PHYSICAL REVIEW D **70,** 113004 (2004)

NuTeV anomaly, lepton universality, and nonuniversal neutrino-gauge couplings

Will Loinaz*

Department of Physics, Amherst College, Amherst, Massachusetts 01002, USA
Department of Physics, University of Massachusetts, Amherst, Massachusetts 01003, USA

Naotoshi Okamura†

Korea Institute for Advanced Study, 207-43 Cheongnyangni 2-dong, Dongdaemun-gu, Seoul 130-722, Korea

Saifuddin Rayyan‡ and Tatsu Takeuchi§

Institute for Particle Physics and Astrophysics, Physics Department, Virginia Tech, Blacksburg, Virginia 24061, USA

L. C. R. Wijewardhana‖

Department of Physics, University of Cincinnati, Cincinnati, Ohio 45221-0011, USA

(Received 31 March 2004; published 2 December 2004)

In previous studies we found that models with flavor-universal suppression of the neutrino-gauge couplings are compatible with NuTeV and Z-pole data. In this paper we expand our analysis to obtain constraints on flavor-dependent coupling suppression by including lepton universality data from W, τ, π and K decays in fits to model parameters. We find that the data are consistent with a variety of patterns of coupling suppression. In particular, in scenarios in which the suppression arises from the mixing of light neutrinos with heavy gauge singlet states (neutrissimos), we find patterns of flavor-dependent coupling suppression which are also consistent with constraints from $\mu \to e\gamma$.

DOI: 10.1103/PhysRevD.70.113004 PACS numbers: 14.60.St, 12.15.Lk, 13.15.+g, 14.60.Pq

I. INTRODUCTION

Recent analysis of ν_μ ($\bar{\nu}_\mu$) scattering data from the NuTeV experiment at Fermilab [1] indicates a value of the effective neutrino-quark coupling parameter g_L^2 which deviates by 3σ from the Standard Model prediction (based on a global fit using non-NuTeV data). The significance of the NuTeV result remains controversial [2], and a critical examination of the initial analysis is ongoing. Several groups are evaluating potential theoretical uncertainties arising from purely Standard Model physics which might be comparable to or larger than the quoted experimental uncertainty of the NuTeV result. Candidate sources of large theoretical uncertainty include next-to-leading-order (NLO) QCD corrections [3], NLO electroweak corrections [4], and parton distribution functions (especially as involves assumptions about sea-quark asymmetries) [5]. The effect of the first has been estimated to be comparable in size to the NuTeV experimental uncertainty, while the latter two might give rise to effects comparable in size to the full NuTeV discrepancy with the Standard Model. Elucidation of the actual impact of these effects on the NuTeV result awaits a reanalysis of the NuTeV data. However, it remains a distinct possibility that the discrepancy with the Standard Model prediction is genuine and that its resolution lies in physics beyond the Standard Model [6]. It is this possibility that we investigate here.

In a previous paper [7], we demonstrated that the Z-pole data from e^+e^- colliders [8] and the ν_μ ($\bar{\nu}_\mu$) scattering data from NuTeV [1] are compatible if (1) the Higgs boson is heavy and (2) the $Z\nu_\ell\nu_\ell$ and $W\ell\nu_\ell$ ($\ell = e, \mu, \tau$) couplings are suppressed by a factors $(1-\varepsilon_\ell)$ and $(1-\varepsilon_\ell/2)$, respectively. We also showed that such suppressions could arise from neutrinos mixing with heavy gauge singlet (neutrissimo) states [9–12].

In Ref. [7], it was assumed that the suppression parameters were flavor-universal: $\varepsilon_e = \varepsilon_\mu = \varepsilon_\tau \equiv \varepsilon$. The value of ε required to fit the data was

$$\varepsilon = 0.0030 \pm 0.0010. \tag{1}$$

However, in seesaw models [13] of neutrino masses and mixings such a large universal ε implies a prohibitively large rate of $\mu \to e\gamma$ [10–12]. To bring the models into agreement with experiment the assumption of universality must be relaxed; either ϵ_e or ϵ_μ, but not both, must be strongly suppressed.[1] Further, in most models flavor-universal suppressions require considerable fine tuning. It is thus natural to ask what patterns of flavor nonuniversal suppressions are consistent with the data. If the suppression parameters can be flavor-dependent, one must also ask whether the preferred values of the ε_ℓ are all positive, i.e., are all the neutrino-gauge couplings suppressed? Negative ε_ℓ indicates an enhancement of the

*Electronic address: loinaz@alumni.princeton.edu
†Electronic address: nokamura@kias.re.kr
‡Electronic address: srayyan@vt.edu
§Electronic address: takeuchi@vt.edu
‖Electronic address: rohana@physics.uc.edu

[1] Note that this restriction applies only to seesaw models with equal numbers of sterile and active neutrinos.

$W\ell\nu_\ell$ and $Z\nu_\ell\nu_\ell$ couplings which cannot be arranged via neutrino-mixing.

In addition to the Z-pole and NuTeV data, there is a wealth of experimental data bounding lepton universality violation in the charged channel from W, π, K, and τ-decays [14]. In the following, we analyze the constraints that Z-pole and NuTeV data and the lepton universality bounds impose on neutrino-mixing models by fitting the data with flavor-dependent suppression parameters ε_ℓ ($\ell = e, \mu, \tau$) along with the S, T, and U oblique correction parameters [15]. We perform fits in which all six parameters float independently, and we also fit to models in which one or more of the ε_ℓ are assumed to be strongly suppressed. As in the flavor-universal case, the data require a negative T parameter and a positive U parameter. However, we find that the data are consistent with a variety of patterns of suppression parameters, including patterns compatible with $\mu \to e\gamma$ data.

II. CONSTRAINTS ON LEPTON UNIVERSALITY

Here, we survey current experimental constraints on lepton universality. For a comprehensive review on the subject, see Ref. [16]. We parametrize the couplings of the $W^\pm$'s with the leptons as

$$\mathcal{L} = \sum_{\ell=e,\mu,\tau} \frac{g_\ell}{\sqrt{2}} W_\mu^+ \bar{\nu}_\ell \gamma^\mu \left(\frac{1-\gamma_5}{2}\right)\ell^- + \text{h.c.} \qquad (2)$$

The Standard Model assumes $g_e = g_\mu = g_\tau = g$. Although experimental limits on the ratios g_μ/g_e, g_τ/g_μ, and g_τ/g_e have been calculated and tabulated as recently as fall 2002 by Pich [17], we repeat the exercise here to incorporate more recent data and to obtain the correlations among the limits necessary for our analysis.

A. W-decay

The decay width of the W at tree level is

$$\Gamma(W \to \ell\bar{\nu}_\ell) = \frac{g_\ell^2 M_W}{48\pi}\left(1 - \frac{m_\ell^2}{M_W^2}\right)^2\left(1 + \frac{m_\ell^2}{2M_W^2}\right). \qquad (3)$$

The branching fractions of the W into the three lepton generations have been measured at LEP-II to be (Ref. [8], page 74)

$$\begin{aligned} B(W \to e\bar{\nu}_e) &= 10.59 \pm 0.17\%, \\ B(W \to \mu\bar{\nu}_\mu) &= 10.55 \pm 0.16\%, \\ B(W \to \tau\bar{\nu}_\tau) &= 11.20 \pm 0.22\%, \end{aligned} \qquad (4)$$

with correlations shown in Table I. From this data, we find (Ref. [8], page 73)

$$\begin{aligned} B(W \to \mu\bar{\nu}_\mu)/B(W \to e\bar{\nu}_e) &= 0.997 \pm 0.021, \\ B(W \to \tau\bar{\nu}_\tau)/B(W \to e\bar{\nu}_e) &= 1.058 \pm 0.029, \end{aligned} \qquad (5)$$

TABLE I. Correlations among the W branching fractions measured at LEP-II (Ref. [8], page 182).

	$B(W \to e\bar{\nu}_e)$	$B(W \to \mu\bar{\nu}_\mu)$	$B(W \to \tau\bar{\nu}_\tau)$
$B(W \to e\bar{\nu}_e)$	1.000	0.092	−0.196
$B(W \to \mu\bar{\nu}_\mu)$		1.000	−0.148
$B(W \to \tau\bar{\nu}_\tau)$			1.000

with a correlation of +0.44 between the two ratios. Using Eq. (3), this translates into

$$\begin{aligned} (g_\mu/g_e)_W &= 0.999 \pm 0.011, \\ (g_\tau/g_e)_W &= 1.029 \pm 0.014, \end{aligned} \qquad (6)$$

with a correlation of +0.44. The central values have shifted slightly from Ref. [17] due to the update of the W branching fractions from LEP-II.

