gies in the battlefield, and the more abstract problem of routing packets through a computer network when failures cause a packet to be routed to the nearest "host."

We have presented a model for a hazardous environment and demonstrated its use in evaluating alternate strategies for navigation using a rule for evading danger by rerouting to the nearest shelter. While the model is simple, it serves to provide insights into the influence of the probabilistic parameters of the dynamic environment on path planning. It may be considered as a step toward treating more general and realistic environments in which all static components (e.g., the obstacles and the shelters) are known while the dynamic components that also influence the path are unknown and can only be detected on-line.

REFERENCES


An Application of the Generalized Neyman–Pearson Fuzzy Test to Stochastic-Signal Detection

Jae Cheol Son, Iickho Song, Sun Yong Kim, and Seong Ill Park

Abstract—In the article an application of the fuzzy testing of hypothesis to the stochastic-signal detection problem is considered when the signal-to-noise ratio approaches zero. We first obtain the general relationship between the test statistic of the locally optimum fuzzy detector and that of the locally optimum detector. Based on this result, the test statistic and structures of the locally optimum fuzzy detector for stochastic signals are obtained. Several aspects of the locally optimum fuzzy nonlinearity for stochastic signals are also described. Finally, performance characteristics of the locally optimum fuzzy detector are briefly discussed.

I. INTRODUCTION

Signal detection schemes based on fuzzy set theory have recently been investigated in the literature [5], [9], [11], [12]. These detection schemes seem to be from a practical standpoint, appealing since the noise distribution is often not precisely known. Previously, we discussed [11], [12], the weak known-signal detection problem with fuzzy information based on the techniques of fuzzy testing of statistical hypothesis (e.g., [3]) and found the detector structure. The performance characteristics of the detector were also compared to those of the combined system of the quantizer and locally optimum (LO) detector (i.e., the LO detector). The rationale to consider fuzzy information in the quantizer-detector structure may be explained as follows. In parametric signal detection, we normally assume that the characteristic of the noise is completely known. The self-noise of the physical detection processor however, is in general, neglected although it does not seem to be a reasonable assumption in some cases. If we take the effect of the self-noise into account, a confidence in the output value of the quantizer would be diminished. Obviously, we may partially alleviate this situation by estimating the statistical characteristics of the self-noise. However, it seems that to employ fuzzy testing of a statistical hypothesis is a more convenient and practical method for signal detection problems since the exact analysis of the quantization error is, in practice, cumbersome and time consuming. The assumption of known signals in [11], [12] is a realistic one since it is not difficult to find many examples that can be modeled as the known-signal detection problem in modern communication systems.

It is assumed in this work that the inputs to both the LOQ and LOF detection processors are $Q = Q(Y)$, where $Q(\cdot)$ is the quantizer characteristic. The LOQ detector makes a decision based on $Q$, while the LOF detector makes a decision viewing $Q$ as fuzzy information. The optimum design of nonuniform quantizers in the context of data quantization for the weak stochastic-signal detection problem was already discussed in [1]. In Fig. 1(a), a typical input–output characteristic of the $m$-level quantizer for stochastic signal detection is shown in which we assume that the quantizer characteristic is even-symmetric. The parameters $(a, b, c, d)$ and $\{1, 0\}$ in Fig. 1(a) are called the breakpoints and quantization levels of the quantizer, respectively.

It should be noted that a considerable amount of study (e.g., [7], [8], [10], [14]) has been devoted to detection of stochastic (or random) signals under various noise circumstances. This is because it is convenient and reasonable to assume less about the signal than is required for known or parametric assumptions when the representation of the desired signal is difficult. For example, in acoustical applications, random dispersion due to turbulence and inhomogeneities in propagation media and insufficient understanding of the signal generating mechanism may lead us to adopt the stochastic-signal model [7].

