Unified Performance Analysis of Two-Hop Fixed Gain Relay Systems with Beamforming

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Abstract—We present a unified performance analysis of a beamforming system in which the source and the destination are both equipped with multiple antennas while the relay is equipped with a single antenna. By assuming general correlation structures with arbitrary eigenvalue multiplicities at the source and the destination, the exact outage probability is derived for maximal ratio transmission (MRT)/maximal ratio combining (MRC) fixed gain relay network. Our new results unify several previously reported cases as well as new additional ones. Furthermore we derive the exact outage probability for transmit antenna selection (TAS/MRC) system and provide a performance comparison of the two systems. To gain insights, we also present high signal-to-noise ratio (SNR) expressions which clearly show the impact of number of transmit/receive antennas and correlation on the outage probability. Accuracy of the analytical results will be verified by Monte Carlo simulation at the end.

I. INTRODUCTION

The performance of the amplify-and-forward (AF) relay system in which the source and destination are both equipped with multiple antennas while the relay is equipped with a single antenna has been investigated in numerous prior works (see eg. [1]–[7]). Diversity transmission techniques such as MRT and MRC has been deployed to improve the performance of the systems. Authors in [1] and [2] analyzed the channel state information (CSI)-assisted relay network with beamforming. Recently, in [3] and [4], the performance of a dual-hop fixed gain network over independent Rayleigh and Nakagami-m fading environments has been analyzed. The performance of fixed gain relaying with exponential correlation has been studied in [7]. TAS/MRC system for CSI assisted relay with correlation effects has been analyzed in [8].

Use of multiple antennas increases the system performance and diversity gain but the desired level cannot be achieved when the antenna correlation is present. Hence this detrimental effect of antenna correlation has to be quantified. To the best of our knowledge, only special cases of correlation in fixed gain relaying has been considered in the literature. This paper aims to provide a unified analysis of MRT/MRC AF fixed gain relay system. Such a system may occur in several practical situations, for example when two base stations equipped with multiple antennas communicate with each other via a single antenna relay node. To this end, a general fading model with arbitrary eigenvalue distributions for the transmit and receive correlation matrices are assumed. Specifically we derive the exact outage probability for fixed gain relay schemes. In order to obtain more insight, we also derive simple asymptotic expressions. Our investigations cover several cases presented previously, as well as new additional ones (e.g.: uniform correlation). Furthermore this paper provides the exact outage probability of TAS/MRC system and presents a comparison of performance of the two systems.

II. SYSTEM MODEL

We consider an AF two hop fixed gain communication system where a source(S), equipped with $n_T$ antennas communicates with a destination(D), equipped with $n_R$ antennas, via a single antenna relay(R). Due to high shadowing we assume $S$ does not have a direct link to the $D$. Hence, in this network, the diversity gain expected from the use of multiple antennas is of significant interest since cooperative diversity can no longer be achieved.

The channel state information is assumed to be available at $S$ and $D$. The communication from source to destination via the relay takes place in two time slots. In the first time slot, the source beamforms its signal to the relay. With the spatial correlation effects at the antennas of $S$, the received signal at $R$ can be written as

$$ y_R = \sqrt{P_1} \Delta_i |w_T|^2 x + n_1, \quad (1) $$

where $x$ and $n_1$ are the transmit signal at $S$ and zero mean additive white Gaussian noise (AWGN) at the $R$, satisfying $E(|x|^2) = 1$ and $E(|n_1|^2) = N_0$. $P_1$ denotes the transmit power. $\Delta = \Phi_S^H h_1$ for MRT/MRC system and $\Delta = h_1^T$ for TAS/MRC system, $n_T \times 1$ channel vector from $S$ to $R$ is $h_1 = [h_1^1, \ldots, h_1^{n_T}]^T$, where $h_1^j$ is a Rayleigh fading entry and $(\cdot)^T$ and $(\cdot)^H$ denotes the Transpose and Hermitian transpose, respectively and $|h_1^j| = \max_{1 \leq i \leq n_T} |h_1^j|$. $\Phi_T$ is the $n_T \times n_T$ matrix, which models the spatial correlation at $S$. The weight vector $w_T$ is chosen according to the principles of transmit beamforming, as $\Delta_{\|w_T\|^2}$ where $\|\cdot\|_F$ is the Frobenius norm.

