Buyer’s optimal ordering policy and payment policy under supplier credit

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Abstract

This paper tries to incorporate both Chung and Huang (2003b) and Huang and Chung (2003) to develop the buyer’s inventory model. That is, we want to investigate the buyer’s optimal cycle time and optimal payment time under supplier’s trade credit policy and cash discount policy. Mathematical models have been derived for obtaining the optimal cycle time and optimal payment time for item under supplier’s trade credit and cash discount so that the annual total relevant cost is minimized. Furthermore, numerical examples are given to illustrate the results developed in this paper and we can also obtain a lot of managerial implications.

Keywords: Inventory, trade credit, cash discount
1. Introduction

The timing of cash flows of an investment proposal is important because the sooner the money becomes available, the sooner it can be used for other worthwhile purposes. Hence, the arrangement of capital of enterprise is an important issue to enterprise itself. In real world, the supplier often makes use of trade credit policy to promote his/her commodities. Before the end of trade credit period, the buyer can sell the goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of trade credit period. The effect of supplier credit policy on inventory problem has received the attention of many researchers. Goyal (1985) established a single-item inventory model under permissible delay in payments. Aggarwal and Jaggi (1995) considered the inventory model with an exponential deterioration rate under the condition of permissible delay in payments. Chung (1998) developed an alternative approach to determine the economic order quantity under condition of permissible delay in payments. Jamal et al. (1997) and Chang and Dye (2001) extended this issue with allowable shortage. Jamal et al. (2000) and Sarker et al. (2000) addressed the optimal payment time under permissible delay in payment with deterioration. Chang et al. (2002) discussed this topic with deteriorating items under time-value of money and finite time horizon. Teng (2002) assumed that the selling price is not equal to the purchasing price to modify Goyal’s model (1985). Huang (2003) extended this issue under two levels of trade credit and developed an efficient solution-finding procedure to determine the optimal lot-sizing policy of the retailer. Chung and Huang (2003a) investigated this issue within EPQ framework and developed an efficient solution-finding procedure to determine the optimal cycle time for the retailer. Shinn and Hwang (2003) determined the retailer’s optimal price and order size simultaneously under the condition of order-size-dependent delay in payments. They assumed that the length of the credit period is a function of the retailer’s order size, and also the quantity demanded is a function of the selling price.

Therefore, it makes economic sense for the buyer to delay the settlement of the replenishment account up to the last moment of the trade credit period allowed by the supplier. From the viewpoint of the supplier, the supplier hopes that the payment is paid from buyer as soon as possible. It can avoid the possibility of resulting in bad debt. So, in most business transactions, the supplier will offer the credit terms mixing cash discount and trade credit to the buyer. The buyer can obtain the cash discount when the payment is paid before cash discount period offered by the supplier. Otherwise, the buyer will pay full payment within the trade credit period. In general, the cash discount period is shorter than the trade credit period. For example, the supplier provides discount off the price if the payment is made within $M_1$ period, otherwise the full payment is due within $M_2$.
period, this usually denoted as “r / M₁, M₂”. Recently, Huang and Chung (2003) investigated the inventory policy under cash discount and trade credit.

In addition, the payment rule of the buyer in all previous articles followed Goyal’s assumption (1985). Goyal (1985) implicitly assumed that the buyer starts paying for higher interest charges on the items in stock and returns money of the remaining balance immediately when the items are sold behind the end of the credit period. Chung and Huang (2003b) adopted alternative payment rule to develop the inventory model and obtain different result. Chung and Huang (2003b) assumed that the buyer needs cash for use to business transactions but the buyer does not like to pay higher interest charges too much to the bank. At the end of the credit period, the buyer will make a partial payment on total purchasing cost and pay off the remaining balance by loan from the bank. Then the buyer does not return money to the bank until the end of the inventory cycle. What the above statement describes are the alternatives of capitals arrangements of enterprises. Based on considerations of profits, costs and developments of enterprises, enterprises may invest their capitals to the best advantage. Recently, Huang (2004) adopted the payment rule discussed in Chung and Huang (2003b) and assumed finite replenishment rate to investigate the buyer’s inventory problem.