In this work an application of the fuzzy testing of the hypothesis for detection of stochastic signals when signal-to-noise ratio approaches zero is considered as a natural extension of our previous studies [11], [12]. In Section II, the observation model and some
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mathematical preliminaries will be given. In Section III, we will obtain a general relationship between the locally optimum fuzzy (LOF) detector test statistic and the LO detector test statistic. Based on the result, the test statistic and structure of the LOF detector for stochastic signals are obtained in Section IV. Finally, performance characteristics of the LOF detector are briefly discussed in Section V. In addition, several conclusions are drawn in Section VI. A. Observation Model

Let us consider the well-known signal-detection problem that can be expressed by the following hypotheses:

\[ H_0: Y_i = W_i \quad \text{(signal is absent)} \]

versus

\[ H_1: Y_i = \theta S_i + W_i \quad \text{(signal is present)} \]

for \( i = 1, 2, \ldots, n \). In (1) and (2), \( Y_i \) is the observation, \( S_i \) is the stochastic signal component, and \( W_i \) is the purely-additive noise (PAN) component at the \( i \)th sampling instant. The positive real quantity \( \theta \) is the amplitude parameter that controls the signal strength. The stochastic-signal component \( S \) is a random variable with finite mean \( \mu \) and variance \( \sigma^2 \), \( i = 1, 2, \ldots, n \). The joint probability density function (pdf) of \( S = (S_1, S_2, \ldots, S_n) \) is denoted by \( f_S \); and the covariance function of \( S_i \) is denoted by \( E[(S_i - \mu_S)(S_j - \mu_S)] = \rho_{ij} \). The PAN components \( W_i, i = 1, 2, \ldots, n \), are assumed to be independent and identically distributed (i.i.d.) with common continuous pdf \( f_w \). The pdf \( f_w \) is assumed to be zero-mean and even-symmetric. It is also assumed that the stochastic signal components are statistically independent of the PAN components. Based on these descriptions, we can express the conditional joint pdf of \( Y = (Y_1, Y_2, \ldots, Y_n) \) assuming \( \theta \) as

\[ f_Y(y \mid \theta) = \sum_{S} f_S(s) \prod_{i=1}^{n} f_w(y_i - \theta s_i) dS \]

where \( \bar{y} = (y_1, y_2, \ldots, y_n), \bar{s} = (s_1, s_2, \ldots, s_n), \) and \( X^n \) is the Euclidean \( n \)-dimensional space.

B. Assumptions

In order to handle the observation \( Y \) as fuzzy information, let us introduce some definitions. Let \( (X^n, B_X, F) \) be a probability space where \( X^n \) is the Euclidean \( n \)-dimensional space, \( B_X \) is the Borel field, and \( F \) is a probability measure over \( X^n \). A fuzzy information system \( \tau \) is a fuzzy partition of the real line \( X \) by means of fuzzy events \( \{ \tau \} \). An \( n \)-tuple of elements in \( \tau, k = (k_1, k_2, \ldots, k_n), k_i \in \tau, i = 1, 2, \ldots, n \), is called the sample fuzzy information of size \( n \) based on which a decision shall be made. The set consisting of all possible sample fuzzy information is called the fuzzy random sample of size \( n \) and is denoted by \( \tau^n \). The probability distribution of \( \tau^n \) is given by

\[ P(\tilde{\tau}) = \int_{X^n} \lambda_{\tau}(\tilde{y}) f_\tau(\tilde{y}) d\tilde{y} \]

where \( \lambda_{\tau}(\tilde{y}) \) is called the membership function of \( \tau \). In this work, we will assume as in [3], [11]-[13] that

\[ \lambda_{\tau}(\tilde{y}) = \prod_{i=1}^{n} \lambda_{\tau_i}(y_i) \]

which means that the membership function at each sample of fuzzy information is a function of the membership functions of the fuzzy information \( \kappa_i, i = 1, 2, \ldots, n \). From (4) we see that the conditional probability of \( \tilde{x} \) assuming \( \theta \) can be expressed as

\[ P(\tilde{\tau} \mid \theta) = \int_{X^n} \lambda_{\tau}(\tilde{y}) f_\tau(\tilde{y}) \mid \theta \ di \tilde{y} \]