After the first time slot, the received signal $y_D$ at the $R$ is multiplied by a gain, $G$, and then during the second time slot retransmitted to the $D$. The received signal at $D$ is given by

$$ y_D = \sqrt{P_2} \Phi_R^2 h_2 G y_R + n_2 $$
$$ = \sqrt{P_2} \Phi_R^2 h_2 G \left( \sqrt{P_1} \Delta |w_T|^2 x + n_1 \right) + n_2, \quad (2) $$

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where $P_2$ is the relay transmit power, $n_R \times 1$ channel vector from $R$ to $D$ is $h_2 = [h_{21}, \ldots, h_{2n_R}]^T$, where $h_{2i}$ being a Rayleigh fading entry and $n_2$ is the AWGN $n_R \times 1$ noise vector having the variance of $N_0I_{n_R}$. Where $I_n$ is the identity matrix of size $n$. $\Phi_R$ is the $n_R \times n_R$ matrix, which models the spatial correlation at $D$. At the $D$, according to principle of MRC, we multiply the received signal by a $1 \times n_R$ weight vector $w_R$, $w_R^\top = \frac{\Phi_R^\top h_2}{\|\Phi_R^\top h_2\|}$, then after some manipulations, the resulting end-to-end SNR, can be written as \[ \gamma_{\text{end}} = \frac{P_1\|\Delta\|^2}{N_0} \frac{\|\Phi_R^\top h_2\|^2}{N_0} + \frac{1}{C^2N_0}. \] (3)

### A. Fixed Gain Relaying

End-to-end SNR of the fixed gain relay network can be easily obtain as,

\[ \gamma_{eq} = \frac{\gamma_1\gamma_2}{\gamma_2 + C}, \] (4)

where $\gamma_1 = \|\Delta\|^2\rho_1$, $\gamma_2 = \|\Phi_R^\top h_2\|^2\rho_2$, and $\rho_1$ and $\rho_2$ are equal to $P_1/N_0$ and $P_2/N_0$ respectively. $C$ is the fixed gain.

Let the distinct real eigenvalues of the transmit correlation matrix, $\Phi_T$ be denoted by $\varphi_1, \varphi_2, \ldots, \varphi_r$ with multiplicities $\nu_1, \nu_2, \ldots, \nu_r$ respectively such that $\sum_{i=1}^{r} \nu_i = n_T$. Let the distinct real eigenvalues of the receive correlation matrix, $\Phi_R$ be denoted by $\sigma_1, \sigma_2, \ldots, \sigma_r$ with multiplicities $\eta_1, \eta_2, \ldots, \eta_r$ respectively such that $\sum_{i=1}^{r} \eta_i = n_R$.

The pdf of the random variable ($RV$), $\gamma_2$ can be obtained by using [9] and the cdf, $F_{\gamma_2}(z) = \int_{0}^{z} p_{\gamma_2}(z)dz$ can be derived by using [12, Eq.2.321.2] as follows. Further pdf and cdf of $\gamma_2$ can be found by replacing corresponding parameters.

\[ p_{\gamma_2}(z) = \sum_{i=1}^{t} \sum_{j=1}^{\nu_i} \frac{\omega_{i,j}}{(j - 1)!}(\rho_1 \varphi_i)^{j-1}z^{-j}e^{-\frac{z}{\rho_1 \varphi_i}}, \] (5)

\[ F_{\gamma_2}(z) = 1 - \sum_{i=1}^{t} \sum_{j=1}^{\nu_i} \sum_{k=0}^{j-1} \omega_{i,j,k} \frac{\Lambda}{\rho_1 \varphi_i} e^{-\frac{\Lambda}{\rho_1 \varphi_i}}, \] (6)

where $\omega_{i,j}$ is defined in II-B. There are two methods of selecting a fixed gain as described in the technical literature. With the following fixed gain

\[ G = \sqrt{\frac{1}{P_1\|\Delta\|^2} \left[ \frac{1}{N_0} + N_0 \right]}, \] (7)

