Compensation and Incentive Modeling in Performance-based Contracts for After-market Service

H. Mirzahosseinian, R. Piplani
Systems and Engineering Management Division
Nanyang Technological University, Singapore
mrpiplani@ntu.edu.sg, Ph. +65 67905601

Abstract- Performance-based contracts, as part of a service support strategy in capital-intensive industries, aim to compensate for performance outcomes. This goal can be achieved by defining effective performance metrics, where the performance is linked to the objective defined in the contract, and applying appropriate incentives to motivate the supplier towards that performance. In this research, we introduce a new approach to incentivizing the supplier using a performance-based contract. We define an availability-related performance metric based on the time average of backorders. Firstly, the contract is modeled based on the objective of the customer and the supplier. Then, the resulting compensation model is compared with the existing models which are based on traditional measures of system performance, such as average number of backorders. The findings indicate that definition of the compensation model based on time average of backorders leads to lower average number of backorders and average total downtime of the system.

Keywords: Performance-based contract, after-market service, Time backorder, Availability.

1. Introduction
PBL is a preferred approach in capital-intensive industries where the systems and subsystems require high availability and are expensive to maintain. In such industries, the failed parts need to be repaired and cannot be scrapped because of their high cost and long lifetime. Therefore, a repairable parts inventory approach is required to support such systems.

Over the last 40+ years researchers have developed many fundamental models in repairable parts inventory, such as METRIC, MOD-METRIC and VARI-METRIC. Recently some studies have been done to model the repairable parts inventory systems under performance-based contracts (Kim et al. 2007a,b; Nowicki et al. 2008; Oner et al. 2010), but the models still have restrictive assumptions which decrease supplier’s flexibility and may give indication of unrealistic performance to the customers. Mirzahosseinian and Piplani (2011) model the PBL system as a queuing network by enhancing the classical repairable parts inventory model. The results show that the base stock level of spare parts has negligible effect on the system availability.

The barriers and enablers for successful implementation of PBL have been studied by many researchers (Devries 2004; Berkowitz et al. 2005). Devries (2004) research, as a domination study in this area, indicated that performance, metrics and incentives were the most frequent enablers of PBL and enhancing them can lead to its successful implementation. While the performance of a system under PBL can be defined in different ways, Sherbrooke (1968) introduced the backorder as the most reasonable performance measure in comparison to other measures such as fill rate, ready rate, and operational rate.

Incentive and insurance are two integral parts of a compensation model in the agency models, which specifies the agent payment in accordance with the delivered outcome at the end of contract. Incentives can be monetary or non-monetary, positive or negative. While sharing the supplier’s costs as a positive incentive widely is used (McAfee and McMillan 1986, Kim et al. 2007a), utilizing both positive and negative incentives at the same time can satisfy both the parties by reducing their risk under uncertain conditions (Kim et al. 2007a).
Some compensation models in PBL consider only partial reimbursement for the cost of spare parts, which are provided for and held by the supplier. Compensating the supplier partially for his total costs (including the cost for improving the repair and failure rates) may incentivize the supplier to improve all parts of the system rather than focusing on just one part. On the other hand, negative incentive should be related to an element which directly affects the defined performance. The average number of backorders has been used as a negative incentive in the earlier compensation models when the desired system performance was availability. This approach decreases the number of backorders but does not influence the length of backorders.

Against this backdrop, the following questions become important in the successful implementation of a PBL contract: What is the impact of the incentive, used in the compensation model, on the contract terms and the outcome performance? How can the customer motivate the supplier to improve the failure and repair rates, as two effective parameters on availability of the system, rather than only increase the base stock?

In this paper, we model an inventory system for repairable parts, operating under a PBL contract. Two compensation models are defined to reimburse the supplier at the end of the contract. The contract is modeled based on the objective of the parties and the system constraints. The findings indicate that defining the compensation model according to the time average of backorders leads to lower average number of backorders and average total down time of systems, simultaneously.

The rest of the paper is organized as follows: Section 2 presents the structure of the inventory system. The system metrics and the performance are discussed in Section 3. The performance contracting is modeled in Section 4. Section 5 presents a numerical study and parametric analysis of the proposed model is conducted in Section 6. Section 7 presents the conclusions and future research directions.

2. Repairable parts inventory system

We consider a single echelon repairable part inventory system, similar to the one in Mirzahosseinian and Piplani (2011) (Fig. 1). The system is based on the following assumptions:

1) The system demand for each component occurs according to a Poisson process with the variable rate \( \lambda(z) \), where \( z \) is the number of operational systems. 2) The replacement time of components from repair facility to the warehouse is exponentially distributed with the variable rate \( \mu(y, m) \), which is a function of the number of components in repair facility \( y \) and the number of servers \( m \). 3) A one-for-one base stock \( S \) replenishment policy is used.

