An experimental study with teachers in training," R. M. Garrett, D. Satterly, D. Gil Perez, and J. Martinez-Torregrosa, Int. J. Sci. Educ. **12**, 1–12 (1990). The authors describe a seminar for preservice teachers intended to change their concept of a "problem" from the bare, abstract questions posed in textbooks to more open-ended questions based on real-world situations. Results assessing shifts in teachers' attitudes about problems are presented. (I)

21. Understanding Basic Mechanics, F. Reif (Wiley, New York, 1995). In this introductory physics textbook, Reif devotes an entire chapter to problem solving in physics. He defines a problem as "a task which requires one to devise a sequence of actions leading from some initial situation to some specified goal" (p. 52). This definition differs from most of the others cited here in that it does not explicitly state that a problem should pose a challenge to the solver. (E)

22. "Problem solving," in **Cognitive Psychology and its Implications, 5th ed.**, J. R. Anderson (Worth, New York, 2000), pp. 239–278. In addition to providing an overview of problem-solving research from a cognitive-science perspective, this chapter describes three essential features of "problem solving" including goal directedness (the behavior must be directed toward accomplishing a specific goal), subgoal decomposition (the original goal must be broken down into subtasks), and operator application (the behavior consists of actions that achieve each of the subgoals). (I)

IV. COGNITIVE ASPECTS

Much of the research in problem solving has been performed by cognitive psychologists in an effort to learn more about how the brain works. Introductory physics problems have often been used as the context for these studies. This section contains examples from three types of cognitive research into problem solving: (1) the generation and employment of mental representations; (2) the use of strategies and heuristics; and (3) the role of cognitive load during the problem-solving process.

A. Use of representations

Various studies have examined the relationship between the mental and physical representations people generate while solving problems and problem-solving performance.

23. "The role of problem representation in physics," J. H. Larkin, in **Mental Models**, edited by D. Gentner and A. L. Stevens (Erlbaum, Hillsdale, NJ, 1983), pp. 75–99. This chapter focuses on the creation of useful mathematical representations based on the problem description. Expert and novice mathematical representations are compared and contrasted. (I)

24. "Prescribing effective human problem-solving processes: Problem description in physics," J. I. Heller and F. Reif, Cogn. Instruct. **1**, 177–216 (1984). The authors describe a prescribed problem-solving method that involves (a) problem redescription, (b) solution generation, and (c) solution assessment. Students who followed the prescribed method (involving the generation of multiple representations) showed markedly improved problem-solving performance compared to unguided students. (A)

25. "The importance of an enhanced problem representation," E. R. Savelsbergh, T. De Jong, and M. G. M. Ferguson-Hessler, in **Instructional Technology Memorandum Series** (Univ. of Twente, Enschede, Netherlands, 1997). This study yielded an elaborate description of the problem-solving process and the role of nonmathematical representations in that process. The experimental design was similar to that in ref. 52 but included a greater number of students and did not use problems with intentionally distracting surface features. The findings showed that stronger students made more efficient use of problem representations. (A)

26. "Learning by understanding: The role of multiple representations in learning algebra," M. E. Brenner, R. E. Mayer, B. Moseley, T. Brar, R. Durán, B. S. Reed, and D. Webb, Am. Ed. Res. J. **34**, 663–689 (1997).

The authors describe a quasi-experimental design that provides evidence from middle school mathematics classes that the use of multiple representations enhances student learning. (I)

27. ''Practicing representation,'' J. G. Greeno and P. H. Rogers, Phi Delta Kappan **78**, 361–367 (January 1997). This article elaborates the rationale for using multiple representations in teaching and learning problem solving and includes examples of the creation and spontaneous use of multiple representations in student learning. (E)

28. Learning with Multiple Representations, edited by M. W. van Someren, P. Reimann, H. P. A. Boshuizen, and T. de Jong (Pergamon, Amsterdam, 1998). The second half of this comprehensive book is dedicated to problem solving using multiple representations. Although only one of the chapters is about solving problems in physics, all are relevant to problem solving in general. (A)

29. ''Learning to relate quantitative and qualitative problem representations in a model-based setting for collaborative problem solving,'' R. Ploetzner, E. Fehse, C. Kneser, and H. Spada, J. Learning Sci. **8**, 177–214 (2000). The authors describe an experimental study showing that concept maps can be a useful alternative representation in teaching problem solving. (A)

B. Strategies and knowledge structures

Solving a problem involves several types of mental tasks, such as creating representations of the problem (see Sec. IV A), recalling knowledge relevant to solving the problem, keeping track of the problem goal, and monitoring whether the results obtained are consistent with other known information. The articles in this section describe various thought processes involved in solving problems.

30. How To Solve It, G. Polya (Doubleday, Garden City, NY, 1945). Recognizing that novice students frequently could not even begin to solve a problem, Polya created a four-step process to guide the student through creating and using various representations of a problem to help in solving it. His problem-solving strategy is the forerunner of all subsequent linear problem-solving strategies constructed to help students solve physics problems. (E)

31. ''Acquisition of problem solving skill,'' J. R. Anderson, J. G. Greeno, P. J. Kline, and D. M. Neves, in **Cognitive Skills and Their Acquisition**, edited by J. R. Anderson (Erlbaum, Hillsdale, NJ, 1981), pp. 191–230. This chapter discusses the acquisition, compilation, and optimization of the cognitive skills necessary to solve problems involving the construction of proofs in geometry. (I)

32. ''Knowledge structures and problem solving in physics,'' F. Reif and J. I. Heller, Educ. Psychol. **17**, 102–127 (1982). In this theoretical exploration, Reif and Heller examine the process of solving a problem. They divide it into three phases: the description phase; the search-for-a-solution phase; and the assessing-the-solution phase, in which each phase involves the creation of a new representation for the problem. (I)

In the following two articles, the authors examined the role of the solver's knowledge organization in problem solving. When students were taught to organize their knowledge into hierarchical structures or to use concept maps, their ability to recall and to use this knowledge to solve problems was enhanced. Furthermore, such students were able to transfer their knowledge-structuring skill to nonphysics contexts.