B. τ and μ decay

The decay widths of the τ and μ into lighter leptons, including radiative corrections [18,19], are:

$$\begin{aligned} \Gamma[\tau \to \mu\bar{\nu}_\mu\nu_\tau(\gamma)] &= \frac{g_\tau^2 g_\mu^2}{64M_W^4}\frac{m_\tau^5}{96\pi^3} f\left(\frac{m_\mu^2}{m_\tau^2}\right)\delta_W^\tau \delta_\gamma^\tau, \\ \Gamma[\tau \to e\bar{\nu}_e\nu_\tau(\gamma)] &= \frac{g_\tau^2 g_e^2}{64M_W^4}\frac{m_\tau^5}{96\pi^3} f\left(\frac{m_e^2}{m_\tau^2}\right)\delta_W^\tau \delta_\gamma^\tau, \\ \Gamma[\mu \to e\bar{\nu}_e\nu_\mu(\gamma)] &= \frac{g_\mu^2 g_e^2}{64M_W^4}\frac{m_\mu^5}{96\pi^3} f\left(\frac{m_e^2}{m_\mu^2}\right)\delta_W^\mu \delta_\gamma^\mu = \frac{1}{\tau_\mu}, \end{aligned} \qquad (7)$$

in which $f(x)$ is the phase space factor

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x, \qquad (8)$$

δ_W^ℓ is the W propagator correction

$$\delta_W^\ell = \left(1 + \frac{3}{5}\frac{m_\ell^2}{M_W^2}\right), \qquad (9)$$

δ_γ^ℓ is the radiative correction from photons

$$\delta_\gamma^\ell = 1 + \frac{\alpha(m_\ell)}{2\pi}\left(\frac{25}{4} - \pi^2\right), \qquad (10)$$

and the values of the running QED coupling constant at relevant energies are [19]

$$\begin{aligned} \alpha^{-1}(m_\mu) &= \alpha^{-1} - \frac{2}{3\pi}\ln\frac{m_\mu}{m_e} + \frac{1}{6\pi} \approx 136.0, \\ \alpha^{-1}(m_\tau) &\approx 133.3 \end{aligned} \qquad (11)$$

The numerical values of these corrections are shown in Table II. The ratios of the coupling constants can be extracted using the relations

TABLE II. The corrections to the leptonic decay widths of the τ and μ.

	Phase space	W propagator	Photon
$\Gamma(\tau \to \mu\bar{\nu}_\mu\nu_\tau)$	$f(m_\mu^2/m_\tau^2) = 0.9726$	$\delta_W^\tau = 1.0003$	$\delta_\gamma^\tau = 0.9957$
$\Gamma(\tau \to e\bar{\nu}_e\nu_\tau)$	$f(m_e^2/m_\tau^2) = 1.0000$		
$\Gamma(\mu \to e\bar{\nu}_e\nu_\mu)$	$f(m_e^2/m_\mu^2) = 0.9998$	$\delta_W^\mu = 1.0000$	$\delta_\gamma^\mu = 0.9958$

$$\begin{aligned}\frac{\Gamma[\tau\to\mu\bar{\nu}_\mu\nu_\tau(\gamma)]}{\Gamma[\tau\to e\bar{\nu}_e\nu_\tau(\gamma)]} &= \frac{B[\tau\to\mu\bar{\nu}_\mu\nu_\tau(\gamma)]}{B[\tau\to e\bar{\nu}_e\nu_\tau(\gamma)]} = \frac{g_\mu^2}{g_e^2}\frac{f(m_\mu^2/m_\tau^2)}{f(m_e^2/m_\tau^2)},\\ \frac{\Gamma[\tau\to\mu\bar{\nu}_\mu\nu_\tau(\gamma)]}{\Gamma[\mu\to e\bar{\nu}_e\nu_\mu(\gamma)]} &= \frac{\tau_\mu}{\tau_\tau}B[\tau\to\mu\bar{\nu}_\mu\nu_\tau(\gamma)] = \frac{g_\tau^2}{g_e^2}\frac{m_\tau^5}{m_\mu^5}\frac{f(m_\mu^2/m_\tau^2)}{f(m_e^2/m_\mu^2)}\frac{\delta_W^\tau}{\delta_W^\mu}\frac{\delta_\gamma^\tau}{\delta_\gamma^\mu}.\end{aligned} \tag{12}$$

The latest world averages for the quantities appearing in these equations are listed in Table III, which yield

$$\begin{aligned}(g_\mu/g_e)_\tau &= 0.9999 \pm 0.0021,\\ (g_\tau/g_e)_{\tau\mu} &= 1.0004 \pm 0.0022,\end{aligned} \tag{13}$$

with a correlation of 0.51 due to the inputs $B[\tau\to\mu\bar{\nu}_\mu\nu_\tau(\gamma)]$ and m_τ common to both ratios.

C. Pion and τ decay

At tree level, the widths of charged π-decay into leptons are

$$\begin{aligned}\Gamma(\pi\to e\bar{\nu}_e) &= \frac{g_e^2 g_{ud}^2}{256\pi}\frac{f_\pi^2}{M_W^4}m_e^2 m_\pi\left(1-\frac{m_e^2}{m_\pi^2}\right)^2,\\ \Gamma(\pi\to \mu\bar{\nu}_\mu) &= \frac{g_\mu^2 g_{ud}^2}{256\pi}\frac{f_\pi^2}{M_W^4}m_\mu^2 m_\pi\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2,\end{aligned} \tag{14}$$

while that of τ-decay into $\pi\nu_\tau$ is

$$\Gamma(\tau\to\pi\nu_\tau) = \frac{g_\tau^2 g_{ud}^2}{512\pi}\frac{f_\pi^2}{M_W^4}m_\tau^3\left(1-\frac{m_\pi^2}{m_\tau^2}\right)^2, \tag{15}$$

where $g_{ud} = g|V_{ud}|$, and the pion decay constant f_π is normalized as (Ref. [23], page 439)

$$\langle 0|\bar{u}\gamma_\mu\gamma_5 d(0)|\pi^-(\mathbf{q})\rangle = iq_\mu f_\pi. \tag{16}$$

Taking ratios, we find

$$\begin{aligned}R^0_{e/\mu} &\equiv \frac{\Gamma(\pi\to e\bar{\nu}_e)}{\Gamma(\pi\to\mu\bar{\nu}_\mu)} = \frac{B(\pi\to e\bar{\nu}_e)}{B(\pi\to\mu\bar{\nu}_\mu)} = \frac{g_e^2}{g_\mu^2}\frac{m_e^2}{m_\mu^2}\frac{(1-m_e^2/m_\pi^2)^2}{(1-m_\mu^2/m_\pi^2)^2},\\ R^0_{\tau/\pi} &\equiv \frac{\Gamma(\tau\to\pi\nu_\tau)}{\Gamma(\pi\to\mu\bar{\nu}_\mu)} = \frac{\tau_\pi}{\tau_\tau}\frac{B(\tau\to\pi\nu_\tau)}{B(\pi\to\mu\bar{\nu}_\mu)} = \frac{g_\tau^2}{g_\mu^2}\frac{m_\tau^3}{2m_\mu^2 m_\pi}\frac{(1-m_\pi^2/m_\tau^2)^2}{(1-m_\mu^2/m_\pi^2)^2}.\end{aligned} \tag{17}$$

Radiative corrections to these relations have been calcu-

TABLE III. The world averages of masses, life times, and branching fractions used in this analysis. The branching fractions subsume the decays with γ's.