III. DETECTOR TEST STATISTICS

Based on the fuzzy set theoretic extension of the generalized Neyman-Pearson lemma, \( \tilde{x} \) was shown in [13] that the test statistic of the LOF detector can be expressed as

\[ d^* P(\tilde{x} \mid \theta) \]

\[ T_{LOF}(\tilde{x}) = \frac{d^* P(\tilde{x} \mid \theta)}{d P(\tilde{x} \mid \theta = 0)} \]

where \( d \) is the first nonzero derivative of \( P(\tilde{x} \mid \theta) \) at \( \theta = 0 \). Using (6) and (7), it can be shown as in Appendix I that the following
relationship holds between the LOF detector test statistic $T_{LOF}(\tilde{\text{x}})$ and the LO detector test statistic $T_{LO}(\tilde{\text{y}})$:

$$T_{LOF}(\tilde{\text{x}}) = \sum_{j=1}^{n} \lambda_j(\tilde{\text{y}}) T_{LO}(\tilde{\text{y}}) \prod_{i=1}^{n} f_{\text{w}}(y_i) \, d\tilde{\text{y}}$$

$$= \prod_{i=1}^{n} P(\text{x}_i \mid \theta = 0).$$

In this section, we derive the LOF detector test statistic for stochastic signals using (8), and obtain the corresponding detector structure. As we shall see, it is interesting that there is more than one LOF detector test statistic for stochastic signals depending on the statistics of the stochastic signals, whereas that for known signals was shown to be unique [12]. More specifically, we will show that there are three different LOF detector test statistics for stochastic signals in the following subsections.

A. The Case of Nonzero Mean Stochastic Signals

Let us first assume that at least one of $\mu_i$, $i = 1, 2, \cdots, n$, is not zero. Then it is shown in Appendix I that the LOF detector test statistic (8) becomes

$$T_{LOF}(\tilde{\text{x}}) = \sum_{i=1}^{n} \mu_i \mathbf{g}_{LOF}(\kappa_i)$$

where

$$\mathbf{g}_{LOF}(\kappa_i) = \frac{E\{X_i(\text{y})\}}{E\{X_i(\text{y})\}}$$

is an LOF nonlinearity for stochastic signals. It is interesting to note that this nonlinearity is exactly the same as the LOF nonlinearity for known signals [12]. In (10), $E\{\cdot\}$ denotes the statistical expectation with respect to $f_{\text{w}}$.

From (9), we see that the test statistic is in the form of the generalized correlator detectors [6]. We also see that in this case the LOF detector test statistic depends only on the mean values of the stochastic signals. This implies that if a stochastic-signal component has a nonzero mean, no other statistical characteristic of the stochastic-signal components than the mean is necessary in constructing the LOF detector, and that the test statistic is exactly the same as that for known-signal detection with $\mu_i$ replaced with known-signal components. Note that a similar observation can also be made in the LO detection of stochastic signals for the case of nonzero mean [6].

B. The Case of Zero-Mean Stochastic Signals

Now let us assume that $\mu_i$, $i = 1, 2, \cdots, n$, are all zero. If we assume that the stochastic signals are correlated, then it can be shown as in Appendix I that the LOF detector test statistic is

$$T_{LOF}(\tilde{\text{x}}) = \sum_{i=1}^{n} \sum_{j=1}^{n} K_{\text{S}}(i,j) \mathbf{g}_{LOF}(\kappa_i) \mathbf{g}_{LOF}(\kappa_j)$$

$$+ \sum_{i=1}^{n} \sigma_i^2 \mathbf{g}_{LOF}(\kappa_i)$$

where

$$\mathbf{g}_{LOF}(\kappa_i) = \frac{\int_{-\infty}^{\infty} \lambda_i(\text{y}) f_{\text{w}}(\text{y}) \, d\text{y}}{P(\kappa_i \mid \theta = 0)}$$

is also an LOF nonlinearity for stochastic signals. Immediately, if the stochastic signals are uncorrelated, (11) becomes

$$T_{LOF}(\tilde{\text{x}}) = \sum_{i=1}^{n} \sigma_i^2 \mathbf{g}_{LOF}(\kappa_i).$$

From (11) and (13), we see that when the stochastic-signal components are all zero-mean, only the second-order statistics of the stochastic-signal components are crucial in making a decision. Again, we see that this observation is also valid in the LO detection of stochastic signals with zero-mean [6], [8].

