Topological Majorana States in Zigzag Chains of Plasmonic Nanoparticles

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ABSTRACT: We propose a simple realization of topological edge states in zigzag chains of plasmonic nanoparticles, mimicking the Kitaev model of Majorana fermions. We demonstrate the one-to-one correspondence between the coupled dipole equations in the zigzag plasmonic chain and the Bogoliubov-de-Gennes equations for the quantum wire on top of superconductor and support the analytical theory by the full-wave electromagnetic simulations. We reveal that localized plasmons can be excited selectively at both edges of the zigzag chain of plasmonic nanoparticles depending on the incident plane wave polarization.

KEYWORDS: topological insulators, edge states, plasmonics, Majorana fermions

The study of topological insulators is one of the most rapidly developing areas of condensed matter physics. Such structures possess bandgaps in the bulk and special edge/surface states inside the gap. Contrary to traditional Tamm states, these edge states are topologically protected. This means that they are stable against a wide class of perturbations that keep a general symmetry of the system, for example, time-reversal symmetry can be probed by analyzing the scattering spectra. This means that they are potential candidates to realize robust qubits. Here, we uncover additional symmetries of the zigzag chains, stemming from the vector nature of the dipole–dipole interaction, and show that plasmonic structures are promising candidates to mimic fascinating Majorana physics.
contains only three eigenfrequencies, along the wire, while the original Kitaev eigenmodes are homogeneous system of equations for coupled plasmonic particular case when the properties of the system eq 5 can be understood in the special case, the role of particle-hole excitations is taken by the two electron annihilation (creation) operators for the site c

\[ \left( \omega - \omega_0 \right) p_j = t (u_{j+1} + u_{j-1}) + \Delta (v_{j+1} - v_{j-1}) \]

\[ \left( \omega - \omega_0 \right) v_j = -t (v_{j+1} + v_{j-1}) - \Delta (u_{j+1} - u_{j-1}) \]

where

\[ t = \frac{3 \Gamma_0}{2 L^3}, \quad \Delta = -\frac{\Gamma_1}{2 L^3} \]

Equations 5 have a striking similarity to the Bogoliubov-de-Gennes equations of Kitaev’s model, describing the quasi-particles in the quantum wire lying on top of the superconductor. In the original Kitaev’s Hamiltonian \( H = (1/2) \sum \left( t c_j^\dagger c_{j+1} + \Delta c_j^\dagger c_{j+1} \right) + h.c. \), the constant \( t \) is the transfer integral describing the motion along the wire, while \( \Delta \) is the superconducting gap, and \( c_j^\dagger \) are the electron annihilation (creation) operators for the site \( j \). In our case, the role of particle-hole excitations is taken by the two polarization degrees of freedom. Similar models also appear for coupled waveguides chains of defects inside the photonic crystals, and conjugated polymer chains. The general properties of the system eq 5 can be understood in the special particular case when \( \Delta = -t \). For such parameters, the spectrum contains only three eigenfrequencies, \( \omega_0 \pm 2t \) and \( \omega_0 \). All the 2N corresponding eigenmodes for the zigzag with \( N \) disks are schematically illustrated in Figure 2. The mode shown in Figure 2a, Figure 2d has the frequency

\[ \omega_{\text{bright}} = \omega_0 + 2t \]

and the total degeneracy \( N - 1 \). Because the dipole momenta on the neighboring disks are parallel to each other, this mode is bright, that is, it can be excited by a normally incident plane wave. Conversely, the mode in Figure 2c,e is dark, has the frequency \( \omega_{\text{dark}} = \omega_0 + 2t \), and the same total degeneracy \( N - 1 \). The remaining eigenmode is shown in Figure 2b. It has the frequency corresponding to the single-particle resonance. This is an edge eigenmode with nonzero momenta only at the first and last disks. The mode is doubly degenerate, the \( y \)-polarized dipole momentum corresponds to the first disk, \( x \)-polarized to the last one. Indeed, while for \( j = 2, \ldots, N - 1 \) the \( y \)-polarized dipole momentum is coupled to one of the modes at Figure 2a or c, the \( y \)- and \( x \)-polarized modes of the first and last particles remain uncoupled and, hence, are localized.