Observable	World Average	Reference
m_e (MeV)	$0.510998918 \pm 0.000000044$	[20]
m_μ (MeV)	$105.6583692 \pm 0.0000094$	[20]
$\tau_\mu(s)$	$(2.19703 \pm 0.00004) \times 10^{-6}$	[20]
m_τ (MeV)	$1776.99^{+0.29}_{-0.26}$	[20]
$\tau_\tau(s)$	$(290.6 \pm 0.9) \times 10^{-15}$	[21] Fig. 1
$B(\tau\to e\bar{\nu}_e\nu_\tau)$	$17.823 \pm 0.051\%$	[22] Fig. 8
$B(\tau\to \mu\bar{\nu}_\mu\nu_\tau)$	$17.331 \pm 0.054\%$	[22] Fig. 9
$B(\tau\to\pi\nu_\tau)$	$10.975 \pm 0.065\%$	[20], [21] Fig. 3, [22] Table III
$B(\tau\to K\nu_\tau)$	$0.686 \pm 0.023\%$	[20]
m_π (MeV)	139.57018 ± 0.00035	[20]
$\tau_\pi(s)$	$(2.6033 \pm 0.0005) \times 10^{-8}$	[20]
$B(\pi\to\mu\bar{\nu}_\mu)$	$99.98770 \pm 0.00004\%$	[20]
$B(\pi\to e\bar{\nu}_e)$	$(1.230 \pm 0.004) \times 10^{-4}$	[20]
m_K (MeV)	493.677 ± 0.016	[20]
$\tau_K(s)$	$(1.2384 \pm 0.0024) \times 10^{-8}$	[20]
$B(K\to\mu\bar{\nu}_\mu)$	$63.43 \pm 0.17\%$	[20]

lated in Ref. [24] and modify them to

$$R_{e/\mu} = \frac{B[\pi \to e\bar{\nu}_e(\gamma)]}{B[\pi \to \mu\bar{\nu}_\mu(\gamma)]} = R^0_{e/\mu}(1 + \delta R_{e/\mu}),$$
$$R_{\tau/\pi} = \frac{\tau_\pi}{\tau_\tau}\frac{B[\tau \to \pi\nu_\tau(\gamma)]}{B[\pi \to \mu\bar{\nu}_\mu(\gamma)]} = R^0_{\tau/\pi}(1 + \delta R_{\tau/\pi}), \tag{18}$$

with

$$\delta R_{e/\mu} = -0.0374 \pm 0.0001,$$
$$\delta R_{\tau/\pi} = +0.0016^{+0.0009}_{-0.0014}. \tag{19}$$

The uncertainty in these corrections is due to the uncertainty from strong interaction effects. With these relations and the experimental data listed in Table III, we obtain

$$(g_\mu/g_e)_\pi = 1.0021 \pm 0.0016,$$
$$(g_\tau/g_\mu)_{\pi\tau} = 1.0030 \pm 0.0034. \tag{20}$$

The correlation between the two is virtually zero due to the accuracy of the common inputs m_μ, m_π, and $B(\pi \to \mu\bar{\nu}_\mu)$. There is a correlation of +0.33 between $(g_\tau/g_\mu)_{\pi\tau}$ and $(g_\tau/g_e)_{\tau\mu}$ of Eq. (13) arising from the common inputs τ_τ and m_τ. A few comments are in order:

(i) Our limit on $(g_\mu/g_e)_\pi$ differs from that of Pich and Silva [25] who use for the value of $B[\pi \to e\bar{\nu}_e(\gamma)]$ the weighted average of the results from TRIUMF [26] and PSI [27], $(1.2310 \pm 0.0037) \times 10^{-4}$. If we use this value instead of the average from the Review of Particle Properties [20] listed in Table III, we obtain $(g_\mu/g_e)_\pi = 1.0017 \pm 0.0015$ in agreement with Ref. [25].

(ii) The experimental value of $B(\tau \to \pi\nu_\tau)$ listed in Table III is the average of CLEO and the four LEP experiments. CLEO [28], OPAL [29], DELPHI [30], and L3 [31] report the semiexclusive branching fraction $B(\tau \to h\nu_\tau)$, where $h = \pi$ or K, as

$$\begin{aligned} &\text{CLEO}: B(\tau \to h\nu_\tau) = 11.52\% \pm 0.05\% \pm 0.12\% \\ &\text{OPAL}: \quad = 11.98\% \pm 0.13\% \pm 0.16\% \\ &\text{DELPHI}: \quad = 11.601\% \pm 0.120\% \pm 0.116\% \\ &\text{L3}: \quad = 12.09\% \pm 0.12\% \pm 0.10\%. \end{aligned} \tag{21}$$

The CLEO and OPAL values are published and used in the average of the Review of Particle Properties [20]. Adding the statistical and systematic errors in quadrature and taking the weighted average of these four numbers, we obtain

$$\text{average without ALEPH:}\quad B(\tau \to h\nu_\tau) = 11.752 \pm 0.079\%. \tag{22}$$

As noted by Gan [21], the agreement among these four measurements is poor: χ^2/d.o.f. = 9.9/3, where d.o.f. is degrees of freedom. This stands in stark contrast to the situation in fall 2002 when the agreement among CLEO and the four LEP experiments was much better (χ^2/d.o.f. = 5.09/4) [32]. The source of the difference is the new L3 value [31] which has a much higher central value and smaller error bar than before [34]. Subtracting the world average $B(\tau \to K\nu_\tau) = 0.686 \pm 0.023\%$ [20], we obtain

$$\text{average without ALEPH:}\quad B(\tau \to \pi\nu_\tau) = 11.066 \pm 0.082\%. \tag{23}$$

ALEPH ([22], Table 3) reports the value of the exclusive branching fraction $B(\tau \to \pi\nu_\tau)$ as

$$\text{ALEPH:}\quad B(\tau \to \pi\nu_\tau) = 10.828 \pm 0.070 \pm 0.078\%, \tag{24}$$

which does not agree particularly well with Eq. (23) either. Although this ALEPH value is excluded from the world average by Gan in Ref. [21] as preliminary, we include it in our analysis since it was included in the previous analysis by Pich [17] (with the caveat that it is subject to change). The weighted average with Eq. (23) is

$$\text{world average:}\quad B(\tau \to \pi\nu_\tau) = 10.975 \pm 0.065\%, \tag{25}$$

which is the value used to obtain Eq. (20). The associated χ^2/d.o.f. is 13.1/4, so is unimproved with the inclusion of the ALEPH result. If we exclude the ALEPH value and use Eq. (23) instead, we obtain $(g_\tau/g_\mu)_{\pi\tau} = 1.0072 \pm 0.0041$. Whether we choose to include or exclude the ALEPH value has little effect on the final outcome of our analysis. Therefore, we only present the result of the analysis with ALEPH included hereafter [33].

The current state of agreement among the data determining $B(\tau \to \pi\nu_\tau)$ is clearly unsatisfactory. Additional data, perhaps from new experiments at CLEO [35], are needed to provide a definitive value.

D. Kaon and τ decay

Paralleling the treatment of pion decays, we can extract g_τ/g_μ from kaon decays. The tree level decay widths involving kaons are

$$\Gamma(K \to \mu\bar{\nu}_\mu) = \frac{g_\mu^2 g_{us}^2}{256\pi}\frac{f_K^2}{M_W^4}m_\mu^2 m_K\left(1 - \frac{m_\mu^2}{m_K^2}\right)^2, \tag{26}$$

TABLE IV. Limits on lepton universality from various processes.

Processes	Constraint
$W \to e\bar{\nu}_e$	$(g_\mu/g_e)_W = 0.999 \pm 0.011$
$W \to \mu\bar{\nu}_\mu$	$(g_\tau/g_e)_W = 1.029 \pm 0.014$
$W \to \tau\bar{\nu}_\tau$	
$\mu \to e\bar{\nu}_e\nu_\mu$	$(g_\mu/g_e)_\tau = 0.9999 \pm 0.0021$
$\tau \to e\bar{\nu}_e\nu_\tau$	$(g_\tau/g_e)_{\tau\mu} = 1.0004 \pm 0.0022$
$\tau \to \mu\bar{\nu}_\mu\nu_\tau$	
$\pi \to \mu\bar{\nu}_\mu$	$(g_\mu/g_e)_\pi = 1.0021 \pm 0.0016$
$\pi \to e\bar{\nu}_e$	$(g_\tau/g_\mu)_{\pi\tau} = 1.0030 \pm 0.0034$
$\tau \to \pi\nu_\tau$	
$K \to \mu\bar{\nu}_\mu$	$(g_\tau/g_\mu)_{K\tau} = 0.979 \pm 0.017$
$\tau \to K\nu_\tau$	

and

$$\Gamma(\tau \to K\nu_\tau) = \frac{g_\tau^2 g_{us}^2}{512\pi} \frac{f_K^2}{M_W^4} m_\tau^3 \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2, \quad (27)$$

where $g_{us} = g|V_{us}|$, and the kaon decay constant f_K is normalized as (Ref. [23], page 439)

$$\langle 0|\bar{u}\gamma_\mu\gamma_5 s(0)|K^-(\mathbf{q})\rangle = iq_\mu f_K. \quad (28)$$