This paper tries to adopt the payment rule discussed in Chung and Huang (2003b) and the cash discount discussed in Huang and Chung (2003) to develop the buyer’s inventory model. Then, we want to determine the optimal cycle time and optimal payment time for the buyer under cash discount and trade credit so that the annual total relevant cost is minimized. This means the operation/production department, market department and finance department in an enterprise jointly to determine these decisions. Therefore, the decisions making involves inventory, marketing and financing issues. So, we investigate this integrated model is very important and valuable to the enterprise.

2. Model formulation

For convenience, following notation and assumptions similar to Chung and Huang (2003b) and Huang and Chung (2003) and will be used throughout in this paper.

2.1 Notation:

\( D = \) demand rate per year  
\( A = \) cost of placing one order  
\( c = \) unit purchasing price  
\( s = \) unit selling price
\( h \) = unit stock holding cost per year excluding interest charges

\( I_e \) = interest earned per $ per year

\( I_p \) = interest charges per $ investment in inventory per year

\( r \) = cash discount rate, \( 0 < r < 1 \)

\( M_1 \) = the period of cash discount in years

\( M_2 \) = the period of permissible delay in payments in years, \( M_1 < M_2 \)

\( T \) = the cycle time in years

\( TRC_{11}(T) \) = the total relevant cost per unit time when payment is paid at time \( M_1 \) and \( T \geq M_1 \)

\( TRC_{12}(T) \) = the total relevant cost per unit time when payment is paid at time \( M_1 \) and \( T \leq M_1 \)

\( TRC_1(T) \) = the total relevant cost per unit time when payment is paid at time \( M_1 \) and \( T > 0 \)

\[
TRC_1(T) = \begin{cases} 
TRC_{11}(T) & \text{if } T \geq M_1 \\
TRC_{12}(T) & \text{if } T \leq M_1 
\end{cases}
\]

\( TRC_{21}(T) \) = the total relevant cost per unit time when payment is paid at time \( M_2 \) and \( T \geq M_2 \)

\( TRC_{22}(T) \) = the total relevant cost per unit time when payment is paid at time \( M_2 \) and \( T \leq M_2 \)

\( TRC_2(T) \) = the total relevant cost per unit time when payment is paid at time \( M_2 \) and \( T > 0 \)

\[
TRC_2(T) = \begin{cases} 
TRC_{21}(T) & \text{if } T \geq M_2 \\
TRC_{22}(T) & \text{if } T \leq M_2 
\end{cases}
\]

\( TRC(T) \) = the total relevant cost per unit time when \( T > 0 \)

\[
TRC(T) = \begin{cases} 
TRC_1(T) & \text{if the payment is paid at time } M_1 \\
TRC_2(T) & \text{if the payment is paid at time } M_2 
\end{cases}
\]

\( T_1^* \) = the optimal cycle time of \( TRC_1(T) \)

\( T_2^* \) = the optimal cycle time of \( TRC_2(T) \)

\( T^* \) = the optimal cycle time of \( TRC(T) \)

\( Q^* \) = the optimal order quantity = \( DT^* \).

2.2 Assumptions:

(1) Demand rate is known and remains constant.

(2) Shortages are not allowed.

(3) Time horizon is infinite.

(4) Replenishments are instantaneous.

(5) \( s \geq c \) and \( I_p \geq I_e \).

(6) Supplier offers a cash discount if payment is paid within \( M_1 \), otherwise the full payment is paid within \( M_2 \).
(7) When the buyer must pay the amount of purchasing cost to the supplier, the buyer will make a partial payment $cDM$ on total purchasing cost to the supplier and pay off the remaining balance by loan from the bank. When $T \geq M_1$ or $M_2$, the buyer returns money to the bank at the end of the inventory cycle. However, when $T \leq M_1$ or $M_2$, the buyer returns money to the bank at $T = M_1$ or $M_2$.

(8) If the credit period is not longer than the cycle length, the buyer can sell the items, accumulate sales revenue and earn interest throughout the inventory cycle.

2.3 The model:

The annual total relevant cost consists of the following elements.