33. ''Effects of knowledge organization on task performance,'' B.-S. Eylon and F. Reif, Cogn. Instruct. **1**, 5–44 (1984). (I)

34. ''From problem-solving to a knowledge structure: An example from the domain of electromagnetism,'' E. Bagno and B.-S. Eylon, Am. J. Phys. **65**, 726–736 (1997). (I)

35. Mathematical Problem Solving, A. H. Schoenfeld (Academic, Orlando, FL, 1985). This book describes Schoenfeld's principles, research, and practices about problem solving. He shows that good problem solvers monitor their progress and work cyclically among the various mental representations they generate to solve a problem. For example, experts evaluate their qualitative analysis of a problem even before they finish planning a solution. (I)

36. ''The impact of goal-specificity on strategy use and the acquisition of problem structure,'' R. Vollmeyer, B. D. Burns, and K. J. Holyoak, Cogn. Sci. **20**, 75–100 (1996). The absence of a well-defined goal makes a problem more challenging, although not necessarily a bad experience for learners. To help students learn from solving goal-free problems, a systematic strategy (such as the vary-one-thing-at-a-time strategy used in this study) should be taught. Such instruction helped students learn the patterns in and implications of their solutions, which in turn helped them solve subsequent problems successfully. (I)

C. Cognitive load and working memory

The physics-textbook problems used in many problem-solving studies can be solved by experts nearly instantaneously and effortlessly. However, meaningful problem solving requires significant mental exertion (a large ''cognitive load'') on the part of the solver. Articles in this section examine the problem-solving process from a cognitive-load perspective. Overloading the mind, either accidentally or by design, hinders one's ability to solve problems.

37. ''The magical number seven, plus or minus two: some limits on our capacity for processing information,'' G. A. Miller, Psychol. Rev. **63**, 81–97 (1956). A well-known result from cognitive science is that the size of the human working memory (a rough measure of the number of different things a person can keep track of at one time) is 7 ± 2 slots. This paper, which presents the research leading to this result, is a seminal work in cognitive science. (A)

38. ''Chunking in recall of symbolic drawings,'' D. E. Egan and B. J. Schwartz, Mem. Cogn. **7**, 149–158 (1979). The authors describe three experiments exploring the abilities of skilled electronics technicians and novices for memorizing circuit diagrams. The skilled technicians were able to recall a larger portion of a meaningful diagram with more accuracy than novices. However, when random circuit diagrams were used, both skilled technicians and novices showed approximately the same level of recall. These results are consistent with a model in which the skilled technicians memorized diagrams in ''chunks'' consisting of multiple circuit elements grouped into functional units. This enabled them to recall more elements from a circuit drawing than novices, who had fewer chunks to call upon, even though both experts and novices had similar numbers of working memory slots. Similar findings with expert and novice chess players memorizing board positions can be found in Ref. 6. (A)

39. ''Cognitive load during problem solving: Effects on learning,'' J. Sweller, Cogn. Sci. **12**, 257–285 (1988). This article suggests that one reason students find solving physics problems difficult, despite all of the practice problems assigned, is that their chosen method of solution (applying a means-ends analysis, see Sec. V A) requires a heavy cognitive load. This limits a students ability to learn from the experience. (I)

40. ''Non-linear analysis of the effect of working-memory capacity on organic-synthesis problem solving,'' D. Stamovlasis and G. Tsaparlis, Chem. Educ.: Res. Prac. Eur. **1**, 375–380 (2000). The authors determined the working memory capacity of some students and demonstrated that if this capacity was exceeded while trying to solve a problem, the students failed at the task. (I)

41. ''The relationships among working memory, math anxiety, and performance,'' M. A. Ashcraft and E. P. Kirk, J. Exp. Psychol. Gen. **130**, 224–237 (2001). This study demonstrated that having math anxiety effectively decreases the number of working memory slots available to a person solving a math problem, even when that person possesses the math skills necessary for solving that problem. There may be similar implications for people with science anxiety attempting to solve physics problems. (A)

42. ''Learning perceptual chunks for problem decomposition,'' P. C. R. Lane, P. C. H. Cheng, and F. Gobet, in **Proceedings of the Twenty-Third Annual Conference of the Cognitive Science Society**, edited by J. D. Moore and K. Stenning (Erlbaum, Mahwah, NJ, 2001), pp. 528–533. The authors describe a study in which they used information about time delays between solution steps to infer information about subjects' use of chunks in solving electric-circuit problems. The results were consistent with those of a computational model for using perceptual chunks to solve problems with diagrams. A similar study using timing data to infer information about chunked memory in novices and experts is reported in Ref. 47. (A)

V. EXPERT-NOVICE CHARACTERISTICS

Although it is evident that experts in a field can solve problems more successfully than novices, a more important question is why this is so. Is it simply because experts have more experience than novices? Is it because experts organize their knowledge differently than novices? Is it because they solve problems in a different way than novices? Many studies have been performed to describe and classify differences between expert and novice problem solvers. Distinctions have been found in a number of areas including knowledge organization, qualitative analysis, depth of experience, and problem representation. A knowledge of how each of these factors influences problem-solving performance can help guide instruction in problem solving.

A. Differences in procedures

Experts and novices solve problems very differently. Identifying and classifying the differences in their procedures can provide insights into the problem-solving process. These differences include the problem-solving strategies employed and the use of planning and qualitative analyses of problem situations.