$E_X[.]$ denotes the expectation operation with respect to the $RV$, $X$. The constant $C_1$ is given by

\[ C = C_1 = \left( E_{\gamma_2} \left[ \frac{1}{(\gamma_2 + 1)} \right] \right)^{-1}. \] (8)

Performing the required expectation with the help of [12, Eq. (9.211.4)] yields the following closed-form solution for $C_1$,

\[ C_1 = \left( \sum_{i=1}^{t} \sum_{j=1}^{\nu_i} \omega_{i,j} \Psi \left( j, j; \frac{1}{\rho_1 \varphi_i} \right) \right)^{-1}. \] (9)

$\Psi(a, b, c)$ denotes the Tricomi confluent hypergeometric function defined in [12, Eq. (9.210.2)].

With the fixed gain factor selected as

\[ G = \sqrt{\frac{1}{P_1\|\Delta\|^2}} \left[ \frac{1}{N_0} + N_0 \right], \] (10)

the constant $C_2$ can be evaluated as

\[ C = C_2 = E_{\gamma_2}(\gamma_1) + 1 \]

\[ = \rho_1 \sum_{i=1}^{t} \sum_{j=1}^{\nu_i} \omega_{i,j} \delta_{i} + 1. \] (11)

### B. Fading Channel Model

**Exponential Correlation:** In the special case of an exponential matrix, we have all the eigen values distinct. For the source we have $t = n_T$, $\nu_i = 1; \forall i$ and for the destination $r = n_R$, $\eta_u = 1; \forall u$. Simplifying [9, Eq. (7)] yields

\[ \omega_{i,j} = \delta_{i}^{\gamma_1 - 1} \prod_{k=1, k\neq i}^{t} (\delta - \delta_{k})^{\gamma_1}, \]

\[ i = 1, j = 1 \]

\[ i = 2, 1 \leq j \leq n_T - 1. \] (12)

$\omega_{u,v}$ can be written by replacing $\phi_i$ and $n_T$ with $\sigma_u$ and $n_U$.

**Uniform Correlation:** In the special case of an uniform correlation [10, Eq. 6a] with parameter $\rho$, we have $\phi_i = 1 + (n_T - 1)\rho$ with multiplicity $\nu_1 = 1$ and $\phi_2 = 1 - \rho$ with multiplicity $\nu_2 = n_T - 1$. Simplifying [9, Eq. (7)] yields

\[ \omega_{i,j} = \left\{ \begin{array}{ll}
\frac{1}{(1-\delta)^{n_T-1}}, & \text{if } i = 1, j = 1 \\
\frac{1}{(1-\delta)^{n_T-1}}, & \text{if } i = 2, 1 \leq j \leq n_T - 1.
\end{array} \right. \] (13)

where $\delta = \frac{\rho}{\rho_1}$. Similarly $\omega_{u,v}$ can be written by replacing $\phi_i$ and $n_T$ with $\sigma_u$ and $n_U$ respectively.

**Independent:** In this special case we have all the eigenvalues equal to 1. For the source we have $t = 1$ with $\nu_i = n_T$ and for the destination $r = 1$ with $\eta_u = n_R$. Simplifying [9, Eq. (7)] yields

\[ \omega_{i,j} = \left\{ \begin{array}{ll}
1, & \text{if } i = 1, j = n_T \\
0, & \text{if } i = 1, 1 \leq j \leq n_T - 1.
\end{array} \right. \] (14)

Similarly $\omega_{u,v}$ can be written by replacing $n_T$ with $n_R$.