(1) Annual ordering cost $= \frac{A}{T}$.

(2) Annual stock holding cost (excluding interest charges) $= \frac{DTh}{2}$.

(3) Annual purchasing cost:

Case 1: Payment is paid at time $M_1$, the annual purchasing cost $= c(1-r)D$.

Case 2: Payment is paid at time $M_2$, the annual purchasing cost $= cD$.

Since the supplier offers a cash discount if payment is paid within $M_1$, there are two payment policies for the buyer. First, the payment is paid at time $M_1$ to get the cash discount, Case 1. Second, the payment is paid at time $M_2$ not to get the cash discount, Case 2. So the interest payable and interest earned, we will discuss these two cases as follows.

(4) Cost of interest charges for the items kept in stock per year:

Case 1: Payment is paid at time $M_1$

Case 1.1 $T \geq M_1$, as shown in Figure 1.

Cost of interest charges for the items kept in stock per year $= c(1-r)I_p D(T - M_1)^3 / T$.

Case 1.2 $T \leq M_1$

In this case, cost of interest charges for the items kept in stock per year $= 0$.

Case 2: Payment is paid at time $M_2$

Case 2.1 $T \geq M_2$, as shown in Figure 1.

Cost of interest charges for the items kept in stock per year $= cI_p D(T - M_2)^3 / T$.

Case 2.2 $T \leq M_2$

In this case, cost of interest charges for the items kept in stock per year $= 0$.

(5) Interests earned per year:
Case 1: Payment is paid at time $M_1$

Case 1.1 $T \geq M_1$, as shown in Figure 2.

Interests earned per year = $\int_{0}^{T} [s \int_{M_1}^{T} D(t - M_1)dt + DM_1(s - c)(T - M_1)]/T$

$$= DI_s \left( \frac{sT}{2} - cM_1 + \frac{cM_1^2}{T} \right).$$

Case 1.2 $T \leq M_1$, as shown in Figure 3.

Interests earned per year = $sI_e \left[ \int_{0}^{T} D(t - M_1)dt + DT(M_1 - T) \right]/T = DsI_e(M_1 - \frac{T}{2}).$

Case 2: Payment is paid at time $M_2$

Case 2.1 $T \geq M_2$, as shown in Figure 2.

Interests earned per year = $\int_{0}^{T} [s \int_{M_2}^{T} D(t - M_2)dt + DM_2(s - c)(T - M_2)]/T$

$$= DI_s \left( \frac{sT}{2} - cM_2 + \frac{cM_2^2}{T} \right).$$

Case 2.2 $T \leq M_2$, as shown in Figure 3.

Interests earned per year = $sI_e \left[ \int_{0}^{T} D(t - M_2)dt + DT(M_2 - T) \right]/T = DsI_e(M_2 - \frac{T}{2}).$

[Insert Figures 1, 2 and 3 here]

From the above arguments, the annual total relevant cost for the buyer can be expressed as:

Annual total relevant cost = ordering cost + stock-holding cost + purchasing cost + interest payable

− interest earned.

We show that the annual total relevant cost is given by

Case 1: Payment is paid at time $M_1$

$$TRC_1(T) = \begin{cases} \frac{A}{T} + \frac{DTh}{2} + c(1 - r)D + \frac{c(1 - r)D(T - M_1)^2}{T} - DI_s \left( \frac{sT}{2} - cM_1 + \frac{cM_1^2}{T} \right) & \text{if } M_1 \leq T, \\ \frac{A}{T} + \frac{DTh}{2} + c(1 - r)D - DsI_e(M_1 - \frac{T}{2}) & \text{if } 0 < T \leq M_1 \end{cases}$$

Let
\[ TRC_{11}(T) = \frac{A}{T} + \frac{DTh}{2} + c(1-r)D + \frac{c(1-r)I_p D(T-M_1)^2}{T} - DI_e\left(\frac{sT}{2} - cM_1 + \frac{cM_1^2}{T}\right) \]  

and

\[ TRC_{12}(T) = \frac{A}{T} + \frac{DTh}{2} + c(1-r)D - DsI_e(M_1 - \frac{T}{2}). \]  

At \( T=M_1 \), we find \( TRC_{11}(M_1)=TRC_{12}(M_1) \). Hence \( TRC_1(T) \) is continuous and well-defined. All \( TRC_{11}(T), \ TRC_{12}(T) \) and \( TRC_1(T) \) are well-defined on \( T>0 \).