![Figure 1. Closed-loop inventory system (Mirzahosseinian and Piplani, 2011)](image)

The inventory level in the warehouse is represented by ‘\( x \)’, which can change from \(-N \) to \( S \) (\(-N \leq x \leq S\)), with negative value of ‘\( x \)’ representing the number of backorders.

\[
\lambda(z) = z\lambda \quad \text{for} \quad 0 \leq z \leq N, \quad \mu(y, m) = [\text{Min}(y, m)]\mu \quad \text{for} \quad 0 \leq y \leq S + N
\]
3. System Performance and the Metrics

Given the steady-state probabilities and following Kim et al. (2007a), the average number of backorder and the availability of the entire set of systems can be computed as follows:

\[
E(B|\lambda, \mu, S) = \left\{ \sum_{x=0}^{N-1} x\pi_x \right\} A_u = 1 - E(B) / N
\]  

(3)

As per Little’s law, the average time spent by a failed part in the repair system, \(E(D)\):

\[
E(D|\lambda, \mu, S) = \frac{E[y]}{E[\lambda(z)]} = \frac{\sum_{x=0}^{N-1} (S-x)\pi_x}{\sum_{x=0}^{N-1} \lambda(N) \pi_x + \sum_{x=N}^{\infty} \lambda(N+x) \pi_x}
\]  

(4)

Therefore, the time average of total backorders or total down time, \(E(DT)\), is computed by multiplying the average number of backorder \(E(B)\) by the average delay, \(E(D)\).

4. Performance Based Contract

In this section, we discuss the modeling of performance contract in detail.

4.1 Supplier’s costs

The supplier’s total cost consists of subsystem design, production, holding, repair facility and repair activities costs.

4.1.1 Subsystem design cost

We assume that baseline reliability for each subsystem \((1/\lambda_{\text{max}})\) does not incur any design cost. Also, an upper bound is considered for subsystem reliability \((1/\lambda_{\text{min}})\) that is infinitely expensive for the supplier. According to these assumptions, the following design cost function is used for subsystem (see Mettas and Kallenberg 2000; Huang et al. 2007 and Oner et al. 2010 for more details):

\[
C_p(\lambda) = B_1 \left[ \exp \left( f_1 \left( \frac{1}{\lambda} - \frac{1}{\lambda_{\text{max}}} \right) \right) - 1 \right] \quad 1/\lambda_{\text{max}} \leq 1/\lambda < 1/\lambda_{\text{min}}
\]  

(5)

where \(B_1\) is a positive parameter and \(f_1\) is interpreted as the feasibility of increasing the subsystem reliability; its value can vary between 0 and 1.

4.1.2 Production cost

The production cost of each unit has fixed \((A_1)\) and variable components which change based on the desired reliability level (see Huang et al. 2007 and Oner et al. 2010). The unit production cost function is formulated as:

\[
c_p(\lambda) = A_1 + B_2 \left[ (1/\lambda) - (1/\lambda_{\text{max}}) \right] \quad 1/\lambda_{\text{max}} \leq 1/\lambda < 1/\lambda_{\text{min}} \quad B_2 > 0
\]  

(6)

Then, the total production cost \((C_p)\), for setting the base stock level \((S)\), is computed by \(S \times c_p\).

4.1.3 Holding cost

Let \(h\) be the inventory holding cost per unit per time incurred by the supplier at the warehouse. Given the steady-state probability, the average of on-hand inventories in the warehouse, \(E(I)\), can be modeled as follows:
\[ E(I \mid \lambda, \mu, S) = \sum_{x=1}^{S} x \pi_x \]  

(7)

Then, the average inventory holding cost, \( C_H \), is represented by \( h \times E(I) \).

### 4.1.4 Repair facility cost

Equation (8) denotes the cost associated with increasing the repair rate (see Mettas and Kallenberg 2000; Cassady et al. 2004).

\[ C_{RF}(\mu) = B_3 \left[ \exp \left( f_2 \frac{\mu - \mu_{\text{min}}}{\mu_{\text{max}} - \mu} \right) \right] \quad \mu_{\text{min}} \leq \mu < \mu_{\text{max}} \quad B_3 > 0 \]  

(8)

Similar to \( f_1 \), \( f_2 \) represents the difficulty in increasing the repair rate, which can vary between 0 and 1, due to complexity, limited resources and technology, etc.