C. Detector Structures

In Figs. 2-4, we show schematic diagrams of the structures of the LOF stochastic-signal detectors for the three cases considered in the two subsections above.

The LOF detector structure in Fig. 3 for the correlated stochastic signals is obtained as follows. Let us assume that the stochastic signal is wide-sense stationary. That is, $\mu_i = \mu$, $i = 1, 2, \cdots, n$, and $K_{\text{S}}(i,j) = K_{\text{S}}(|i-j|)$. Since $K_{\text{S}}(k)$ is nonnegative definite, we have [6]

$$K_{\text{S}}(k) = \sum_{l=-\infty}^{\infty} c_{l} c_{l}^*$$

where $c_l = 0$ for $l < 0$ and $*$ represents the complex conjugate. Here $(c_l)_{l=-\infty}^{\infty}$ is the impulse response of the discrete-time filter with frequency response $H(\omega)$ satisfying $|H(\omega)|^2 = \Phi_{s}(\omega)$, where $\Phi_{s}(\omega)$ is the signal power spectral density. In the same way as in [6], we see that (11) can now be expressed as

$$T_{LOF}(\tilde{\text{x}}) = \sum_{i=1}^{n} \left| \sum_{l=1}^{n} \mathbf{g}_{LOF}(\kappa_l) c_{l-1} \right|^2$$

$$+ \sum_{i=1}^{n} \left\{ \mathbf{g}_{LOF}(\kappa_i) - \mathbf{g}_{LOF}(\kappa_i) \right\}$$

where the corresponding block diagram of the structure is shown in Fig. 3.
Comparing the structures shown in Figs. 2-4 with the LO detector structures, we can conclude that the detector structures of the LOF detectors are identical to those of the LO detectors [6] except for the detector nonlinearities.

IV. LOCALLY OPTIMUM FUZZY NONLINEARITY FOR STOCHASTIC SIGNALS

One of the important factors that characterize the detector structure is the detector nonlinearity. Hence more details on the characteristics of the LOF nonlinearities would be helpful and important in describing and analyzing LOF detectors. Since the characteristics of the nonlinearity \( g_{LOF} \) was already discussed in [12], we will discuss several characteristics of the nonlinearity \( h_{LOF} \) in this section.

Let us first consider an alternative expressions of (12). If we apply integration by parts to (12) twice, we have

\[
h_{LOF}(\xi_j) = \frac{E\{\lambda^*_i(y)\}}{E\{h_o(y)\}}. \tag{16}
\]

For trapezoidal membership functions, it is shown in Appendix II that we have Property 1 for the LOF nonlinearity \( h_{LOF} \).

Property 1: If we consider the trapezoidal membership function, an alternative form of \( h_{LOF}(\cdot) \) is, for any \( \pm z \), \( i = 1, 2, \ldots, m-2 \),

\[
h_{LOF}(\pm z_i) = \frac{D(b_i) - D(b_{i-1})}{G(b_{i-1}) - G(b_i)} \tag{17}
\]

where

\[
D(\eta) = \int_{-\Delta/2}^{\Delta/2} f_{w}(\eta - \Delta/2) - f_{w}(\eta + \Delta/2) \tag{18}
\]

and

\[
G(\eta) = \int_{-\Delta/2}^{\Delta/2} F_{w}(\xi) d\xi
\]

with \( F_{w} \) being the cumulative distribution function (cdf) of the additional noise.

From (17) we see that the LOF nonlinearity \( h_{LOF} \) depends only on the values of the pdf at the four points \( b_i \pm (\Delta/2) \) and \( b_{i-1} \pm (\Delta/2) \), and of the noise cdf for the two intervals of length \( \Delta \), \( [b_i - (\Delta/2), b_i + (\Delta/2)] \) and \( [b_{i-1} - (\Delta/2), b_{i-1} + (\Delta/2)] \), in which the membership grade varies. Expressions for the LOF nonlinearity \( h_{LOF} \) for \( \pm \tau \) can also be obtained to be, respectively,

\[
h_{LOF}(\pm \tau) = \frac{D(b_i)}{2} - G(b_i) \tag{20}
\]

and

\[
h_{LOF}(\pm \tau) = \frac{D(b_i)}{2} - G(b_i) \tag{20}
\]

In obtaining (20) we used (A3.3) in Appendix III.