It has been shown that the structure of the eigenmodes remains qualitatively the same for arbitrary values of \( \Delta \). Particularly, the degenerate bright and dark modes at \( \omega = \omega_0 \pm 2t \) evolve into the two allowed bands with the dispersion \( \omega(k) = \omega_0 \pm 2 (t^2 \cos^2 K + \Delta^2 \sin^2 K)^{1/2} \). Here, \( K \) is the Bloch wavevector, \( u_j = e^{iK} u_j \). The frequency of the double-degenerate state 8 is equal to \( \omega_0 \) independent of \( \Delta \). For \( \Delta |t| \neq 0 \) this state is exponentially localized at the structure edges, the case when \( \Delta = -t/3 \) corresponds to the edge states decaying as \( |p| \alpha \propto 1/2L^3 \).

Appearance of the edge states may be understood as the topological transition taking place when the chain geometry changes from a line to a zigzag. Such zigzag distortion is a particular kind of the Peierls phase transition.

The simple analysis above suggests the presence of localized polarization-degenerate plasmonic eigenmodes in the zigzag chain at the frequency corresponding to the resonance of the single disk. To confirm this hypothesis we numerically simulate the electric field induced in the structure by the plane wave \( E_0 e^{i\omega t} \), propagating along \( z \) direction at different frequencies. The calculation has been performed using the CST Microwave Studio software for touching nanodisks \( L = 2R \). Electric field is linearly polarized, \( E_0 = \cos \phi \hat{x} + \sin \phi \hat{y} \) with \( \phi = 45^\circ \). The results of calculation are demonstrated in Figure 3. Thick/black curve in the panel (a) shows numerically calculated extinction cross section for the zigzag chain with 11 nanodisks. Thin/red curve presents the cross section for a single disk, which has the dipole resonance at the energy \( E \approx 2.1 \) eV. The cross section for the zigzag chain has a complex multipeak structure. Main peak in the spectrum is blue-shifted from the single disk resonance, in

**Figure 1.** Schematics of a zigzag array of plasmonic nanoparticles.

\[ \hat{G}(r) = \frac{3\pi}{r} \]  

We also focus on the normal wave incidence, when the field propagates along \( z \) direction. In this case, all the excited dipole momenta lie in the \( xy \) plane and can be described in the following basis

\[ p_j = u_j e_j(j) + u_j e_{\text{c}}(j) \]

\[ e_j(j) = -\cos \phi_j \hat{x} + \sin \phi_j \hat{y}, \quad e_{\text{c}}(j) = -\sin \phi_j \hat{x} - \cos \phi_j \hat{y} \]

\[ \phi_j = \pi j/2 \]

We also use the pole approximation for the particle polarizability \( \alpha_0 = \Gamma_0/(\omega_0 - \omega) \), valid near the plasmon resonance frequency \( \omega_\rho \). Substituting 4 and 3 into 2, we obtain the homogeneous system of equations for coupled plasmonic eigenmodes

\[ (\omega - \omega_0) u_j = t(u_{j+1} + u_{j-1}) + \Delta (v_{j+1} - v_{j-1}) \]

\[ (\omega - \omega_0) v_j = -t(v_{j+1} + v_{j-1}) - \Delta (u_{j+1} - u_{j-1}) \]

where

\[ t = \frac{3 \Gamma_0}{2L^3}, \quad \Delta = -\frac{\Gamma_1}{2L^3} \]

**Figure 2.** Illustration of the dipole eigenmodes of eq 5 for \( \Delta = -t \).

\[ \omega_{\text{loc}} = \omega_0 \]
agreement with eq 7 for the bright mode eigenfrequency. The results of the analysis of the electric field distribution, excited inside the cluster at the different frequencies, are presented in the panels (b–e). Panels (b)–(d) show the false color maps of the electric field in the xy plane, while panel (e) shows the frequency dependence of the induced electric field at the center of each particle. At low (b) or large (d) frequencies, the induced electric field is relatively homogeneously distributed inside the cluster. However, at the central frequency (panel c) the field has strong maxima at the first and last particles. This is also demonstrated by Figure 3e. Black and dark blue curves, corresponding to \( j = 1 \) and \( j = 11 \) have maxima at the single particle resonance \( E \approx 2.1 \text{ eV} \), which is a direct manifestation of the excitation of localized states.

Next, we examine the polarization dependence of the structure response. Figure 4 presents the spatial distribution of the polarization, excited at \( E = 2.1 \text{ eV} \) for three different values of the angle \( \phi = 0, 45^\circ, \) and \( 90^\circ \) between the electric field direction and the x axis. Panel (a) shows the polarization dependence of the extinction cross section. The black line presents the extinction cross section for \( \phi = 45^\circ \), green curve shows the result for \( \phi = 0^\circ \). Since the structure has mirror plane \( x = -y \), the cross section for \( \phi = 0^\circ \) and \( 90^\circ \) is the same. Panels (b–e) illustrate the polarization dependence of the induced electric field pattern.