Taking the ratio yields

$$R^0_{\tau/K} \equiv \frac{\Gamma(\tau \to K\nu_\tau)}{\Gamma(\pi \to \mu\bar{\nu}_\mu)} = \frac{\tau_K}{\tau_\tau} \frac{B(\tau \to K\nu_\tau)}{B(K \to \mu\bar{\nu}_\mu)} = \frac{g_\tau^2}{g_\mu^2} \frac{m_\tau^3}{2m_\mu^2 m_K} \frac{(1 - m_K^2/m_\tau^2)^2}{(1 - m_\mu^2/m_K^2)^2}. \quad (29)$$

Radiative corrections modify this to

$$R_{\tau/K} = \frac{\tau_K}{\tau_\tau} \frac{B[\tau \to K\nu_\tau(\gamma)]}{B[K \to \mu\bar{\nu}_\mu(\gamma)]} = R^0_{\tau/K}(1 + \delta R_{\tau/K}), \quad (30)$$

with [24]

$$\delta R_{\tau/K} = +0.0090^{+0.0017}_{-0.0026}. \quad (31)$$

Using this relation and the data listed in Table III, we obtain

$$(g_\tau/g_\mu)_{K\tau} = 0.979 \pm 0.017, \quad (32)$$

which agrees with Ref. [17]. This has a correlation of +0.07 with $(g_\tau/g_e)_{\tau\mu}$ of Eq. (13), and a correlation of +0.04 with $(g_\tau/g_\mu)_{\pi\tau}$ of Eq. (20), arising from the common inputs τ_τ and m_τ.

We tabulate our results in Tables IV and V.

III. Z-POLE, NUTEV, AND W MASS DATA

For the Z-pole and NuTeV data, we use the same set as in Ref. [7], namely $\Gamma_{\rm lept}$, $\Gamma_{\rm inv}/\Gamma_{\rm lept}$, and $\sin^2\theta_{\rm eff}^{\rm lept}$ from e^+e^- colliders, g_L^2 and g_R^2 from NuTeV. Of these, only the value of $\sin^2\theta_{\rm eff}^{\rm lept}$ has been updated since our last

TABLE V. Correlations among the lepton universality constraints.

	$(g_\mu/g_e)_W$	$(g_\tau/g_e)_W$	$(g_\mu/g_e)_\tau$	$(g_\tau/g_e)_{\tau\mu}$	$(g_\mu/g_e)_\pi$	$(g_\tau/g_\mu)_{\pi\tau}$	$(g_\tau/g_\mu)_{K\tau}$
$(g_\mu/g_e)_W$	1.00	0.44					
$(g_\tau/g_e)_W$		1.00					
$(g_\mu/g_e)_\tau$			1.00	0.51	0.00	0.00	0.00
$(g_\tau/g_e)_{\tau\mu}$				1.00	0.00	0.33	0.07
$(g_\mu/g_e)_\pi$					1.00	0.00	0.00
$(g_\tau/g_\mu)_{\pi\tau}$						1.00	0.04
$(g_\tau/g_\mu)_{K\tau}$							1.00

TABLE VI. The observables used in this analysis in addition to the lepton universality data. The measured value of $\sin^2\theta_{\rm eff}^{\rm lept}$ and the W mass have been updated in Ref. [8] since the analysis in Ref. [7]. The SM predictions are ZFITTER [36] outputs with inputs of $M_{\rm top} = 178.0$ GeV [37], $M_{\rm Higgs} = 115$ GeV, $\alpha_s(M_Z) = 0.119$, and $\Delta\alpha_{\rm had}^{(5)} = 0.02755$ [39].

Observable	SM prediction	Measured Value	Reference
M_Z	Input	91.1875 ± 0.0021 GeV	[8] page 8
$\Gamma_{\rm lept}$	84.034 MeV	83.984 ± 0.086 MeV	[8] page 9
$\Gamma_{\rm inv}/\Gamma_{\rm lept}$	5.972	5.942 ± 0.016	[8] page 8
$\sin^2\theta_{\rm eff}^{\rm lept}$	0.23133	0.23150 ± 0.00016	[8] page 142
g_L^2	0.3040	0.3002 ± 0.0012	Average of [1,38]
g_R^2	0.0304	0.0310 ± 0.0010	Average of [1,38]
M_W	80.399 GeV	80.426 ± 0.034 GeV	[8] page 146

analysis in Ref. [7]. The W mass has also been updated by LEP-II. We list the values in Table VI. There is a correlation of 0.17 between $\Gamma_{\rm lept}$ and $\Gamma_{\rm inv}/\Gamma_{\rm lept}$; other correlations are negligible.

IV. THE CORRECTIONS

Suppression of the neutrino-gauge couplings modifies the relation between the Fermi constant G_F and the muon decay constant G_μ to

$$G_F = G_\mu\left(1 + \frac{\varepsilon_e + \varepsilon_\mu}{2}\right). \quad (33)$$

Since G_μ is used as an input in calculating the SM predictions, all observables whose SM prediction depends on G_F will receive this correction through G_F. Of the observables that measure the $Z\nu_\ell\nu_\ell$ and $W\ell\nu_\ell$ vertices directly, the Z invisible width is corrected by an additional factor of

$$1 - \frac{2}{3}(\varepsilon_e + \varepsilon_\mu + \varepsilon_\tau), \quad (34)$$

while the NuTeV parameters g_L^2 and g_R^2 receive an additional correction of $(1 - \varepsilon_\mu)$. The dependence of the observables on the oblique correction parameters S, T, and U can be found elsewhere [15].

Numerically, the observables are corrected as follows:

$$\begin{aligned}
\frac{\Gamma_{\rm lept}}{[\Gamma_{\rm lept}]_{\rm SM}} &= 1 - 0.0021S + 0.0093T + 0.60\varepsilon_e + 0.60\varepsilon_\mu,\\
\frac{\Gamma_{\rm inv}/\Gamma_{\rm lept}}{[\Gamma_{\rm inv}/\Gamma_{\rm lept}]_{\rm SM}} &= 1 + 0.0021S - 0.0015T - 0.76\varepsilon_e - 0.76\varepsilon_\mu - 0.67\varepsilon_\tau,\\
\frac{\sin^2\theta_{\rm eff}^{\rm lept}}{[\sin^2\theta_{\rm eff}^{\rm lept}]_{\rm SM}} &= 1 + 0.016S - 0.011T - 0.72\varepsilon_e - 0.72\varepsilon_\mu,\\
\frac{g_L^2}{[g_L^2]_{\rm SM}} &= 1 - 0.0090S + 0.022T + 0.41\varepsilon_e - 0.59\varepsilon_\mu,\\
\frac{g_R^2}{[g_R^2]_{\rm SM}} &= 1 + 0.031S - 0.0067T - 1.4\varepsilon_e - 2.4\varepsilon_\mu,\\
\frac{M_W}{[M_W]_{\rm SM}} &= 1 - 0.0036S + 0.0056T + 0.0042U + 0.11\varepsilon_e + 0.11\varepsilon_\mu.
\end{aligned} \quad (35)$$

Here, $[*]_{\rm SM}$ is the usual SM prediction of the observable $*$ using G_μ as input.

Despite the fact that six observables are available to the fit, this set of data is not sufficient to fix all six parameters. This is because the ratio g_L^2/g_R^2 depends on the fit parameters only through $\sin^2\theta_{\rm eff}^{\rm lept}$, and thus can only constrain the exact same linear combination of S, T, ε_e, and ε_μ as $\sin^2\theta_{\rm eff}^{\rm lept}$. In fitting the parameters to the observables, the linear combination

$$\alpha T + 2(\varepsilon_\mu - 2\varepsilon_e), \quad (36)$$

remains unconstrained. Therefore, we can constrain only five of the six fit parameters with the Z-pole and NuTeV data.

The linear combinations constrained by the lepton universality bounds from W, τ, π, and K-decays are

$$\frac{g_\mu}{g_e} = 1 + \frac{\varepsilon_e - \varepsilon_\mu}{2}, \qquad \frac{g_\tau}{g_\mu} = 1 + \frac{\varepsilon_\mu - \varepsilon_\tau}{2}, \qquad \frac{g_\tau}{g_e} = 1 + \frac{\varepsilon_e - \varepsilon_\tau}{2}. \quad (37)$$

Since there are only two independent observables but three fit parameters, only two independent linear combinations can be simultaneously constrained by the lepton universality data. However, when these bounds are combined with the Z-pole and NuTeV data, all three ε_ℓ can be constrained independently.