We see that when the membership function is trapezoidal the expressions (16) and (17) are more convenient to handle than (12) since we can calculate the numerator of (16) and (17) with ease. In (17) we see that the LOF nonlinearity \( h_{LOF} \) depends only on the values of the pdf at the four points \( b_i \pm (\Delta/2) \) and \( b_{i-1} \pm (\Delta/2) \), and of the noise cdf for the two intervals of length \( \Delta \), \( [b_i - (\Delta/2), b_i + (\Delta/2)] \) and \( [b_{i-1} - (\Delta/2), b_{i-1} + (\Delta/2)] \), in which the membership grade varies. Expressions for the LOF nonlinearity \( h_{LOF} \) for \( \pm \tau \) can also be obtained to be, respectively,

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TABLE II
VALUES OF \( G(\cdot) \) FOR THE BREAKPOINTS OF THE FOUR-LEVEL OPTIMUM QUANTIZER AND VARIOUS VALUES OF INCREDIBILITY IN THE STANDARD NORMAL NOISE (\( \Delta_{\text{max}} = 0.644 \))

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<td>( \Delta_{\text{max}} )</td>
<td>0.53215</td>
<td>0.60072</td>
<td>0.63659</td>
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TABLE III
VALUES OF \( h_{\text{LOF}}(\cdot) \) AND RELATIVE DEVIATIONS FOR THE FOUR-LEVEL OPTIMUM QUANTIZER AND VARIOUS VALUES OF INCREDIBILITY IN THE STANDARD NORMAL NOISE (\( \Delta_{\text{max}} = 0.644 \))

<table>
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<tr>
<th>( \Delta )</th>
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The LO nonlinearity

\[
h_{\text{LO}}(y) = \frac{f_a(y)}{f_a(-y)}
\]

is an even function of \( y \) when \( f_a \) is even. It is noteworthy that the same observation can be found for \( h_{\text{LOF}} \).

Property 3: If \( h_{\text{LO}}(y) = h_{\text{LO}}(-y) \) and \( f_a(y) \) is even, the nonlinearity \( h_{\text{LO}}(\cdot) \) is an even function of \( y \).

The proof of Property 3 can also be found in Appendix II. Properties 2 and 3 imply that the LO and LOF nonlinearities are of similar characteristic.

V. PERFORMANCE CHARACTERISTIC

In this section we examine some performance characteristics of the LOF detector for stochastic signals obtained in Section III and compare them with those of the LOQ detector. Specifically, we performed three computer simulations, letting \( n = 50, m = 4 \), and the false-alarm probability (\( P_{fa} \)) equal \( 10^{-2} \). Each simulation to obtain the detection probabilities (\( P_d \)) of the LOQ and LOF detectors was accomplished by \( 10^5 \) Monte Carlo runs on a Trigem SPARC-station SDT-200 at the Korea Advanced Institute of Science and Technology, so that the relative error is about 0.1 percent. For simplicity, we assumed that the stochastic-signal components are i.i.d. with the standard normal pdf. We also assumed that the pdf of the PAN components is standard normal. To generate the stochastic signal and PAN components we used the GGNML subroutine of the IMSL.

In the first simulation, we assumed the ideal situation; that is, it was assumed that we had the perfect statistical information on the stochastic signal and PAN components and no self-noise is present. For the LOF detectors, we considered two values of \( \Delta \), 0.1 and 0.4. The detector thresholds and \( P_d \) were obtained through Monte Carlo simulations. Note that Monte Carlo simulation is one of the conventional and reasonable methods, although there is no doubt that the method is based on a heuristic approach.

Fig. 5 shows the plots of the detection probabilities of the LOQ and LOF detectors as functions of the stochastic-signal strength parameter \( \theta \). From Fig. 5 we can see that there is no difference among the performance characteristics of the detectors. This seems to be due to the fact that the order of the ordered fuzzy information space is preserved, as pointed out in [11]. However, we will not verify whether the order is really preserved or not since the verification for this case is indeed tedious and time-consuming work. We see that the LOF detector can replace the LOQ detector in the ideal situation although the LOF detector regards the output of the quantizer as fuzzy information.