43. ‘‘Individual differences in solving physics problems,’’ D. P. Simon and H. A. Simon, in **Children’s Thinking: What Develops?** edited by R. S. Siegler (Erlbaum, Hillsdale, NJ, 1978), pp. 325–361. The authors provide in-depth analyses of think-aloud problem-solving sessions, one from an expert and one from a novice. From these sessions, the authors infer that the experts used a ‘‘knowledge-development’’ approach, starting from the given information and working forwards to find the desired quantity, while novices used a ‘‘means-end’’ approach, starting from the desired quantity and working backwards to connect it to the given information. They also describe the role of physical intuition in the problem-solving process. (I)

44. ‘‘A tale of two protocols,’’ D. P. Simon and H. A. Simon, in **Cognitive Process Instruction**, edited by J. Lochhead and J. S. Clement (Franklin Institute, Philadelphia, 1979), pp. 119–132. This is an in-depth study of two experts solving a ‘‘real-world’’ problem. The experts differed on the overall goals of problem solving and the context in which they approached the problem. (E)

45. ‘‘Processing information for effective problem solving,’’ J. H. Larkin, Eng. Educ. **70**, 285–288 (December 1979). Larkin describes the expert practice of performing a qualitative analysis of problem situations and the role of chunking in the qualitative analysis. Chunking of knowledge is vital for efficient use of working memory slots (see Sec. IV C). (E)

46. ‘‘Expert and novice performance in solving physics problems,’’ J. H. Larkin, J. McDermott, D. P. Simon, and H. A. Simon, Science **208**, 1335–1342 (1980). This review article examines the differences between the procedures experts and novices use to solve problems. The authors describe the role problem representation plays in the solution process, and they relate the generation of representations to the problem solver’s knowledge organization and ability to conduct an effective search of the knowledge space. (I)

47. ‘‘A view from physics,’’ K. Schultz and J. Lockhead, in **Toward a Unified Theory of Problem Solving: Views From the Content Domains**, edited by M. U. Smith (Erlbaum, Hillsdale, NJ, 1991), pp. 99–114. This chapter gives an overview of several relevant issues that have come to light through expert-novice studies, along with some instructional implications. The authors discuss expert-novice differences with regards to use of representations, use of analogies, ability to self-monitor, and transfer of skills from one context to another. They also highlight the need for ‘‘real-world’’ problems in teaching good problem-solving skills to novices. (I)

48. ‘‘Individual differences within problem-solving strategies used in physics,’’ A. S. Dhillon, Sci. Educ. **82**, 379–405 (1998). The author conducted think-aloud problem-solving sessions with 13 subjects (7 experts, 6 novices) and identified the subjects’ activities in the problem-solving process. Dhillon then classified these activities into strategies that are similar to the ones identified in Ref. 52. (A)

49. ‘‘New light on novice-expert differences in physics problem solving,’’ A. G. Priest and R. O. Lindsay, B. J. Psychol. **83**, 389–405 (1992). Priest and Lindsay question the validity of the claim that experts use a ‘‘knowledge-development’’ problem-solving approach and that novices ‘‘work backwards’’ (see Ref. 43, for example). They conducted two experiments that show no differences between novices and experts in the direction of inferences made, but that experts are better able to plan solutions without using equations. (A)

50. ‘‘Common sense clarified: Intuitive knowledge and its role in physics expertise,’’ B. Sherin, NARST Annual Meeting 1999. (http://www.educ.sfu.ca/narstsite/conference/sherin/sherin.html) Sherin presents data to support his theory that intuitive knowledge plays a role in students’ problem solving throughout their transition from novice to expert problem solvers. (I)

51. ‘‘When physical intuition fails,’’ C. Singh, Am. J. Phys. **70**, 1103–1109 (2002). One of the limitations of many expert-novice studies is that the experts (usually university physics professors) are typically given problems that they already know how to solve (introductory physics textbook problems). In this study, the performance of university physics professors on a difficult problem was examined. Although they still tended to approach the problem in a systematic way, some aspects of their performance came to resemble the problem-solving behaviors of novices. (E)

B. Differences in knowledge structures

In addition to following different problem-solving procedures, another important difference between expert and novice problem solvers is the structure of their knowledge and the depth of their understanding of the relevant concepts.

52. ‘‘Categorization and representation of physics problems by experts and novices,’’ M. T. H. Chi, P. J. Feltovich, and R. Glaser, Cogn. Sci. **5**, 121–152 (1981). This seminal article in expert-novice problem solving includes experimental results of a problem-categorization task in which experts categorized problems based on their deep structure (the physics principles applicable to solving the problem) while novices categorized them based on their surface features (such as the particular objects mentioned in the problem description). (I)

53. ‘‘Expertise in problem solving,’’ M. T. H. Chi, R. Glaser, and E. Rees, in **Advances in the Psychology of Human Intelligence, Vol. 1**, edited by R. J. Sternberg (Erlbaum, Hillsdale, NJ, 1982), pp. 7–75. This review paper describes models of expert problem solving from the perspectives of cognitive science and artificial intelligence and summarizes the experimental evidence in support of those models. (A)

54. ‘‘Self-Explanations: How students study and use examples in learning to solve problems,’’ M. T. H. Chi, M. Bassok, M. W. Lewis, P. Reimann, and R. Glaser, Cogn. Sci. **13**, 145–182 (1989). This paper describes a study in which students were asked to talk aloud while studying worked examples from introductory physics. The authors found differences between the self-explanations generated by ‘‘good’’ and ‘‘poor’’ students which are similar to some of the differences between expert and novice problem-solving behavior. (I)

55. ‘‘The relationship between problem categorization and problem solving among experts and novices,’’ P. T. Hardiman, R. Dufresne, and J. P. Mestre, Mem. Cogn. **17**, 627–638 (1989). This article questions the assertion that experts categorize problems by their deep structure and that novices categorize problems by surface features. In one experiment, the authors found that similarities in the surface features of problems could interfere with experts’ tendency to classify problems by deep structure and that novices were capable of classifying problems by deep structure under certain conditions. In a second experiment, the authors found that novices at similar levels of experience employed different types of reasoning in classifying problems and that novices who classified problems in a more expert-like manner were more proficient in solving problems. (I)