**Case 2**: Payment is paid at time \( M_2 \)

\[
TRC_2(T) = \begin{cases} 
\frac{A}{T} + \frac{DTh}{2} + cD + \frac{cI_p D(T-M_2)^2}{T} - DI_e\left(\frac{sT}{2} - cM_2 + \frac{cM_2^2}{T}\right) & \text{if } M_2 \leq T \\
\frac{A}{T} + \frac{DTh}{2} + cD - DsI_e(M_2 - \frac{T}{2}) & \text{if } 0 < T \leq M_2
\end{cases}
\]  

Let

\[ TRC_{21}(T) = \frac{A}{T} + \frac{DTh}{2} + cD + \frac{cI_p D(T-M_2)^2}{T} - DI_e\left(\frac{sT}{2} - cM_2 + \frac{cM_2^2}{T}\right) \]  

and

\[ TRC_{22}(T) = \frac{A}{T} + \frac{DTh}{2} + cD - DsI_e(M_2 - \frac{T}{2}). \]  

At \( T=M_2 \), we find \( TRC_{21}(M_2)=TRC_{22}(M_2) \). Hence \( TRC_2(T) \) is continuous and well-defined. All \( TRC_{21}(T), \ TRC_{22}(T) \) and \( TRC_2(T) \) are well-defined on \( T>0 \).

When the cash discount is neglected, our model is reduced to that of Chung and Huang (2003b).

### 2.4 Optimality conditions:

From equations (2), (3), (5) and (6) yield

\[ TRC_{11}'(T) = -\frac{[A + cDM_1^2[(1-r)I_p - I_e]]}{T^2} + \frac{D[h + 2c(1-r)I_p - sI_e]}{2}, \]  

\[ TRC_{11}''(T) = \frac{2[A + cDM_1^2[(1-r)I_p - I_e]]}{T^3}, \]  

\[ TRC_{21}'(T) = -\frac{[A + cDM_2^2(I_p - I_e)]}{T^2} + \frac{D(h + 2cI_p - sI_e)}{2}, \]  

\[ TRC_{21}''(T) = \frac{2[A + cDM_2^2(I_p - I_e)]}{T^3} > 0, \]  

\[ TRC_{21}'''(T) = \frac{2[A + cDM_2^2(I_p - I_e)]}{T^4} > 0. \]
\[ TRC_{12}'(T) = TRC_{22}'(T) = \frac{-A}{T^2} + \frac{D(h + sI_e)}{2} \] (11)

and

\[ TRC_{12}''(T) = TRC_{22}''(T) = \frac{2A}{T^3} > 0. \] (12)

Equations (10) and (12) imply that all \( TRC_{12}(T), \ TRC_{21}(T) \) and \( TRC_{22}(T) \) are convex on \( T > 0 \).

Equation (8) implies \( TRC_{11}(T) \) is convex on \( T > 0 \) if \( A + cDM_1^2(1-r)I_p - I_e > 0 \).

However, \( TRC_{11}(M_i) \neq TRC_{12}(M_i) \) and \( TRC_{21}(M_2) \neq TRC_{22}(M_2) \). So, both \( TRC_{1}(T) \) and \( TRC_{2}(T) \) are piecewise convex but not convex in general if \( A + cDM_1^2[(1-r)I_p - I_e] > 0 \).