### III. Exact Outage Probability

The outage probability, $P_{\text{out}}$ is an important performance measure defined as the probability that $\gamma_{eq}$ drops below a predefined SNR threshold $\Lambda$. The outage probability can be written as

\[ P_{\text{out}} = \Pr(\gamma_{eq} < \Lambda) = F_{\gamma_{eq}}(\Lambda) \] (15)
A. Fixed Gain Relaying for MRT/MRC

Cdf of $\gamma_{eq}$ for fixed gain relay can be calculated as, [3, Eq.22]

$$F_{\gamma_{eq}}(\Lambda) = \int_0^\infty \Pr \left( \gamma_1 < \Lambda + \frac{CA}{\gamma_2} \right) p_{\gamma_2}(z)dz$$ (16)

Substituting (6) and (5) into (16) and by applying the Binomial theorem yields,

$$F_{\gamma_{eq}}(\Lambda) = 1 - \sum_{i=1}^t \sum_{j=1}^{\nu_i} \omega_{i,j} e^{-\frac{\Lambda}{\rho_1}} \sum_{u=1}^{r} \sum_{v=1}^{\eta_u} \frac{\omega_{u,v}}{(v-1)!}(\rho_1 \sigma_u)^v$$

$$\times \sum_{k=0}^{j-1} \frac{\Lambda^k}{k!(\rho_1 \phi_i)^k} \int_0^\infty \left( 1 + \frac{C}{z} \right) z^{v-1} e^{-\frac{CA}{\rho_1 \phi_i}} \omega_{u,v} dz$$ (17)

After some mathematical manipulations and using [12, Eq. (3.471.9)] the outage probability can be expressed in closed-form as

$$F_{\gamma_{eq}}(\Lambda) = 1 - 2 \sum_{i=1}^t \sum_{j=1}^{\nu_i} \sum_{u=1}^{r} \sum_{v=1}^{\eta_u} \sum_{k=0}^{j-1} k! \omega_{i,j} e^{-\frac{\Lambda}{\rho_1 \phi_i}} \omega_{u,v}$$

$$\times \frac{C^{v+k-n} \Lambda^{v+k-n}}{v^{v+k}(\rho_2 \sigma_u)^{v+k-n}} \int_0^\infty \left( 1 + \frac{C}{z} \right) z^{v-n} e^{-\frac{CA}{\rho_2 \phi_i}} K_{v+n-k} \left( 2 \sqrt{\frac{CA}{\rho_2 \phi_i}} \right)$$ (18)

where $K_{v}(\cdot)$ is the $v^{th}$ modified Bessel function of second kind given in [12, Eq. (8.407)]

B. Fixed Gain Relaying for TAS/MRC

Here we analyze fixed gain relay system of S-R independent channel with TAS and R-D general correlation channel with MRC. Cdf of $\gamma_1$ for TAS and independent channel can be derived using [11] and binomial expansion as,

$$F_{\gamma_1}(z) = 1 - \sum_{p=1}^{n_T} (-1)^{p+1} \left( \frac{n_T}{p} \right) e^{\frac{-zp}{\rho_1}}$$ (19)

By differentiating (19), w.r.t $z$ we obtain the pdf of $\gamma_1$ as,

$$p_{\gamma_1}(z) = \frac{1}{\rho_1} \sum_{p=1}^{n_T} (-1)^{p+1} \left( \frac{n_T}{p} \right) p e^{\frac{-zp}{\rho_1}}$$ (20)

Using (20) to find the required expectation mentioned in (8) and (11), we can find the fixed gain $CT_1$ and $CT_2$ as,

$$CT_1 = \rho_1 \left[ \sum_{p=1}^{n_T} \left( \frac{n_T}{p} \right) p(-1)^{p+1} \right]^{-1}$$

$$\times \text{Ei} \left( -\frac{p}{\rho_1} \right)$$ (21)

where $\text{Ei}(x)$ is the exponential integral of $x$.