56. ‘‘Constraining novices to perform expertlike problem analyses: Effects on schema acquisition,’’ R. J. Dufresne, W. J. Gerace, P. T. Hardiman, and J. P. Mestre, J. Learn. Sci. **2**, 307–331 (1992). The authors conducted three experiments in which students, using a Hierarchical Analysis Tool (HAT), were constrained to perform expert-like problem analyses. They found that students were then able to match solutions to similar problems based on deep structure of the solution, and reason about similarities between problem solutions in expert-like ways. The students' problem-solving performance also improved when using HAT. (I)

57. ''Comprehension of arithmetic word problems: A comparison of successful and unsuccessful problem solvers,'' M. Hegarty, R. E. Mayer, and C. A. Monk, J. Ed. Psychol. **87**, 18–32 (1995). This paper describes two experiments designed to measure differences in problem comprehension between expert and novice problem solvers. Successful problem solvers understood the deep structure of the problems, while unsuccessful problem solvers understood surface features of the problems. (A)

VI. PROBLEM SOLVING IN MATHEMATICS

Like much of physics instruction, mathematics instruction places an emphasis on problem solving and has a research community studying problem solving. Because of the similarities between physics and mathematics problems, results from mathematics problem-solving research can be useful to physicists.

A. Mathematics as the language of physics

Much of the practical applicability of physics stems from the use of mathematical tools to quantify predictions. Not surprisingly, mathematics education researchers have uncovered some issues that may affect the learning of physics.

58. ''Some misconceptions concerning the concept of variable. Are you careful about defining your variables?'' P. Rosnick, Mathe. Teach. **74**, 418–420,450 (1981). The author summarizes work in mathematics education research focused on how an inconsistent or careless definition of variables can cause difficulties for students in solving problems. For example, if S represents the number of students and P represents the number of professors, many students interpret the equation $6S=P$ as indicating that there are six students for each professor (interpreting S as students instead of the number of students and P as professors rather than the number of professors). Differences in usage, which are apparent to an expert because of context, can be confusing to novices. (E)

59. ''Cognitive science and algebra learning,'' R. H. Wenger, in **Cognitive Science and Mathematics Education**, edited by A. H. Schoenfeld (Erlbaum, Hillsdale, NJ, 1987), pp. 217–252. This chapter describes how students substitute numbers into equations to help reduce the cognitive load while solving problems. It also demonstrates that students are very good at solving for x in algebraic equations, but struggle if the equations do not have an x to solve for. (I)

60. ''When good teaching leads to bad results: The disasters of 'well-taught' mathematics courses,'' A. H. Schoenfeld, Educ. Psychol. **23**, 145–166 (1988). This case study presents many unintentional outcomes of instruction, including the belief of many high school students that math problems that cannot be solved in 12 minutes are impossible. This particular result has natural implications for physics homework assignments. (E)

61. ''Learning to think mathematically: problem solving, metacognition, and sense making in mathematics,'' A. H. Schoenfeld, in **NCTM Handbook of Research on Mathematics Teaching and Learning**, edited by D. A. Grouws (Macmillan, New York, NY, 1992), pp. 334–370. Schoenfeld is one of the leading authorities in mathematical problem-solving research and this review is a must-read for anyone interested in the field. The chapter begins by reviewing various definitions of ''problem'' and ''problem solving.'' It next examines how the use of different types of problems can reflect instructional goals and determine what students learn. Finally, a theoretical framework for human problem solving is proposed that includes five key aspects: knowledge base, problem-solving strategies, monitoring and control, beliefs and affects, and practices. (I)

62. ''How students understand physics equations,'' B. L. Sherin, Cogn. Instruct. **19**, 479–541 (2001). Sherin describes how students create and interpret equations in physics. Sherin's central claim is that the meanings created by students for equations may or may not be mathematically and physically correct. For example, students frequently say that forces ''balance,'' but often do not connect this balancing concept with Newton's second law. As a result, their solution looks right and is mathematically correct, but hides a conceptual deficiency. (A)

63. ''Symbols and the Bifurcation between Procedural and Conceptual Thinking,'' D. Tall, E. Gray, M. Bin Ali, L. Crowley, P. DeMarois, M. McGowen, D. Pitta, M. Pinto, M. Thomas, and Y. Yusof, Can. J. Sci. Math. Tech. Educ. **1**, 80–104 (2001). This is a review article on research from contemporary mathematics education. Of particular interest to physics teachers is a discussion of the ''procept,'' which includes knowledge beyond a mere definition of a concept and the simple procedures suggested by mathematical symbols. For example, a procept for Newton's second law would include an understanding of how to apply the law, the conditions under which it is applicable, and operational definitions for related concepts such as force, mass, and acceleration. (A)

B. Transfer from mathematics to other contexts

A common perception among physics instructors is that taking more mathematics classes would make students better problem solvers. However, the difficulty of transferring knowledge across domains, such as from mathematics to physics, is documented in the cognitive-science literature.