In the second simulation, we again assumed that we have the perfect statistical information on the stochastic-signal and PAN components. We assumed, however, that some self-noise is present in the second simulation. We let the self-noise be normal with mean zero and variance (or power) 0.01. (Note that the variance of the PAN is assumed to be 1). Now let us denote the LOQ detector for noise with variance \( \gamma \) by LOQ(\( \gamma \)). In the second simulation we also used Monte Carlo simulations to find the detection probabilities and thresholds for the detectors. Table IV shows the detection probabilities of the LOQ and LOF detectors as functions of \( \theta \). From Table IV we see that when we actually have self-noise of variance 0.01 in addition to the PAN of variance 1, LOQ(1.01) slightly outperforms LOQ(1) since it takes the effect of the self-noise into account. The performance of the three LOF detectors is also slightly inferior to that of LOQ(1.01). It is noteworthy that there is no difference among the performance characteristics of LOQ(1) and LOF detectors as in the first simulation.

In the third simulation, we assumed the same environment as in the second simulation. However, we used an approximate approach to find the thresholds in the third simulation, since finding the exact thresholds through the Monte Carlo simulations is too time consuming and thus physically cumbersome to implement. To find the thresholds of the LOQ detectors, we used the approximate value

\[
\text{threshold} = Z_{\alpha} \left[ \sum_{x \leq r} P(x) = \theta = 0 \right] \left| h_{\text{LO}}(r) \right|^{1/2}
\]

where \( Z_{\alpha} \) is the \( 100(1 - \alpha) \)th percentile of the standard normal distribution. Equation (24) can be obtained with the central limit theorem. The detector threshold of LOQ(1) was also obtained based on the asymptotic approximation. It should be noted that some errors can be made by this asymptotic approximation. In Fig. 6 we show the plots of the detection probabilities of the LOQ and LOF detectors as functions of \( \theta \). From Fig. 6 we first see that the power function of LOQ(1) is larger than that of LOQ(1.01) for all values of \( \theta \geq 0 \), since LOQ(1) does not take the effect of
the self-noise into account. We also see that the LOF detectors have intermediate performance characteristics between LOQ(1) and LOQ(1.01), and that as $\Delta$ becomes large the performance of the LOF detector approaches that of LOQ(1.01). These results are primarily due to the fact that we calculated the thresholds based on the Zadeh's definition of probability (4) and that the probability mass function (pmf) of the quantizer output level for the LOF detector is more similar to the pmf of the quantizer output level for LOQ(1.01) than to that for LOQ(1).

It should be noted here that one may attempt to overcome this self-noise problem by estimating the overall noise variance. Even if an almost exact value of the noise variance is estimated, one should adjust all the values of the breakpoints and output levels of the quantizer. On the other hand we could just adjust the values of the output levels based on the incredibility when the fuzzy set theoretic approach is used.

VI. CONCLUSION

In this work we obtained the locally optimum fuzzy detector test statistics and detector structures for stochastic signals. Several aspects of the locally optimum fuzzy nonlinearity for stochastic signals were discussed. We also examined the performance characteristics of the locally optimum fuzzy detector and showed that the locally optimum fuzzy detector has a property of robustness.

The assumption of the self-noise can be considered in a different point of view. That is, the same procedure as those in the second and third computer simulations in Section V can be applied to a situation in which the actual noise variance is slightly larger than the estimated noise variance.

APPENDIX I
DERIVATION OF THE LOF TEST STATISTIC

Differentiating $v$ times (6) with respect to $\theta$, and then letting $\theta = 0$, we have

$$\frac{d^v P(\xi | \theta)}{d\theta^v} \bigg|_{\theta = 0} = \int_{\chi^*} \lambda_z(\bar{y}) \frac{df_\xi(\bar{y} | \theta)}{d\theta^v} \bigg|_{\theta = 0} d\bar{y}. \quad (25)$$