3. Decision rule of the optimal cycle time and optimal payment time

Let \( TRC_{ij}(T_{ij}^*) = 0 \) for all \( i = 1, 2 \) and \( j = 1, 2 \). We can obtain

\[ T_{11}^* = \sqrt{\frac{2\{A + cDM_1^2[(1-r)I_p - I_e]\}}{D[h + 2c(1-r)I_p - sI_e]}}, \]

if \( A + cDM_1^2[(1-r)I_p - I_e] > 0 \) and \( h + 2c(1-r)I_p - sI_e > 0 \) (13)

\[ T_{21}^* = \sqrt{\frac{2[A + cDM_2^2(I_p - I_e)]}{D(h + 2cI_p - sI_e)}} \] if \( h + 2cI_p - sI_e > 0 \) (14)

and

\[ T_{12}^* = T_{22}^* = \sqrt{\frac{2A}{D(h + sI_e)}}. \] (15)

Equation (13) implies that the optimal value of \( T \) for the case of \( T \geq M_1 \), that is \( T_{11}^* \geq M_1 \). We substitute Equation (13) into \( T_{11}^* \geq M_1 \), then we can obtain the optimal value of \( T \)

if and only if \( -2A + DM_1^2[h + (2c - s)I_e] \leq 0 \). (16)

Likewise, Equation (15) implies that the optimal value of \( T \) for the case of \( T \leq M_1 \), that is \( T_{12}^* \leq M_1 \). We substitute Equation (15) into \( T_{12}^* \leq M_1 \), then we can obtain the optimal value of \( T \)

if and only if \( -2A + DM_1^2(h + sI_e) \geq 0 \). (17)

In a similar fashion, we can obtain following results:

\( T_{21}^* \geq M_2 \) if and only if \( -2A + DM_2^2[h + (2c - s)I_e] \leq 0 \), (18)
\[ T_{22}* \leq M_2 \text{ if and only if } -2A + DM_2^2 (h + sI_c) \geq 0. \] (19)

Furthermore, to simplify, we let

\[
\Delta_{11} = -2A + DM_1^2 [h + (2c - s)I_c],
\]
(20)

\[
\Delta_{12} = -2A + DM_1^2 (h + sI_c),
\]
(21)

\[
\Delta_{21} = -2A + DM_2^2 [h + (2c - s)I_c]
\]
(22)

and

\[
\Delta_{22} = -2A + DM_2^2 (h + sI_c).
\]
(23)

Since \( M_1 < M_2 \) and \( c \leq s \), we can get \( \Delta_{11} < \Delta_{21} \), \( \Delta_{12} \leq \Delta_{11} \leq \Delta_{12} \) and \( \Delta_{21} \leq \Delta_{22} \) from Equations (20)-(23). Combining the above arguments all together, the optimal cycle time \( T^* \) and optimal payment time ( \( M_1 \) or \( M_2 \) ) can be obtained as follows.

**Theorem 1:**

(A) If \( \Delta_{11} \geq 0 \), then \( TRC(T^*) = \min\{ TRC_1(T_{12}^*), TRC_2(T_{22}^*) \} \). Hence \( T^* = T_{12}^* = T_{22}^* \) and optimal payment time is \( M_1 \) or \( M_2 \) associated with the possible least cost.

(B) If \( \Delta_{11} < 0 \), \( \Delta_{12} \geq 0 \) and \( \Delta_{21} \geq 0 \), then \( TRC(T^*) = \min\{ TRC_1(T_{11}^*), TRC_1(T_{12}^*), TRC_2(T_{22}^*) \} \). Hence \( T^* \) is \( T_{11}^* \) or \( T_{12}^* = T_{22}^* \), optimal payment time is \( M_1 \) or \( M_2 \) associated with the possible least cost.

(C) If \( \Delta_{12} \geq 0 \) and \( \Delta_{21} < 0 \), then \( TRC(T^*) = \min\{ TRC_1(T_{11}^*), TRC_1(T_{12}^*), TRC_2(T_{21}^*), TRC_2(T_{22}^*) \} \). Hence \( T^* \) is \( T_{11}^*, T_{21}^* \) or \( T_{12}^* = T_{22}^* \), optimal payment time is \( M_1 \) or \( M_2 \) associated with the possible least cost.

(D) If \( \Delta_{12} < 0 \) and \( \Delta_{21} \geq 0 \), then \( TRC(T^*) = \min\{ TRC_1(T_{11}^*), TRC_2(T_{22}^*) \} \). Hence \( T^* \) is \( T_{11}^* \) or \( T_{22}^* \), optimal payment time is \( M_1 \) or \( M_2 \) associated with the possible least cost.