64. ''The initial knowledge state of college physics students,'' I. A. Halloun and D. Hestenes, Am. J. Phys. **53**, 1043–1055 (1985). Among many results discussed in this study, the authors found that a student's mathematics skill (as measured by a test of their own creation) was correlated with student performance on a physics-concept test. (E)

65. Cognition in Practice: Mind, Mathematics and Culture in Everyday Life, J. Lave (Cambridge U. P., Cambridge, UK, 1988). Lave studied the math skills of grocery shoppers and dieters and discovered that the mathematics used by these individuals in natural settings was remarkably accurate (around 95% accuracy). However, when these subjects solved problems using the same mathematics skills in a school setting, their accuracy dropped noticeably (to about 70%). Some mathematics skills, such as the use of fractions, did not seem to transfer, even to similar contexts. (A)

66. ''Are cognitive skills context-bound?'' D. N. Perkins and G. Salomon, Educ. Res. **18**, 16–25 (January–February 1989). This review article explores the issues relevant to and history of research on the transfer of skills. The authors summarize and elaborate on a few general instructional guidelines, such as teaching general principles rather than contextual principles, self-monitoring during problem solving, and teaching general problem-solving heuristics in the context of a wide range of domains. (I)

67. ''Interdomain transfer between isomorphic topics in algebra and physics,'' M. Bassok and K. J. Holyoak, J. Exp. Psychol.: Learn. Mem. Cogn. **15**, 153–166 (1989). Of course, some transfer from mathematics to physics courses can occur. In this study, algebra students could recognize the appropriate mathematics imbedded in a physics word problem. Unfortunately, otherwise successful physics students could not solve a similar algebra problem. This parity-violating result has been the focus of Bassok's work over the past 15 years. (I)

68. How People Learn: Brain, Mind, Experience, and School, J. D. Bransford, A. L. Brown, and R. R. Cocking (National Academy, Washington, DC, 2000). The issue of transfer is well-explored in this summary of cognitive-science results. The transfer of a skill from one domain to another is classified as either ''near transfer'' or ''far transfer,'' depending on the degree of similarity between the original domain and the new domain. Near transfer can be accomplished if taught, but far transfer (for example, applying physics problem-solving skills to motorcycle repair) is difficult for researchers to measure reliably or teach. (I)

VII. ALTERNATIVE PROBLEM TYPES

A growing area of research and development is that of creating and testing new types of problems. The goal of this work is to design problems that encourage students to employ more expert-like problem-solving methods. The different problem types described in the articles and curricular materials below accomplish this goal in a variety of ways. This list is likely to grow substantially in the near future. This section is ordered differently than the rest of the Resource Letter; the resources here are listed alphabetically by problem type.

A. Active Learning Problem Sheets (ALPS)

69. "Learning to think like a physicist: A review of research-based strategies," A. Van Heuvelen, Am. J. Phys. **59**, 891–897 (1991). After reviewing some of the general results of problem-solving research, Van Heuvelen describes the use of his Active Learning Problem Sheets (ALPS kits) in physics instruction. (E)

70. **Active Learning Problem Sheets**, A. Van Heuvelen (Hayden-McNeil, Plymouth, MI, 1996). Two packets of these worksheets are available: one for mechanics and one for electricity and magnetism. The dominant feature of the ALPS packets is that they break problems down into manageable steps so that students learning new concepts can also learn sound problem-solving approaches. The use of multiple qualitative representations is stressed in several ways, including through Jeopardy problems (see Sec. VIII D). There are also brief discussions of problem-solving strategies and hierarchical presentations of knowledge. (E)

B. Context-rich problems

71. "Teaching problem solving through cooperative grouping. Part 2: Designing problems and structuring groups," P. Heller and M. Hollabaugh, Am. J. Phys. **60**, 637–644 (1992). The first half of this paper describes context-rich problems, which were developed for use in classroom group-problem-solving sessions. One goal of these problems is to shift the focus of student discussions from formulas to the applicability of physical concepts and principles in a given situation. Context-rich problems, as the name implies, demand that students solve problems in a more complex and real-world context than is typically seen in traditional problems. Students may have to determine what the target variable is, ignore extraneous information, make estimates for missing information, or make simplifying assumptions. (E)

72. http://groups.physics.umn.edu/physed/Research/CRP/crintro.html. This web site provides information on creating and using context-rich problems and includes a large archive of context-rich problems for many introductory physics topics. (E)

C. Experiment problems

In the following two papers, "experiment problems" are described as problems based on the use of apparatus. Students may be asked to do one or more of the following as part of their solution: define the problem more narrowly, plan a solution, divide the problem into subproblems, make measurements, approximate or estimate needed quantities, design an experiment, or determine how something works. Examples of various types of experiment problems for different topics are included.

73. "Experiment problems for mechanics," A. Van Heuvelen, Phys. Teach. **33**, 276–280 (1999). (E)

74. "Experiment problems for electricity and magnetism," A. Van Heuvelen, L. D. Allen, and P. Mihas, Phys. Teach. **37**, 482–485 (1999). (E)

75. http://www.physics.ohio-state.edu/~physedu/index2.html. This web site contains a selection of experiment problems. (E)

D. Jeopardy problems

76. "Playing physics jeopardy," A. Van Heuvelen and D. Maloney, Am. J. Phys. **67**, 252–256 (1999). Jeopardy problems present the student with an equation, diagram, or graph describing a physical process, and then ask students to generate a physical situation that corresponds to this representation. The authors argue that this reversal of the usual problem-solving process helps students develop a better understanding of physics concepts and promotes the use of multiple representations in problem solving. Jeopardy problems can also prevent students from relying on mathematical formulae and help them see the utility of units. Some Jeopardy problems can be found in Ref. 70. (E)

E. Problem posing

77. "Probing adults' conceptual understanding and transfer of learning via problem posing," J. Mestre, J. Appl. Dev. Psych. **23**, 9–50 (2002). Although this paper describes a series of studies in which problem posing was used as the experimental probe, it is included here because of the applicability of this technique as an instructional tool. In problem posing, students are provided with the beginning of a problem statement, then asked to complete it so that a given concept or principle must be applied to solve it. This approach forces students to think in terms of concepts, rather than equations. (I)

F. Ranking tasks

78. "Ranking tasks: A new type of test item," D. P. Maloney, J. Coll. Sci. Teach. **16**, 510–514 (1987). This article introduces ranking tasks as an alternative problem format. A ranking task presents several variations of a situation and asks students to rank the situations according to one or more parameters, explaining their reasoning. (For example, one problem asks students to rank submerged blocks of various masses and volumes by the buoyant force on them.) Maloney offers suggestions for how to construct, use, and score such problems. He identifies two main strengths of ranking tasks: they are relatively quick to write and administer and they can give the instructor more insight into the students' thought processes than conventional problems. (E)