V. FITS TO LEPTON UNIVERSALITY CONSTRAINTS

The three ratios of the coupling constants contain only two independent degrees of freedom, since the third is the product of the first two. Equivalently, when parameterizing the ratios in terms of the differences of the ε_ℓ, as in Eq. (37), only two (*any* two) sets of the differences can be taken as free parameters; the third can be expressed as the difference between them. We can combine the seven pieces of experimental data in Table IV by fitting with any two of the three parameters

$$\begin{aligned}
\Delta_{e\mu} &\equiv \varepsilon_e - \varepsilon_\mu = \Delta_{e\tau} + \Delta_{\mu\tau},\\
\Delta_{\mu\tau} &\equiv \varepsilon_\mu - \varepsilon_\tau = \Delta_{e\tau} - \Delta_{e\mu},\\
\Delta_{e\tau} &\equiv \varepsilon_e - \varepsilon_\tau = \Delta_{e\mu} - \Delta_{\mu\tau}.
\end{aligned} \quad (38)$$

We obtain

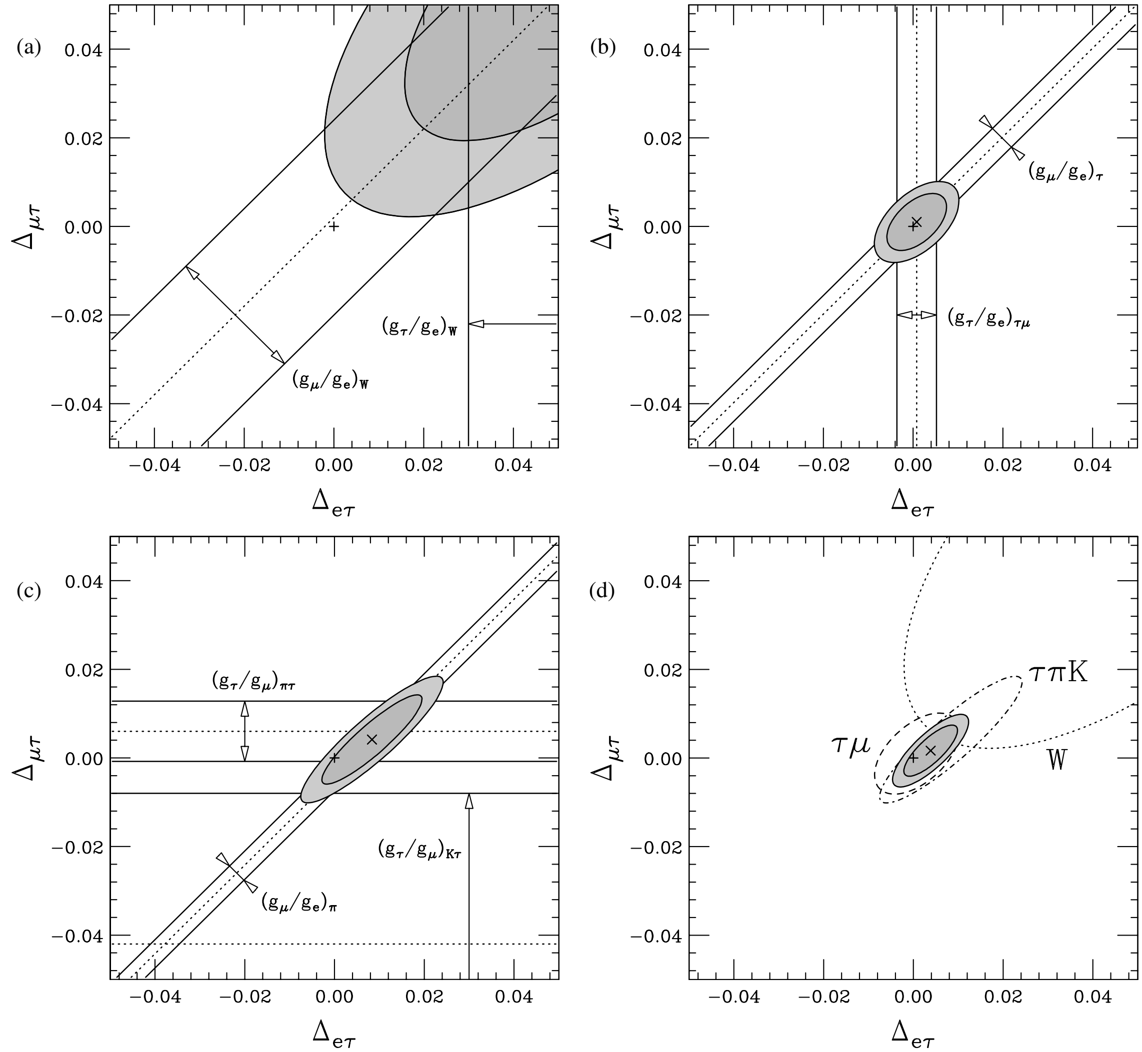

FIG. 1. The limits on $\Delta_{e\tau}$ and $\Delta_{\mu\tau}$ from (a) W-decay, (b) τ-decay, (c) π and K-decay, and (d) all decays combined. The 1σ bands are shown for each coupling constant ratio ignoring correlations. The shaded areas represent the 68% (dark gray) and 90% (light gray) confidence contours including correlations.

$$\Delta_{e\mu} = 0.0022 \pm 0.0025, \qquad \Delta_{\mu\tau} = 0.0017 \pm 0.0038, \qquad \Delta_{e\tau} = 0.0039 \pm 0.0040, \tag{39}$$

with correlations shown in Table VII. In terms of the coupling constant ratios, this translates to

$$\begin{aligned} (g_\mu/g_e) &= 1.0011 \pm 0.0012 \\ (g_\tau/g_e) &= 1.0019 \pm 0.0020, \end{aligned} \tag{40}$$

with a correlation of 0.37. The quality of the fit is unimpressive: the χ^2 is 8.4 for $(7 - 2) = 5$ degrees of freedom. The largest contribution is from $(g_\tau/g_e)_W$ which contributes 4.6. The region of $\Delta_{e\tau}$-$\Delta_{\mu\tau}$ parameter space preferred by the fit is shown in Fig. 1. The 90% confidence contour preferred by the W-decay data hardly overlaps with that of the τ-decay data, which causes the large χ^2.

Since the objective of this paper is to determine whether the Z-pole and NuTeV data are compatible with lepton nonuniversality, it is problematic that the lepton universality constraints are not clearly compatible among themselves. The set of coupling ratios we consider here has an intrinsic χ^2 of 8.4 (or 10.8, if the ALEPH value is excluded from the calculation of $B(\tau \to \pi\nu_\tau)$) which cannot be mitigated in our model. This is in addition to the large χ^2 associated with $B(\tau \to \pi\nu_\tau)$, discussed previously. Further experiments may provide the ultimate resolution of the tension in the data. For now, to prevent

TABLE VII. Correlations among the Δ's from fit.

	$\Delta_{e\mu}$	$\Delta_{\mu\tau}$	$\Delta_{e\tau}$
$\Delta_{e\mu}$	1.00	−0.27	0.37
$\Delta_{\mu\tau}$		1.00	0.80
$\Delta_{e\tau}$			1.00

TABLE VIII. The results of the fits.

fit parameters	A	B	C	D
S	-0.01 ± 0.10	0.00 ± 0.10	0.00 ± 0.10	-0.04 ± 0.10
T	-0.48 ± 0.15	-0.56 ± 0.16	-0.56 ± 0.16	-0.45 ± 0.15
U	0.55 ± 0.16	0.62 ± 0.17	0.62 ± 0.17	0.51 ± 0.16
ε_e	0.0030 ± 0.0010	0.0048 ± 0.0018	0.0049 ± 0.0018	0.0050 ± 0.0018
ε_μ	0.0030 ± 0.0010	0.0027 ± 0.0014	0.0027 ± 0.0014	$\cdots$
ε_τ	0.0030 ± 0.0010	0.0007 ± 0.0028	$\cdots$	0.0012 ± 0.0028
χ^2	2.4	0.91	0.97	4.4
d.o.f.	4	2	3	3
χ^2/d.o.f.	0.6	0.5	0.3	1.5
large χ^2	$\cdots$	$\cdots$	$\cdots$	$\cdots$

fit parameters	E	F	G	H
S	-0.03 ± 0.10	-0.04 ± 0.10	-0.03 ± 0.10	-0.08 ± 0.10
T	-0.30 ± 0.13	-0.46 ± 0.15	-0.30 ± 0.13	-0.18 ± 0.12
U	0.38 ± 0.15	0.52 ± 0.16	0.37 ± 0.15	0.25 ± 0.13
ε_e	$\cdots$	0.0051 ± 0.0018	$\cdots$	$\cdots$
ε_μ	0.0028 ± 0.0014	$\cdots$	0.0029 ± 0.0014	$\cdots$
ε_τ	0.0021 ± 0.0027	$\cdots$	$\cdots$	0.0026 ± 0.0027
χ^2	7.8	4.5	8.3	11.6
d.o.f.	3	4	4	4
χ^2/d.o.f.	2.6	1.1	2.1	2.9
large χ^2	5.1 from (g_μ/g_e) 2.5 from (g_τ/g_e)	$\cdots$	5.3 from (g_μ/g_e)	5.5 from g_L^2 3.0 from (g_τ/g_e)

this large χ^2 among the lepton universality data from obscuring their compatibility with the Z-pole and NuTeV data, we will use the average values obtained in Eq. (40) in our subsequent fits with the caveat that the pair hides a large χ^2.