(E) If \( \Delta_{12} < 0 \), \( \Delta_{21} < 0 \) and \( \Delta_{22} > 0 \), then \( TRC(T^*) = \min\{ TRC_1(T_{11}^*), TRC_2(T_{21}^*), TVC_2(T_{22}^*) \} \). Hence \( T^* \) is \( T_{11}^*, T_{21}^* \) or \( T_{22}^* \), optimal payment time is \( M_1 \) or \( M_2 \) associated with the possible least cost.

(F) If \( \Delta_{22} \leq 0 \), then \( TRC(T^*) = \min\{ TRC_1(T_{11}^*), TRC_2(T_{21}^*) \} \). Hence \( T^* \) is \( T_{11}^* \) or \( T_{21}^* \), optimal payment time is \( M_1 \) or \( M_2 \) associated with the possible least cost.

Theorem 1 immediately determines the optimal cycle time \( T^* \) and optimal payment time ( \( M_1 \) or \( M_2 \) ) after computing the numbers \( \Delta_{11}, \Delta_{12}, \Delta_{21} \) and \( \Delta_{22} \). Theorem 1 is an efficient solution-finding procedure.
4. Numerical examples

To illustrate the results, let us apply the proposed method to solve the following numerical examples. For convenience, the numbers of the parameters are selected randomly. The optimal solutions for different parameters of $r$, $M_1$ and $s$ are shown in Table 1.

The following inferences can be made based on Table 1. For fixed $r$ and $M_1$, the larger the value of $s$ is, the smaller optimal cycle time and optimal order quantity will be. This result implies that the buyer will not order more quantity to take the benefits of the trade credit more frequently when the larger the differences between the unit selling price per item and the unit purchasing price per item.

For fixed $r$ and $s$, the larger the value of $M_1$ is, the smaller the optimal annual total relevant cost as the optimal payment time is $M_1$; however, if the optimal payment time is $M_2$, the optimal annual total relevant cost is independent of the value of $M_1$. For fixed $s$ and $M_1$, the value of $r$ increasing, the buyer will be more possible to pay the payment quickly to get the cash discount. Since this policy can reduce the optimal annual total relevant cost for the buyer.
Table 1 Optimal solutions

Let $A=\$70/order$, $D=2000\text{units/year}$, $c=\$12/\text{unit}$, $h=\$5/\text{unit/year}$, $I_p=\$0.15/\text{$/year}$, $I_e=\$0.12/\text{$/year}$ and $M_2=0.1\text{year}$.

<table>
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<th>r</th>
<th>$M_1$</th>
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<th>$T^*$</th>
<th>$Q^*$</th>
<th>TRC($T^*$)</th>
<th>Optimal payment time</th>
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5. Conclusions

The supplier offers the trade credit to stimulate the demand of the buyer. Particularly, we investigate the effect of the cash discount. The supplier can also use the cash discount policy to attract buyer to pay the payment to shorten the collection period. This paper adopts the payment rule discussed in Chung and Huang (2003b) and the cash discount discussed in Huang and Chung (2003) to develop the buyer’s inventory model and provides a very efficient solution procedure. Theorem 1 effectively determines the optimal cycle time $T^*$ and optimal payment time after computing the numbers $\Delta_{11}$, $\Delta_{12}$, $\Delta_{21}$ and $\Delta_{22}$. Then our model is reduced to the model of Chung and Huang (2003b) when cash discount is neglected. Finally, numerical examples are given to illustrate the theoretical results and we can also obtain a lot of managerial implications. In future research, we would like to extend to allow for shortages or finite replenishment rate.

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$$I(M) = DT - DM$$

Figure 1. Inventory level, $M_1 \leq T$ or $M_2 \leq T$

Figure 2. The total amount of interests earned when $M_1 \leq T$ or $M_2 \leq T$

Figure 3. The total amount of interests earned when $T \leq M_1$ or $T \leq M_2$