79. "Ranking tasks revisited," D. P. Maloney and A. W. Friedel, J. Coll. Sci. Teach. **25**, 205–210 (1996). This article presents several variations of the original ranking-task format and describes in detail several effects these tasks might have on student problem-solving behaviors. In particular, they combat the "mechanical" equation-based ways in which students approach typical physics problems by forcing them to think about relationships between variables and to compare situations. The authors emphasize the importance of having students process information in more than one way. Since ranking tasks are quite different from the usual problems students are asked to solve, they are difficult to solve by applying familiar algorithms. (E)

80. **Ranking Task Exercises in Physics**, T. L. O'Kuma, D. P. Maloney, and C. J. Hieggelke (Prentice Hall, Upper Saddle River, NJ, 2000). This book contains a wide variety of ranking-task problems for topics throughout introductory physics. The foreword describes various uses for ranking tasks, along with a discussion of potential benefits for students' conceptual development. The book includes a CD with the ranking tasks in portable-document format (.pdf). (E)

VIII. CURRICULAR INTERVENTIONS

The difficulties students have with solving typical problems in introductory physics courses are well-documented. Various curricular modifications, ranging from small changes to complete overhauls, have attempted to make students more proficient in problem solving. Common features of successful interventions include explicit teaching of problem-solving heuristics (similar to those in Ref. 30), modeling the use of the heuristics by the instructor, and requiring students to use the heuristics explicitly when solving problems.

The following two papers describe in detail the development and assessment of a problem-solving strategy for students in a thermodynamics and an electromagnetism course.

81. "Linking factual and procedural knowledge in solving science problems: A case study in a thermodynamics course," C. T. C. W. Mettes, A. Pilot, and H. J. Roossink, Instruct. Sci. **10**, 333–361 (1981). (I)

82. ‘‘Teaching problem-solving in physics: A course in electromagnetism,’’ J. H. P. van Weeren, F. F. M. de Mul, M. J. Peters, H. Kramers-Pals, and H. J. Roossink, Am. J. Phys. **50**, 725–732 (1982). (I)

83. ‘‘Prescribing effective human problem-solving processes: Problem description in physics,’’ J. I. Heller and F. Reif, Cogn. Instruct. **1**, 177–216 (1984). The authors found that explicit teaching of a systematic problem-solving method to students, emphasizing the generation of an effective initial problem description, and insuring that students follow such a method, can result in appreciably better problem-solving performance. (A)

The following two papers describe modified curricula in which a qualitative framework is developed for physics principles before quantitative details are added. In each case, students using the new curricula exhibited more expert-like problem-solving behaviors than those in traditionally taught classes. In the first paper, the authors also looked for correlations between problem-solving ability and intelligence, numerical ability, and attitudes towards science, but found none.

84. ‘‘Alternative instructional systems and the development of problem-solving skills in physics,’’ J. Bascones and J. D. Novak, Eur. J. Sci. Educ. **7**, 253–261 (1985). (I)

85. ‘‘Overview, case study physics,’’ A. Van Heuvelen, Am. J. Phys. **59**, 898–907 (1991). (E)

86. ‘‘A WISE strategy for introductory physics,’’ D. S. Wright and C. D. Williams, Phys. Teach. **24**, 211–216 (1986). This paper describes a study in which an explicit problem-solving strategy was taught to students. Students who used the strategy frequently had higher final grades than students who used it only rarely, regardless of their performance on a course pretest to assess their mathematics skills. Students also rated the method as helpful for solving problems. More details can be found in the doctoral dissertation by D. S. Wright, ‘‘Explicitly Structured Physics Instruction,’’ Virginia Polytechnic Institute and State University, 1984. (E)

87. ‘‘Structuring effective worked examples,’’ M. Ward and J. Sweller, Cogn. Instruct. **7**, 1–39 (1990). This paper describes a series of experiments to assess the value of worked examples in helping students learn to solve problems. They found that such examples can be helpful to students, but must be designed to call students’ attention to important solution features and must not require a large cognitive load of the student (see Sec. IV C.). For instance, examples should not use a means-end strategy, give much extraneous information, or force students to integrate information from multiple sources (such as equations and diagrams or diagrams and text). (A)

The next two references describe a classroom intervention to improve students’ problem-solving abilities including explicit teaching of a prescriptive problem-solving strategy, cooperative group work, and use of context-rich problems. The third reference is a problem-solving manual for students resulting from this study. It models the use of the Minnesota Problem-Solving Strategy and other context-specific heuristics and makes explicit the process of solving problems and creating new representations.

88. ‘‘Teaching problem solving through cooperative grouping. Part 1: Group versus individual problem solving,’’ P. Heller, R. Keith, and S. Anderson, Am. J. Phys. **60**, 627-636 (1992). (E)

89. ‘‘Teaching problem solving through cooperative grouping. Part 2: Designing problems and structuring groups,’’ P. Heller and M. Hollabaugh, Am. J. Phys. **60**, 637–644 (1992). (E)

90. The Competent Problem Solver for Introductory Physics, K. Heller and P. Heller (McGraw-Hill Primis, Boston, 2000). (E)

In the following two papers, the authors describe a curricular experiment in which constraining students to perform a qualitative analysis of a physics problem before attempting a quantitative solution resulted in more expert-like performance on problem-categorization tasks and in writing qualitative explanations. In addition, students’ problem-solving performance improved significantly if such qualitative analyses were performed successfully.

91. ‘‘Promoting skilled problem-solving behavior among beginning physics students,’’ J. P. Mestre, R. J. Dufresne, W. J. Gerace, P. T. Hardiman, and J. S. Touger, J. Res. Sci. Teach. **30**, 303–317 (1993). (I)

92. ‘‘Using qualitative problem-solving strategies to highlight the role of conceptual knowledge in solving problems,’’ W. J. Leonard, R. J. Dufresne, and J. P. Mestre, Am. J. Phys. **64**, 1495–1503 (1996). (E).