VI. FITS TO Z-POLE, NUTEV, AND LEPTON UNIVERSALITY DATA

We fit the expressions in Sec. IV to the Z-pole, NuTeV, and W mass data listed in Table VI, and the lepton universality constraint Eq. (40). The S, T, U parameters were used in all fits. Of the three ε_ℓ we performed fits with the following eight combinations of fit parameters:

(A) fit with a flavor-universal ε ($\varepsilon_e = \varepsilon_\mu = \varepsilon_\tau = \varepsilon$),
(B) fit with all three parameters ε_e, ε_μ, and ε_τ,
(C) fit with ε_e and ε_μ,
(D) fit with ε_e and ε_τ,
(E) fit with ε_μ and ε_τ,
(F) fit with ε_e only,
(G) fit with ε_μ only,
(H) fit with ε_τ only.

Fit A with flavor-universal ε is the one performed in Ref. [7] (without the lepton universality constraints). We include it here as a benchmark against which to compare the flavor-dependent fits. The reference Standard Model values were calculated using ZFITTER [36] with the inputs $M_Z = 91.1875$ GeV [8], $M_H = 115$ GeV, $M_t = 178.0$ GeV [37], $\alpha_s(M_Z) = 0.119$, and $\Delta\alpha^{(5)}_{\rm had}(M_Z) = 0.02755$ [39].

The results of these fits have been tabulated in Table VIII, with correlations among the fit parameters shown in Table X. As the values of χ^2 in Table VIII indicate, the quality of the fits A, B, C, D, and F is excellent, while fits E and G are only marginal and fit H fails. The largest contribution to the overall χ^2 for fits E and G is from (g_μ/g_e) (5.1 for E and 5.3 for G) which indicates that these fits are not compatible with lepton universality. For fit H, the largest contributions to the overall χ^2 is from the NuTeV observable g_L^2 (5.5) and (g_τ/g_e) (3.0) which indicates that neither NuTeV nor lepton universality are accommodated. Comparisons of fits B and C, D and F, E and G show that including ϵ_τ in the fits does little to improve the overall χ^2. Indeed, the χ^2 per degree of freedom in actually worse with the inclusion of ϵ_τ.

The constraints placed on the fit parameters by each observable are illustrated in Fig. 2. The one σ bands in each two dimensional plane are plotted assuming that all other fit parameters are set to zero. The gray ellipses are the 68% and 90% confidence regions for fit C, i.e. the five parameter fit with S, T, U, ε_e, and ε_μ, projected to each

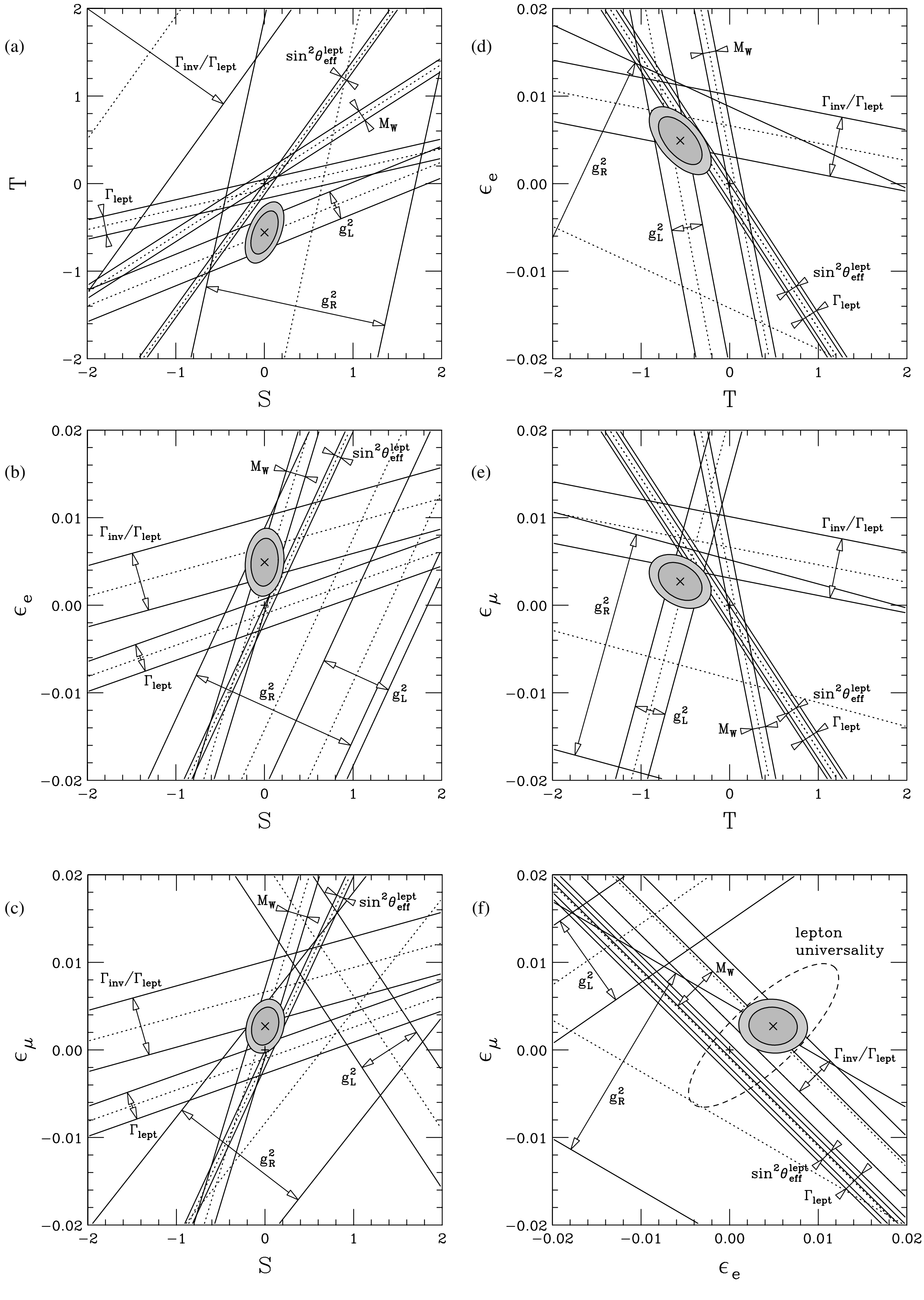

FIG. 2. The 68% and 90% confidence contours projected onto various planes for the five-paremeter fit with S, T, U, ε_e and ε_μ. The bands associated with each observable show the 1σ limits in the respective planes. The origin is the reference SM with $M_{\rm top} = 178.0$ GeV and $M_{\rm Higgs} = 115$ GeV. The dashed contour in 2(f) is the 90% lepton universality bound.