Since

$$\frac{d^v f_\xi(\bar{y} | \theta)}{d\theta^v} \bigg|_{\theta = 0} = \frac{\lambda_z(\bar{y})}{f_\xi(\bar{y} | \theta = 0)}, \quad (26)$$

we have

$$T_{LO}(\bar{y}) = \frac{1}{P(\kappa | \theta = 0)} \int_{\chi^*} \lambda_z(\bar{y}) T_{LO}(\bar{y}) f_\xi(\bar{y} | \theta = 0) d\bar{y}. \quad (27)$$

The relationship (8) can immediately be obtained from (27) noting that $f_\xi(\bar{y} | \theta = 0) = \Pi_{\theta = 0} f_\xi(\bar{y})$ and $P(\kappa | \theta = 0) = \Pi_{\theta = 0} P(\kappa | \theta = 0) = 0$.

Now let us obtain the LOF detector test statistic for stochastic signals. Since

$$T_{LO}(\bar{y}) = \frac{1}{\kappa} \sum_{i=1}^{\kappa} \frac{f_\xi(\bar{y})}{f_\xi(\bar{y})}, \quad (28)$$

the self-noise into account. We also see that the LOF detectors have intermediate performance characteristics between LOQ(1) and LOQ(1.01), and that as $\Delta$ becomes large the performance of the LOF detector approaches that of LOQ(1.01). These results are primarily due to the fact that we calculated the thresholds based on the Zadeh's definition of probability (4) and that the probability mass function (pmf) of the quantizer output level for the LOF detector is more similar to the pmf of the quantizer output level for LOQ(1.01) than to that for LOQ(1).

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$$T_{LO}(\bar{y}) = \frac{1}{\kappa} \sum_{i=1}^{\kappa} \frac{f_\xi(\bar{y})}{f_\xi(\bar{y})}, \quad (28)$$
for the case of nonzero-mean stochastic signals [6], we have from (8) that

$$T_{LO}(z) = \frac{1}{\prod_{i=1}^{n} P(k_i | \theta = 0)} \sum_{i=1}^{n} \sum_{j=1}^{n} K_{ij}(y) \int_{y} \lambda_{z}(\tilde{y}) \right.$$

$$\cdot \sum_{i=1}^{n} \left[ -\mu_{i} \int_{y} f_{w}(\tilde{y}) \prod_{k=1}^{n} f_{w}(y_k) d\tilde{y} \right]$$

$$= \frac{-1}{\prod_{i=1}^{n} P(k_i | \theta = 0)} \sum_{i=1}^{n} \left[ \int_{y} \lambda_{z}(\tilde{y}) f_{w}(\tilde{y}) d\tilde{y} \right]$$

$$= \frac{\prod_{k=1}^{n} P(k_i | \theta = 0)}{\prod_{i=1}^{n} P(k_i | \theta = 0)} \int_{y} \lambda_{z}(\tilde{y}) f_{w}(\tilde{y}) d\tilde{y}$$

from which we have (9).

Now let us consider the case of zero-mean stochastic signals. In this case, it is not difficult to see that $v = 2$. From (8) and the LO detector test statistic for zero-mean stochastic signals

$$T_{LO}(Y) = \sum_{i=1}^{n} \sum_{j=1}^{n} K_{ij}(y) \prod_{k=1}^{n} f_{w}(y_k) d\tilde{y}$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} K_{ij}(y) \prod_{k=1}^{n} f_{w}(y_k) d\tilde{y}$$

and $h_{LO}(y)$ is defined in (23), we have

$$h_{LO}(y) = \frac{f_{w}(y)}{f_{w}(\tilde{y})}$$

$$T_{LO}(z) = \frac{1}{\prod_{i=1}^{n} P(k_i | \theta = 0)} \sum_{i=1}^{n} \sum_{j=1}^{n} K_{ij}(y) \int_{y} \lambda_{z}(\tilde{y}) \right.$$

$$\cdot \sum_{i=1}^{n} \left[ -\mu_{i} \int_{y} f_{w}(\tilde{y}) \prod_{k=1}^{n} f_{w}(y_k) d\tilde{y} \right]$$

$$= \frac{\prod_{k=1}^{n} P(k_i | \theta = 0)}{\prod_{i=1}^{n} P(k_i | \theta = 0)} \int_{y} \lambda_{z}(\tilde{y}) f_{w}(\tilde{y}) d\tilde{y}$$

$$= \frac{\prod_{k=1}^{n} P(k_i | \theta = 0)}{\prod_{i=1}^{n} P(k_i | \theta = 0)} \int_{y} \lambda_{z}(\tilde{y}) f_{w}(\tilde{y}) d\tilde{y}$$