$$CT_2 = \rho_1 \sum_{p=1}^{n_T} \left( \frac{n_T}{p} \right) \frac{1}{p} (-1)^{p+1} + 1$$ (22)

By employing (19) and (5) to (16), and with some simplification we obtain,

$$F_{\gamma_{eq}}(\Lambda) = 1 - \sum_{p=1}^{n_T} \left( \frac{n_T}{p} \right) (-1)^{p+1} e^{-\frac{nz}{\rho_1}} \sum_{u=1}^{r} \sum_{v=1}^{\eta_u} \frac{\omega_{u,v}}{(v-1)!}(\rho_2 \sigma_u)^v$$

$$\times \int_0^\infty z^{v-1} e^{-\frac{z p C T_1}{\rho_1}} \frac{z}{\rho_2 \sigma_u} dz$$ (23)

where $\Gamma(v)$ is the Gamma function of $v$. By using [12, Eq. (3.471.9)] we can simplify (23) as,

$$F_{\gamma_{eq}}(\Lambda) = 1 - 2 \sum_{p=1}^{n_T} \left( \frac{n_T}{p} \right) (-1)^{p+1} e^{-\frac{nz}{\rho_1}} \sum_{u=1}^{r} \sum_{v=1}^{\eta_u} \frac{\omega_{u,v}}{(v-1)!}(\rho_2 \sigma_u)^v$$

$$\times \left( p C T_1 \right)^{\frac{z}{2}} \frac{z}{p C T_1 (\rho_1 \rho_2 \sigma_u)}$$ (24)

IV. HIGH SNR ANALYSIS

A. Fixed Gain Relaying for MRT/MRC

At high SNR we can express fixed gain mentioned in (8) and (11) as, $C = \rho_1 D$. We can rewrite $F_Z(z)$ in (18) by substituting $p_2 = \mu_1 \rho_1$, $z = \frac{\Lambda}{\rho_1 \phi_i}$ and by expanding the modified Bessel function using [12, Eq. 8.446, 8.447.3] with the even property of modified bessel function $K_v(z) = K_{-v}(z)$ and using Maclaurin Series to expand the $e^{\frac{z}{\rho_1 \phi_i}}$. Then substituting the corresponding $\omega_{ij}$ and $\omega_{u,v}$ values, it is observed that $z^n; n < n_T$ terms sum up to zero for $n_T < n_R$ case, $z^n; n < n_R$ terms sum up to zero for $n_R < n_T$ case and similarly for $n_R = n_T = n_{Eq}$. Therefore $F_Z(z)$ can be derived avoiding lower order terms as,

$$F_Z(z) = \left\{ \begin{array}{ll}
\frac{\beta_N z^{n_T}}{n_T} + o(z^{n_T+1}), & n_T < n_R \\
\frac{\beta_{n_T-n_{Eq}}}{n_T-n_{Eq}} + o(z^{n_{Eq}+1}), & n_T = n_{Eq} = n_{Eq} \\
\frac{\beta_N z^{n_R}}{n_R} + o(z^{n_R+1}), & n_T > n_R
\end{array} \right.$$ (25)

where, $o(x) = \lim_{x \to 0} \frac{f(x)}{x} = 0$ and,

$$\beta_N = \frac{n_T}{\rho_1} \sum_{i=1}^t \sum_{j=1}^{\nu_i} \sum_{u=1}^{r} \sum_{v=1}^{\eta_u} \sum_{p=0}^{n_T} \frac{\omega_{i,j,u,v}}{(\Gamma(p))^p} \left( \frac{1}{\phi_i} \right)^N \text{Ei} \left( -\frac{p}{\rho_1} \right)$$ (26)