93. ‘‘Teaching science problem-solving,’’ R. Taconis, M. G. M. Ferguson-Hessler, and H. Broekkamp, J. Res. Sci. Teach. **38**, 442–468 (2001). This is a meta-analysis of 22 previously published articles on teaching problem solving in science classes. The authors found that instruction that incorporated one or more of the common features listed in the introduction to this section contributed to better problem-solving performance in students. Furthermore, having students work in groups did not improve problem solving unless the group work was combined with the above features. (I)

IX. COMPUTERS AND PROBLEM SOLVING

Problem-solving research has benefited from technology. In the early days, researchers used computer programs to model human problem-solving behavior and to test their theories of the cognitive structures of expert and novice problem solvers. More recently, much of the research has shifted to developing computer-aided problem-solving instruction.

A. Computer modeling of human problem solving

To understand basic features of human problem-solving behavior, computer programs were written to simulate the solving of physics problems by experts and novices. Although not a current area of research, such work was the foundation for information-processing models of human problem solving.

94. ‘‘Models of competence in solving physics problems,’’ J. H. Larkin, J. McDermott, D. P. Simon, and H. A. Simon, Cogn. Sci. **4**, 317–345 (1980). This article describes two computer models for solving physics problems. One model used a ‘‘means-end’’ analysis, starting from the desired quantity and working backwards to connect it to the given information. Its performance more closely replicated that of novice problem solvers. The second model used a ‘‘knowledge-development’’ approach, starting from the given information and working forwards to find the desired quantity. The performance of this model more closely resembled that of expert problem solvers. (A)

95. ‘‘Skilled problem solving in physics: a hierarchical planning model,’’ J. H. Larkin, J. Struct. Learn. **6**, 271–297 (1980). The author describes the construction of a computer program that could model the strategic decisions made by an expert problem-solver, including collecting information from a given problem, abstracting that information into a simpler form, and solving the abstracted problem. (A)

The following two references describe computer programs designed to model novice-to-expert shifts in problem solving. These programs (ABLE and EUREKA) could store information from previous problem-solving experiences and use that information to help them solve further problems in a more expert-like manner.

96. ‘‘Cognition of learning physics,’’ J. H. Larkin, Am. J. Phys. **49**, 534–541 (1981). (E)

97. ‘‘Modeling novice-to-expert shifts in problem-solving strategy and knowledge organization,’’ R. Elio and P. B. Scharf, Cogn. Sci. **14**, 579–639 (1990). (A)

98. ‘‘FERMI: A flexible expert reasoner with multi-domain inferencing,’’ J. H. Larkin, F. Reif, J. Carbonell, and A. Gugliotta, Cogn. Sci. **12**, 101–138 (1988). The authors describe a prototype expert-system called FERMI with knowledge organized hierarchically into schemata.

This system could solve problems in multiple physics domains, transfer its knowledge to additional domains, and explain its reasoning in solving a problem. (A)

B. Computer-assisted problem solving

At present, the main function of computers in problem-solving instruction is to provide students with individualized guidance and feedback while practicing solving problems. Such feedback can be either content-based (giving hints about the physics) or process-based (giving hints about what step to perform next). The level of sophistication of these programs ranges from simple helpers based on a linear model of problem solving to programs with built-in artificial intelligence that can assess a student's competence in various problem-solving skills.

The following two articles describe one of the earliest attempts to use computers to help students learn to solve physics problems.

99. ''Free-body diagrams (a PLATO lesson),'' B. A. Sherwood, Am. J. Phys. **39**, 1199–1202 (1971). (E)

100. ''Educational uses of the PLATO computer system,'' S. G. Smith and B. A. Sherwood, Science **192**, 344–352 (1976). (E)

101. ''ALBERT: A physics problem-solving monitor and coach,'' G. E. Oberem, in **International Conference on Computer Assisted Learning in Post-secondary Education**, edited by D. Norrie (Univ. of Calgary, Calgary, Canada, 1987), pp. 179–184. This article describes a computer tutor that can solve or help students to solve kinematics problems involving constant-acceleration. The ALBERT tutor has a natural-language processor and can extract physical information from plain English. (E)

102. ''Teaching scientific thinking skills: Students and computers coaching each other,'' F. Reif and L. A. Scott, Am. J. Phys. **67**, 819–831 (1999). The authors describe the development and testing of computer tutorials designed to teach students a systematic method for applying Newton's second law to solving problems. The performance of students using the tutorials on a class examination was significantly better than that of students who did not have access to the tutorials, and was almost as good as that of students receiving human tutoring. Elaboration and further studies can be found in a Ph.D. dissertation by L. A. Scott, ''Design and assessment of an interactive physics tutoring environment,'' University of Pittsburgh, 2001. (I)

The following two references describe the ANDES tutor, a sophisticated program with artificial intelligence that provides students with a supportive problem-solving environment. ANDES can give students feedback and hints and includes an assessor to track a student's proficiency.

103. ''An intelligent tutor for classical physics,'' K. G. Schulze, R. N. Shelby, D. J. Treacy, M. C. Wintersgill, K. VanLehn, and A. Gertner, J. Electron. Publ. **6**(1), www.press.umich.edu/jep/06-01/schulze.html (2000). (E)

104. ''Minimally invasive tutoring of complex physics problem solving,'' K. VanLehn, C. Lynch, L. Taylor, A. Weinstein, R. Shelby, K. Schulze, D. Treacy, and M. Wintersgill, in **Intelligent Tutoring Systems: 6th International Conference**, edited by S. A. Cerri, G. Gouarderes, and F. Paraguacu (Springer, Berlin, 2002), pp. 367–376. (E).