TABLE IX. The results of the fits using the lepton universality constraint from μ and τ-decay only.

fit parameters	E'	F'
S	-0.03 ± 0.10	0.00 ± 0.10
T	-0.50 ± 0.17	-0.38 ± 0.14
U	0.56 ± 0.18	0.45 ± 0.15
ε_e	0.0058 ± 0.0023	$\cdots$
ε_μ	$\cdots$	0.0048 ± 0.0017
χ^2	4.7	2.9
d.o.f.	4	4
χ^2/d.o.f.	1.2	0.74

plane from the full five-dimensional parameter space. In the case of the M_W band, since M_W serves only to fix U, it exerts no statistical "pull" on the other fit parameters; also, since U is fixed by a single observable we have not included figures with U on an axis. We have also omitted projections onto planes involving ε_τ since ε_τ serves little in improving the quality of the fits and since, other than the lepton universality constraint on (g_τ/g_e), it is constrained by only $\Gamma_{\rm inv}/\Gamma_{\rm lept}$.

Figure 2 clarifies the reason for the failure of fit H. From Fig. 2(a), we see that the NuTeV observable g_L^2 prefers a negative T. To maintain the agreement between the SM predictions and the Z-pole data, the effect of negative T must be absorbed by a corresponding shift in G_F, Eq. (33), by making ε_e and/or ε_μ positive (as indicated in Figs. 2(d) and 2(e)). However, since ε_e and ε_μ are both constrained to zero in fit H, it cannot accommodate g_L^2. Further, the measured value of $\Gamma_{\rm inv}/\Gamma_{\rm lept}$ is smaller than the SM prediction, which demands positive ε_τ, while (g_τ/g_e), Eq. (40), prefers negative ε_τ. Thus, H cannot satisfy (g_τ/g_e) either.

For fits E and G, in which ε_e is constrained to zero, the effect of a negative T is absorbed by a positive ε_μ. However, the experimental value of (g_μ/g_e) prefers ε_μ negative. A tension thus remains between the electroweak data and the lepton universality data in these fits.

VII. DISCUSSION AND CONCLUSIONS

The electroweak data are well-fit by several of the patterns of neutrino-gauge coupling suppressions considered. In all cases considered, the best-fit values of the ε_ℓ are positive, i.e., neutrino-gauge couplings are suppressed with respect to the Standard Model, as is required in models of neutrino-mixing. In models in which ε_e is allowed to be nonzero (A-D, F) the fit to the data is good, and the fit improves if ε_μ is allowed to be nonzero

TABLE X. Correlations among the fit parameters for fits A thought H.

A	S	T	U	ε
S	1.00	0.56	−0.21	0.18
T		1.00	−0.74	−0.64
U			1.00	0.58
ε				1.00

B	S	T	U	ε_e	ε_μ	ε_τ
S	1.00	0.47	−0.15	0.12	0.22	−0.12
T		1.00	−0.77	−0.60	−0.34	0.08
U			1.00	0.54	0.33	−0.09
ε_e				1.00	−0.04	−0.19
ε_μ					1.00	−0.09
ε_τ						1.00

C	S	T	U	ε_e	ε_μ
S	1.00	0.48	−0.17	0.10	0.21
T		1.00	−0.77	−0.59	−0.33
U			1.00	0.53	0.33
ε_e				1.00	−0.06
ε_μ					1.00

D	S	T	U	ε_e	ε_τ
S	1.00	0.59	−0.25	0.13	−0.10
T		1.00	−0.74	−0.65	0.05
U			1.00	0.58	−0.06
ε_e				1.00	−0.19
ε_τ					1.00

E	S	T	U	ε_μ	ε_τ
S	1.00	0.68	−0.26	0.23	−0.10
T		1.00	−0.67	−0.45	−0.04
U			1.00	0.42	0.02
ε_μ				1.00	−0.10
ε_τ					1.00

F	S	T	U	ε_e
S	1.00	0.60	−0.25	0.12
T		1.00	−0.74	−0.65
U			1.00	0.59
ε_e				1.00

G	S	T	U	ε_μ
S	1.00	0.68	−0.26	0.22
T		1.00	−0.67	−0.46
U			1.00	0.42
ε_μ				1.00

H	S	T	U	ε_τ
S	1.00	0.90	−0.40	−0.07
T		1.00	−0.59	−0.10
U			1.00	0.07
ε_τ				1.00

as well (B, C). The fit quality is degraded for models in which ε_e is set to zero (E, G, H), and the fit with ε_τ alone (H) is poor. In general the overall χ^2 is insensitive to the presence of ε_τ as a degree of freedom in the fit. The data prefer the model with only ε_e nonzero (F) to the model with only ε_μ nonzero (G).

Since the $\mu \to e\gamma$ data from MEGA [40] demands that either ε_e or ε_μ is strongly suppressed [10–12], $\varepsilon_e \neq 0$, $\varepsilon_\mu \approx 0$ seems to be the solution preferred by current data. However, we stress that the inconsistency within the lepton universality data makes any such conclusion tentative. For example, fits using only the lepton universality constraint from μ and τ-decay, Eq. (13), which is free of QCD uncertainties, indicate that the fit with only ε_μ is superior to a fit with only ε_e, as shown in Table IX. Therefore, future improvements in the lepton universality data (e.g. better determination of the τ lifetime by Belle and Babar [21], measurement of $B(\pi \to e\bar{\nu}_e)$ at the 0.2% level by PIBETA [41], *etc.*) may ultimately provide a different conclusion. In Fig. 2(f), the current 90% contour overlaps with the ε_e axis but not with the ε_μ axis. If the region preferred by the lepton universality data (dashed contour) is shifted toward the ε_μ axis, $\varepsilon_e \approx 0$, $\varepsilon_\mu \neq 0$ may become a viable solution also.

Langacker [42] has noted that the observed violation of unitarity in the CKM matrix [43] will be aggravated by suppressions of neutrino-gauge couplings. However, if the suppression parameters are allowed to break universality, it is only a nonzero ε_μ that aggravates the CKM unitarity problem. Thus the CKM unitary data actually prefers the $\varepsilon_e \neq 0$, $\varepsilon_\mu \approx 0$ solution (in the sense that it does not make the problem worse). An improved determination of $|V_{ud}|$ is expected from the UCN-A experiment at LANL in the near future [44].

The fits A, B, C, D, and F with excellent χ^2's require T to be negative by more than 3σ, U to be positive by more than 3σ, while S is within 1σ of zero. As discussed in Ref. [7], the S and T parameters can be accommodated within the Standard Model by increasing the Higgs mass to several hundred GeV. The large U parameter arises in part from discrepancy between the Standard Model prediction for the W mass and in part from the shift due to the other fit parameters. Neutrino-mixing alone does not account for this discrepancy between the predicted and observed values of the W mass; the U parameter appears to require new physics. Whether a large U parameter can be generated without a correspondingly large T parameter in some models is an open question that needs to be addressed.

The constraint on the suppression parameters $\varepsilon_\ell(\ell = e, \mu, \tau)$ from muon $g - 2$ [45,46] is weak [47]. Further constraints may be obtained from μ to e conversion in nuclei [48,49], and muonium-antimuonium oscillation [50–52]. These will be discussed in a future publication.

ACKNOWLEDGMENTS

We would like to thank K. K. Gan, Gene Golowich, Randy Johnson, Alexey Pronin, and Kei Takeuchi for helpful communications and discussions. This research was supported in part by the U.S. Department of Energy Grant Nos. DE-FG05-92ER40709, Task A (T. T., N. O., and S. R.), and DE-FG02-84ER40153 (L. C. R. W.).

[1] NuTeV Collaboration, G. P. Zeller *et al.*, Phys. Rev. Lett. **88**, 091802 (2002); Phys. Rev. D **65**, 111103 (2002); K. S. McFarland *et al.*, hep-ex/0205080; G. P. Zeller *et al.*, hep-ex/0207052;M. Tzanov *et al.*, hep-ex/0306035.

[2] S. Davidson, S. Forte, P. Gambino, N. Rius, and A. Strumia, J. High Energy Phys. 02 (2002) 037; S. Davidson, J. Phys. G **29**, 2001 (2003); P. Gambino, hep-ph/0211009.

[3] B. A. Dobrescu and R. K. Ellis, Phys. Rev. D **69**, 114014 (2004); S. Kretzer and M. H. Reno, Phys. Rev. D **69**, 034002 (2004).

[4] K. P. O. Diener, S. Dittmaier, and W. Hollik, Phys. Rev. D **69**, 073005 (2004).

[5] P. Gambino, Int. J. Mod. Phys. A **19**, 808 (2004); S. Kretzer, F. Olness, J. Pumplin, D. Stump, W. K. Tung, and M. H. Reno, Phys. Rev. Lett. **93**, 041802 (2004).

[6] A detailed analysis of the tensions in the precision electroweak data, including NuTeV, is presented in M. S. Chanowitz, Phys. Rev. D **66**, 073002 (2002).