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which is the same as (11).
see that

\[ D(-\eta) + D(\eta) = 0 \]  

(37)

and

\[ G(-\eta) + G(\eta) = \Delta. \]  

(38)

In addition, the values of \( D(\cdot) \) and \( G(\cdot) \) when \( \eta = 0 \) can easily be obtained. From (18), it is no doubt that \( D(0) = 0 \). Since \( f_0 \) is even and zero-mean, we have \( g(0) = \Delta/2 \) from (19) and (38).

Finally, let us introduce some inequalities for \( G(\cdot) \). Under the assumptions for \( f_0 \) in Section II-A, we easily see that

\[ 0 < G(-b_i) < \frac{\Delta}{2} \quad i = 1, 2, \ldots, m - 1. \]  

(39)

and

\[ \frac{\Delta}{2} < G(b_i) < \Delta \quad i = 1, 2, \ldots, m - 1. \]  

(40)

where the breakpoints are assumed to be finite values. From (39) and (40), we immediately have

\[ G(-b_i) < G(b_i) \quad i, j = 1, 2, \ldots, m - 1. \]  

(41)

REFERENCES


Planning Near-Minimum-Length Collision-Free Paths for Robots

Mohamed B. Trabia

Abstract—This paper describes an algorithm for the automatic generation of near-minimum-length collision-free paths for robots. Obstacles are assumed to have polygonal cross sections. This algorithm requires very small data storage. Expansion of obstacles and shrinkage of robots to a point are used to simplify the analysis of the robot collision detection problem. The robot path is considered to be composed of straight line segments. Collision detection criteria between the robot path and the obstacles are discussed. A scheme to search for near-minimum length collision-free paths is presented. This scheme has the benefit of being concise and suitable for real-time implementation, especially for robots working in cluttered environments. Examples to show the above ideas are included.

NOMENCLATURE

\( A_{i,k} \) \( x \) coefficient of the straight line equation of edge \( i \) on obstacle \( k \).

\( B_{i,k} \) \( y \) coefficient of the straight line equation of edge \( i \) on obstacle \( k \).

\( C_{i,k} \) Third coefficient of the straight line equation of edge \( i \) on obstacle \( k \).

\( n_i \) Number of obstacles intersecting the path.

\( n_t \) Total number of obstacles.

\( m \) Average number of obstacle vertices.

\( r \) Radius of a circle enclosing the mobile robot or the robot end-effector.

\( x_{v_i} \) \( x \) coordinate of vertex \( i \) of obstacle \( k \).

\( y_{v_i} \) \( y \) coordinate of vertex \( i \) of obstacle \( k \).

\( x_{p_i} \) \( x \) coordinate of a point on the expanded obstacle \( k \).

\( y_{p_i} \) \( y \) coordinate of a point on the expanded obstacle \( k \).

\( x_{v_i} \) \( x \) coordinate of a vertex \( i \) on the expanded obstacle \( k \).

\( y_{v_i} \) \( y \) coordinate of a vertex \( i \) on the expanded obstacle \( k \).

\( \theta_{v_i} \) Angle describing the outward direction for edge \( i \) of obstacle \( k \).

I. INTRODUCTION

Robots are increasingly used in many applications such as assembly, material handling, welding, and in areas such as nuclear plants, seabeds, and hazardous waste sites. One of the most important areas for robotic applications is the implementation of flexible manufacturing systems. Robots can be mobile or stationary. If a mobile robot is used in rugged terrains, it is usually a legged robot. The problem is relatively smooth, wheeled robots are more suitable since they require relatively simpler control schemes. One of the problems faced by mobile robots is the path planning problem. The path planning problem is defined in this paper as

"find the robot path between given starting and target points such that the robot does not collide with any obstacle located within the robot workspace."

We deal with this problem from a purely kinematic point of view. This means that we do not consider the effects of robot dynamics.