for $v + n - p > 0$,

$$\Theta = \sum_{k=0}^{n_T-n_{Eq}+1} \left( \frac{1}{\rho_1 \sigma_u} \right)^k \frac{1}{\Gamma(n_T-n_{Eq})} \left( \frac{\Gamma(p)^{n_T-n_{Eq}}}{\Gamma(n_T-n_{Eq})} \right)$$

$$+ \sum_{k=0}^{n_T-n_R} \left( \frac{D}{\mu_1 \sigma_u} \right)^k \frac{1}{\Gamma(n_T-n_R)} \left( \frac{\Gamma(p)^{n_T-n_R}}{\Gamma(n_T-n_R)} \right)$$

for $v + n - p = 0$
for $v + n - p < 0$, 

$$\Theta = \sum_{k=0}^{p-v-n-1} \frac{(-1)^{N-v-n+1}(p - v - n - k)!}{k!(N - v - n - k)!}$$

$$\times \left( \frac{D}{\mu \sigma_u} \right)^{v+k} + \sum_{k=0}^{n-p} \frac{(-1)^{N-v-n-k} \Upsilon_{p-v-n} \left( \frac{D}{\mu \sigma_u} \right)^{p-n+k}}{k!(p - n - v + k)!(N - p - k)!}$$

where,

$$\Upsilon_x = \ln \left( \frac{D_z}{\mu \sigma_u \phi_z} \right) - \psi(k + 1) - \psi(x + k + 1) \quad (27)$$

$\psi(x)$ is the Euler psi function defined in [12, Eq.8.36]. From the above expression we can say that the diversity order $= \min[n_T, n_R]$. Now we simplify this general solution to some special cases as follows.

**Exponential Correlation:** With the substitution of desired values of $\omega_{i,j}$ of (12), the general expression, (26) can be reduced to the exponential correlation as follows,

$$\beta_N = N \sum_{i=1}^{n_T} \sum_{u=1}^{n_R} \omega_{i,u} \frac{(-1)^{N+1}}{N!} + \sum_{k=0}^{N-1} \left( \frac{D}{\mu \sigma_u} \right)^{k+1}$$

$$\times \frac{(-1)^{N-k} \Upsilon_1}{k!(N - k - 1)!(k+1)!} \quad (28)$$

Here we can clearly see that diversity order $= \min[n_T, n_R]$. In [7], they considered an approximated modified Bessel function expansion which lead them to omit an important part of equation and hence to decide diversity order to be equal to $n_T$.

**Uniform Correlation:** With the substitution of suitable values of $\omega_{i,j}$ of (13), the general expression, (26) can be reduced to the uniform correlation but further simplification is not possible.

**Independent:** $n_T < n_R$: With the substitution of $\omega_{i,j}$ of the independent case mentioned in (14), the general solution (26) can be simplified as,

$$\beta_{n_T} = n_T \sum_{n=0}^{n_T} \frac{(n_R - n - 1)!}{(n_T - n)!} \left( \frac{D}{\mu} \right)^n \quad (29)$$

$n_T > n_R$: By substituting the of $\omega_{i,j}$ of independent case mentioned in (14), the general solution (26) is reduced to,

$$\beta_{n_R} = n_R \frac{(n_T - n_R - 1)!}{n_R!(n_T - n_R)!} \left( \frac{D}{\mu} \right)^{n_R} \quad (30)$$

$n_T = n_R = n_{Eq}$: Following the same procedure as above, (26) can be simplified as,

$$\beta_{n_{Eq}} = n_{Eq} \left[ \sum_{n=0}^{n_{Eq}-1} \frac{(D/\mu)^n}{1!(n_{Eq} - n)!} \right]$$

$$\times \left( 1 - n_{Eq} \left( \ln \left( \frac{D_z}{\mu} \right) - 2\psi(1) \right) \right) \left( \frac{D}{\mu} \right)^{n_{Eq}} \quad (31)$$