[7] W. Loinaz, N. Okamura, T. Takeuchi, and L. C. R. Wijewardhana, Phys. Rev. D **67**, 073012 (2003); T. Takeuchi, hep-ph/0209109; T. Takeuchi, W. Loinaz, N. Okamura, and L. C. R. Wijewardhana, hep-ph/0304203.

[8] LEP Collaborations, LEP Electroweak Working Group, and SLD Heavy Flavor and Electroweak Groups, hep-ex/0312023.

[9] M. Gronau, C. N. Leung, and J. L. Rosner, Phys. Rev. D **29**, 2539 (1984); J. Bernabeu, A. Santamaria, J. Vidal, A. Mendez, and J.W. Valle, Phys. Lett. B **187**, 303 (1987); K. S. Babu, J. C. Pati, and X. Zhang, Phys. Rev. D **46**, 2190 (1992); L. N. Chang, D. Ng, and J. N. Ng, Phys. Rev. D **50**, 4589 (1994); W. J. Marciano, Phys. Rev. D **60**, 093006 (1999); A. De Gouvea, G. F. Giudice, A. Strumia, and K. Tobe, Nucl. Phys. **B623**, 395 (2002); K. S. Babu and J. C. Pati, hep-ph/0203029.

[10] B.W. Lee, S. Pakvasa, R. E. Shrock, and H. Sugawara, Phys. Rev. Lett. **38**, 937 (1977); **38**, 1230E (1977).

[11] S. L. Glashow, hep-ph/0301250.

[12] W. Loinaz, N. Okamura, S. Rayyan, T. Takeuchi, and L. C. R. Wijewardhana, Phys. Rev. D **68**, 073001 (2003).

[13] T. Yanagida, in *Proceedings of the Workshop on Unified Theory and Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto (KEK Report No. 79-18, 1979), p. 95; M. Gell-Mann, P. Ramond, and S. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979), p. 315; R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).

[14] The first comprehensive study of the impact of neutrino mass and lepton mixing on leptonic and hadronic decays was presented in R. E. Shrock, Phys. Rev. D **24**, 1232 (1981); **24**, 1275 (1981).

[15] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990); Phys. Rev. D **46**, 381 (1992); J. L. Hewett, T. Takeuchi, and S. Thomas, hep-ph/9603391.

[16] A. Pich, hep-ph/9701263; M. Davier, in *25th SLAC Summer Institute on Particle Physics: Physics of Leptons (SSI 97), Stanford, CA* (LAL Report No. 98-68 1997).

[17] A. Pich, Nucl. Phys. B, Proc. Suppl. **123**, 1 (2003).

[18] T. Kinoshita and A. Sirlin, Phys. Rev. **113**, 1652 (1959); A. Sirlin, Rev. Mod. Phys. **50**, 573 (1978; **50**, 905E (1978); T. van Ritbergen and R. G. Stuart, Phys. Rev. Lett. **82**, 488 (1999).

[19] W. J. Marciano and A. Sirlin, Phys. Rev. Lett. **61**, 1815 (1988).

[20] Particle Data Group Collaboration, S. Eidelman *et al.*, Phys. Lett. B **592**, 1 (2004).

[21] K. K. Gan, hep-ex/0311047.

[22] ALEPH Collaboration, M. Davier and C. Z. Yuan, Nucl. Phys. B, Proc. Suppl. **123**, 47 (2003).

[23] Particle Data Group Collaboration, K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).

[24] R. Decker and M. Finkemeier, Nucl. Phys. **B438**, 17 (1995).

[25] A. Pich and J. P. Silva, Phys. Rev. D **52**, 4006 (1995).

[26] D. I. Britton *et al.*, Phys. Rev. Lett. **68**, 3000 (1992).

[27] G. Czapek *et al.*, Phys. Rev. Lett. **70**, 17 (1993).

[28] CLEO Collaboration, A. Anastassov *et al.*, Phys. Rev. D **55**, 2559 (1997); **58**, 119904 (1998).

[29] OPAL Collaboration, K. Ackerstaff *et al.*, Eur. Phys. J. C **4**, 193 (1998).

[30] DELPHI Collaboration, F. Matorras, http://eps2003.physik.rwth-aachen.de/data/papers/20030624164845d.ps.

[31] L3 Collaboration, P. Achard *et al.*, http://l3.web.cern.ch/l3/paper/paper270.ps.

[32] F. Matorras, Nucl. Phys. B, Proc. Suppl. **123**, 13 (2003).

[33] One method used by the Particle Data Group [20] to account for such discrepancies is to rescale the errors by a common factor so that $\chi^2/\mathrm{d.o.f} = 1$. If we apply this recipe to the problem at hand, the errors on the averages are rescaled to:

$$\text{average without ALEPH: } B(\tau \to \pi\nu_\tau) = 11.066 \pm 0.146\%,$$

$$\text{average with ALEPH: } B(\tau \to \pi\nu_\tau) = 10.975 \pm 0.117\%.$$

The resulting errors on the coupling constant ratios are:

$$\text{without ALEPH: } (g_\tau/g_\mu)_{\pi\tau} = 1.0072 \pm 0.0069,$$

$$\text{with ALEPH: } (g_\tau/g_\mu)_{\pi\tau} = 1.0030 \pm 0.0056.$$

This rescaling of the errors is somewhat ad hoc and also has very little effect on the final outcome of our analysis. Therefore, we do not employ this method in this paper.

[34] L3 Collaboration, M. Acciarri *et al.*, Phys. Lett. B **345**, 93 (1995).

[35] CLEO Collaboration, A. Weinstein Nucl. Phys. B, Proc. Suppl. **123**, 107 (2003).

[36] The ZFITTER package, D. Bardin *et al.*, Z. Phys. C **44**, 493 (1989); Nucl. Phys. **B351**, 1 (1991); Phys. Lett. B **255**, 290 (1991); hep-ph/9412201; Comput. Phys. Commun. **133**, 229 (2001).

[37] CDF Collaboration, P. Azzi *et al.*, hep-ex/0404010.

[38] Particle Data Group Collaboration, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).

[39] K. Hagiwara, A. D. Martin, D. Nomura, and T. Teubner, Phys. Rev. D **69**, 093003 (2004).

[40] MEGA Collaboration, M. L. Brooks *et al.*, Phys. Rev. Lett. **83**, 1521 (1999).

[41] PIBETA Collaboration, D. Pocanic, hep-ph/0307258.

[42] P. Langacker, in *Electroweak Physics*, AIP Conf. Proc. No. 698 (AIP, New York, 2004).

[43] H. Abele *et al.*, Phys. Rev. Lett. **88**, 211801 (2002); Eur. Phys. J. C **33**, 1 (2004).

[44] B. Tipton *et al.*, AIP Conf. Proc. No. 539 (AIP, New York, 2000).

[45] g-2 Collaboration, E. Sichtermann, hep-ex/0309008.

[46] E. Ma and D. P. Roy, Phys. Rev. D **65**, 075021 (2002); G. G. Boyarkina and O. M. Boyarkin, Phys. Rev. D **67**, 073023 (2003); K. S. Babu and J. C. Pati, Phys. Rev. D **68**, 035004 (2003); T. Fukuyama, T. Kikuchi, and N. Okada, Phys. Rev. D **68**, 033012 (2003).

[47] S. Rayyan (unpublished).

[48] S. Ahmad *et al.*, Phys. Rev. D **38**, 2102 (1988); MECO Collaboration, J. L. Popp, Nucl. Instrum. Methods Phys. Res., Sect. A **472**, 354 (2000); MECO Collaboration, M. Hebert, Nucl. Phys.**A721**, 461 (2003).

[49] F. Simkovic, V. E. Lyubovitskij, T. Gutsche, A. Faessler, and S. Kovalenko, Phys. Lett. B **544**, 121 (2002); R. Kitano, M. Koike, and Y. Okada, Phys. Rev. D **66**, 096002 (2002); R. Kitano, M. Koike, S. Komine, and Y. Okada, Phys. Lett. B **575**, 300 (2003).

[50] L. Willmann *et al.*, Phys. Rev. Lett. **82**, 49 (1999).

[51] M. Aoki, Nucl. Instrum. Methods Phys. Res., Sect. A **503**, 258 (2003).

[52] A. Halprin, Phys. Rev. Lett. **48**, 1313 (1982); T. E. Clark and S. T. Love, Mod. Phys. Lett. A **19** 297 (2004).