Clearly, the direction of the incident wave polarization controls the distribution of the field at the structure edges. Particularly, for the \( x \) polarization \( (\phi = 0) \) the field is localized at the last particle, \( j = 11 \), while for the \( y \) polarization \( (\phi = 90^\circ) \) the field is localized at the first particle, in agreement with Figure 2b. The effect is clearly manifested both in the color maps of the field (panels b and d) and in the spatial distribution of the field in the particle centers (green and red curves in panel e). The dipole momenta of the particles, extracted from the electric field distribution, are directed oppositely to the electric field (cyan arrows in panels b–d). The case when \( \phi = 45^\circ \) corresponds to equal amplitudes for both particles (panel c and black line in panel e). Hence, the presence of the polarization-degenerate eigenmode at \( E \approx 2.1 \text{ eV} \) allows selective excitation of the first or the last disk. This is an optical analogue of a qubit, based on Majorana fermions, proposed by Kitaev. Here, the polarization switching has a purely classical origin. The truly quantum regime, where the bosonic properties of the edge states become manifest, may demonstrate even richer physics. For instance, similar systems with tunneling-coupled cavities are already used for polarization-entangled photon pair generation and are proposed for quantum computing devices with multipartite entanglement.

It is also instructive to examine the degree of the states localization as function of the number of particles in the chain. The results for \( N = 3, ... , 6, 10, \) and \( 11 \) are shown in Figure 5. This figure demonstrates that the edge states are manifested already for the smallest possible structure with defined edges, that is, for the zigzag chain with \( N = 3 \) particles. Importantly, the localization takes place at the same frequency independent of the structure
length. Localization has a compact character: its strength slowly increases when the number of particles grows but saturates already at $N \approx 5$. The maximum edge-to-center ratio of the field intensities, $|E_1|^2/|E_0|^2$, is about three. Thus, the localization is weaker than exponential, contrary to the traditional Kitaev model. The reason is that the direct correspondence between the Kitaev model and the considered model is valid only for the nearest-neighbor dipole–dipole interactions. The long-ranged electromagnetic coupling between the nanodisks partially suppresses the localization.

Figure 6 examines the robustness of the edge states against the disorder. Panels (a) and (b) are calculated for the ideal structure, where all the disk diameters are equal to $\bar{D} = 50$ nm and the center-to-center distance is $\bar{L} = 60$ nm. Results are qualitatively the same as in Figure 3: at the single disk resonance energy $E \approx 2.15$ eV mainly the first and last particles are excited. We include the disorder both in the positions and in the diameter of the disks. This leads to the inhomogeneous broadening of the dipole resonances and to the modification of the coupling between the disks due to the random distortion of the zigzag chain. Particularly, we assume that the in-plane disk center coordinates $x$ and $y$ and the disk diameter are independent random Gaussian variables, characterized with the same dispersion $\sigma$. Figure 6c–f is calculated for two realizations of the disorder with different strengths, $\sigma = 0.1\bar{D} = 5$ nm and $\sigma = 0.2\bar{D} = 10$ nm. The impact of the disorder on the excited electric field distribution is significant already for $\sigma = 0.1\bar{D}$, see Figure 6a,c. However, the edge states still survive in this case. This is illustrated by Figure 6d, which demonstrates preferential excitation of the edges of the disordered chain at the frequency of the single disk resonance of the ideal structure. Edge states are completely destroyed only for strong disorder, $\sigma = 0.2\bar{D}$, when the chain mode structure is changed completely, see Figure 6e,f.

In summary, we have proposed a simple plasmonic analogue of the concept of Majorana topological edge states in the zigzag chain of metallic nanodisks. We have demonstrated an exact correspondence between Kitaev’s model of a finite-extent quantum wire over superconductor to the model of coupled electric dipoles in the plasmonic chain. This mapping is valid for the dipole–dipole interactions between the nearest neighbors of the chain. The edge states have been shown to be robust against distant interactions and disorder. We have demonstrated the possibility to excite selectively two edges of the zigzag chain by changing the direction of the linear polarization of the incident light. We believe that the study of the topological properties of the polarization-entangled eigenmodes suggests a new way for engineering the properties of plasmonic clusters for novel applications in nanophotonics.

**REFERENCES**