**B. Fixed Gain Relaying for TAS/MRC**

For high SNR we can express fixed gain mentioned in (21) and (22) as, $C_T = \mu x_T D_T$. We can rewrite (24) by substituting $\rho = \mu \rho_1, z = \frac{D_T}{\mu}$ and expanding the modified Bessel function and using Maclaurin Series series to expand $e^{-pz}$. Then by substituting the corresponding $\omega_{u,v}$ values, it is observed that $z^n; n < n_T$ terms sum up to zero for $n_T < n_R$ case, $z^n; n < n_R$ terms sum up to zero for $n_R < n_T$ case and similarly for $n_R = n_{Eq}$. Therefore $F_{Z}(z)$ can be derived avoiding lower order terms as,

$$F_{Z}(z) = \left\{ \begin{array}{ll}
\frac{\partial_n z^n}{n!} + o(z^n+1), & n_T < n_R \\
\frac{\partial^2_n z^n}{n!} + o(z^n+1), & n_T = n_R = n_{Eq} \\
\frac{\partial^3_n z^n}{n!} + o(z^n+1), & n_T > n_R
\end{array} \right. \quad (32)$$

where,

$$\vartheta = N \sum_{p=1}^{n_T} \left( \frac{n_T}{p} \right) \Gamma(v) \sum_{u=1}^{n_R} \omega_{u,v} \sum_{v=1}^{n_T} \frac{(v-k-1)!D_T^k}{k!(N-k)!} \left( \frac{D}{\mu \sigma_u} \right)^{v+k}$$

$$\times \left( -1 \right)^{N+p} \sum_{k=0}^{N-v} \frac{(-1)^{N-k+p+1} \Upsilon_{v}^{T}}{k!(v+k)!(N - v - k)!} \quad (33)$$

$$\Upsilon_x = \ln \left( \frac{p D_T z}{\mu \sigma_u} \right) - \psi(k + 1) - \psi(x + k + 1) \quad (34)$$

From (32) we can conclude that diversity order $= \min[n_T, n_R]$.

**V. NUMERICAL RESULTS AND DISCUSSION**

In this section we analyze and verify the presented theoretical results for fixed gain system with comparison of Monte Carlo simulations. In figures, $\rho$ represents the correlation parameter.

Fig.1 shows fixed gain outage probability for both analytical and high SNR. There is an improvement in the outage when the number of antennas increases. Increases of the number of antennas at S gives a higher rate of improvement than the increases in antennas in D. Moreover it shows that diversity order is equal to $\min[n_T, n_R]$.

Fig.2 shows the fixed gain outage probability for different correlation cases for both analytical and high SNR. We can clearly see that reduction in correlation improves the outage probability. While independent case has the highest improvement in the outage probability, the exponential correlation has a slight improvement in the outage probability than the uniform correlation for the same correlation parameter. Since we used the same number of antennas, we can see that the diversity order does not change with the form of correlation.

Fig.3 shows outage probability comparison between MRT/MRC and TAS/MRC fixed gain systems for $n_T = n_R = 3$ with the assumption that the S-R link is always independent. It shows better performance in MRT/MRC system compared to TAS/MRC system. Further, when correlation decreases both systems improve the outage probability.
correlation cases, Fixed gain

Fig. 1. Fixed gain outage probability for $\mu = 2, \Lambda = 0$ dB, exponential correlation with $\rho = 0.5$.

Fig. 2. Fixed gain outage probability for $\mu = 2, \Lambda = 0$ dB, for different correlation cases, Fixed gain $C_2$.

VI. CONCLUSION

We have analyzed the performance of two hop AF fixed gain MRT/MRC and TAS/MRC relay systems for a general antenna correlation model. We derived the closed form outage probability for two systems and also investigated the asymptotic analysis to obtain the insight and provided Monte Carlo simulations to verify the results. We showed that the diversity order of the system to be always equal to $\min\{n_T, n_R\}$ and evaluated the impact of number of antennas and different correlation models to the system performance. Moreover our analysis shows that MRT/MRC outperforms TAS/MRC system.

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