105. ''Comparison of student performance using web and paper-based homework in college level physics,'' S. W. Bonham, D. L. Deardorff, and R. J. Beichner, J. Res. Sci. Teach. **40**, 1050–1071 (2003). The authors compared students in two large introductory physics courses. In one course, students completed homework on paper and in the other course, students completed homework using a web-based homework system. In an evaluation of the students' learning gains, as measured by course examinations and standardized tests such as the Force and Motion Conceptual Evaluation (FMCE), no differences were found between the two groups. (E)

C. Web resources for problem solving in physics

Several groups have been developing computer-based systems to help students learn to solve problems. These systems vary widely in sophistication and the types of help given to students. Although the extent to which each is based on problem-solving research varies widely, data regarding student use of these systems can aid future research in problem solving.

106. Tycho (http://www.physics.uiuc.edu/Tycho/index.html) Developed at the University of Illinois, Urbana-Champaign, Tycho is an on-line homework system that can give students hints towards solving a problem, in addition to feedback as to whether an answer is correct or not. The hints are physics-based and geared to the specific problem that a student is working on.

107. Cybertutor (http://www.effedtech.com/index.html) Developed at MIT, Cybertutor is similar to Tycho in the type of guidance and feedback it can provide to students to help them solve problems. Some articles on the Cybertutor system can be found at http://relate.mit.edu/.

108. ANDES (http://www.pitt.edu/~vanlehn/andes.html) ANDES is a sophisticated computer-based problem-solving tutor developed jointly at the University of Pittsburgh and the U.S. Naval Academy. ANDES is designed to be minimally invasive, allowing a student to follow his or her own path in solving a problem. However, it also can assess a student's progress at intermediate stages, offer hints and feedback as to how a student should proceed, and solve systems of algebraic equations for the student. The ANDES system is discussed in Refs. 103 and 104.

109. PALs (http://www.gen.umn.edu/faculty_staff/hsu/pal/index.html) The PAL (Personal Assistant for Learning) tutorials are simple, cognitively based tutorials designed to teach students some important scientific-thinking skills. The guidance and feedback the PALs provide to students are geared toward the process of interpreting and applying physics principles and are based on cognitive research. A research study involving the PALs is discussed in Ref. 102.

X. GLOSSARY

Like any subfield of physics, problem-solving researchers have developed specialized terms to describe the concepts in their field precisely and compactly. Since not all of these may be familiar to readers, we define some of the more common terms below.

algorithm An algorithm is a well-specified procedure that one can follow to accomplish some task. It differs from a heuristic in that an algorithm is more prescriptive and detailed in terms of the steps to be followed and is thus less flexibly useful than a heuristic.

chunking When two or more initially separate pieces of knowledge are combined by the brain so that the information in all the pieces can be stored in a single ''slot'' of working memory, they are said to be chunked. Chunking frees up slots in working memory, enabling experts to solve problems more efficiently. For example, after a five-second look at a chessboard from a game in progress, beginning chess players can successfully reproduce the position of only about six pieces while masters can reproduce the entire board nearly error-free. This is because masters store the positions of meaningful or commonly seen groups of chess pieces as a single element of information while novices, who have less experience, must try to remember the board position piece by piece.

cognitive load Cognitive load describes the demand that is made on a person's working memory. A task that has a high cognitive load requires a person to pay attention to many different things at the same time and can reduce the person's effectiveness at carrying out the task. For example, multiplying two seven-digit numbers together in one's head is beyond most people's ability because it requires too many pieces of information to be remembered at one time. A piece of paper and a pencil make the task doable.

heuristic Heuristics in problem solving are generalized procedures that one can use when trying to solve any problem, similar to rules-of-thumb. Much problem-solving instruction involves teaching students a set of heuristics they can use to approach novel problems (for example, draw a picture of the situation or choose an applicable physics principle to use).

metacognition Literally, metacognition means "thinking about thinking." In problem-solving, metacognition usually refers to the process by which a problem-solver monitors his or her performance while solving the problem. The self-monitoring can include checking for mathematical or conceptual errors and assessing whether the results are compatible with other known information about the physical world.

schema Schemata are general representations produced in a person's mind that enable one to categorize a situation. A particular problem-solving schema may consist of the concepts, diagrams, reasoning tools, and procedures relevant to solving a group of problems with common structural characteristics. For example, a schema for solving a dynamics problem could consist of a free-body diagram, Newton's laws, vector-addition procedures, and algebraic manipulations.

structure (deep and surface) These terms are used most commonly when referring to how people classify problems. A classification is said to be based on deep structure when problems are grouped based on the general principle(s) that are applicable to their solution (for example, grouping together all problems that can be solved using conservation of energy). A classification is said to be based on surface structure (or surface features) when problems are grouped based on the objects present in the problem (for example, grouping together all problems involving blocks and pulleys).

transfer (near and far) Transfer is the ability to apply skills or principles learned in one context to another. For example, students learn to solve algebraic equations for the unknown quantity x in a mathematics class. However, in a physics class, these same students often have difficulty solving an equivalent physics equation. Transfer can be classified as near or far depending on whether the two contexts are similar or dissimilar. However, this is a subjective measure with no set standards.

working memory Research has shown that humans can store about seven (plus or minus two) items of information at any given time. This is referred to as "working memory capacity." If this capacity is exceeded, then some of the information in working memory will be lost. For example, when considering a string of random integers between 0 and 9, most people can memorize a string that is 1 to 5 digits long with ease and a string that is 6 to 10 digits long with some effort, but would find it very difficult to memorize a string that is 11 digits or longer without practice or some kind of memory aid.

ACKNOWLEDGMENTS

The authors thank Vince Kuo, David Maloney, Fred Reif, Jonathan Tuminaro, and Kurt VanLehn for their careful reading of the manuscript and useful suggestions. The authors also thank the reviewers for their constructive comments.