### 4.1.5 Repair activities cost

The supplier incurs costs for each failure, which includes transportation cost and repair activities cost (for using resources). The average number of repairs, \( E(R) \), during the lifetime of the contract (\( T \)), can be calculated based on MTTR as:

\[ E(R) = T / MTTR \]  

(9)

If \( r \) denotes the average cost for repairing each failed subsystem, total repair activities cost (\( C_{RA} \)) is \( r \times E(R) \). Thus, the supplier’s total cost (\( TC \)) can be expressed as follows:

\[ TC(\lambda, \mu, S) = C_F(\lambda) + C_p(\lambda, S) + C_H(\lambda, \mu, S) + C_{RF}(\mu) + C_{RA}(\lambda, \mu, S) \]  

(10)

### 4.2 Compensation models

For providing insurance, a fixed payment (\( w \)) is included in the model. The model contains both positive and negative incentives. As per this policy, the customer pays its share (\( 0 \leq \alpha \leq 1 \)) of supplier’s total cost (\( TC \)) and penalizes the supplier based on the average number of backorders.

\[ P_B = w + \alpha(TC) - \beta_1 E(B) \]  

(11)

In equation (11), \( \alpha \) represents a percentage for supplier’s total cost that is compensated for by the customer; parameter \( \beta_1 \) denotes the penalty rate for each backorder. Alternatively, we define an enhanced compensation model in which the supplier is penalized based on the time average of backorders. In equation (12), the parameter \( \beta_2 \) is the penalty rate per unit time backordered.

\[ P_{DT} = w + \alpha(TC) - \beta_2 E(DT) \]  

(12)

### 4.3 Supplier’s problem

The customer offers the compensation model (\( P \)), defined either on the average number of backorders (\( P_B \)) or the time average of backorders (\( P_{DT} \)). The supplier then chooses its own set of controllable variables (\( \lambda, \mu, S \)) to maximize his expected profit.

\[ \phi(\lambda, \mu, S) = P(\lambda, \mu, S \mid w, \alpha, \beta) - TC(\lambda, \mu, S) \]  

(13)

### 4.4 Customer’s problem

The customer determines the contract parameters (\( w, \alpha, \beta \)), in which the penalty rate (\( \beta \)) can be defined either for each backorder or unit time backordered, to minimize its cost subject to the constraints.
\[
\text{Min } \psi(w, \alpha, \beta) = P(w, \alpha, \beta | \lambda^*, \mu^*, S^*) \quad S.t \quad A(\lambda^*, \mu^*, S^*) \geq \hat{A} \quad (AR) \tag{14}
\]

\[
\varphi(\lambda^*, \mu^*, S^*) \geq 0 \quad (IR) \tag{15}
\]

\[
(\lambda^*, \mu^*, S^*) \in \text{arg max } \varphi(\lambda, \mu, S) \quad (IC) \tag{16}
\]

The first constraint (AR) controls the system availability (\(A\)) which should be maintained higher than the minimum acceptable level (\(\hat{A}\)). (IR) constraint ensures the supplier’s participation and the incentive compatibility constraint (IC) shows that the supplier always makes decisions to maximize its own profit.

5. Numerical study

The critical components (engine, propeller, avionics computer) of air vehicle (AV) are considered to support 10 UAV systems with 40 AV’s for one year (Kang et al. 2005). The parameters of the system are tabulated in Table 1.

<table>
<thead>
<tr>
<th>Component</th>
<th>(\lambda) (per year)</th>
<th>(\mu) (hrs)</th>
<th>(S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine</td>
<td>1.92</td>
<td>123 (=15+108)</td>
<td>6</td>
</tr>
<tr>
<td>Propeller</td>
<td>2.88</td>
<td>89 (=5+84)</td>
<td>8</td>
</tr>
<tr>
<td>Avionics</td>
<td>1.44</td>
<td>135 (=15+120)</td>
<td>4</td>
</tr>
</tbody>
</table>

When an engine fails, it requires 4.5 days (2.25 days each way) of transportation delay with 15 hours of repair time, on average. Similarly, the repair lead time calculation of propeller and avionics computer are shown in Table 1. The mean time between failures of an engine, propeller and avionics computer is expected to be 750, 500 and 1000 hours, respectively. By considering the AV flying hours per year (12×120), the annual failure rates of each component can be computed.

6. Parametric Analysis

Two scenarios are considered based on the compensation models (\(P_B, P_{DT}\)) which can be utilized in the contract. Table 2 summarizes the fixed parameters which are used by the parties for each component.

<table>
<thead>
<tr>
<th>Component</th>
<th>(\hat{A})</th>
<th>(A_1)</th>
<th>(B_1)</th>
<th>(B_2)</th>
<th>(B_3)</th>
<th>(f_1)</th>
<th>(f_2)</th>
<th>(r)</th>
<th>(h)</th>
<th>(\lambda_{max})</th>
<th>(\lambda_{min})</th>
<th>(\mu_{max})</th>
<th>(\mu_{min})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine</td>
<td>0.9</td>
<td>$100</td>
<td>$2000</td>
<td>$1</td>
<td>$3000</td>
<td>0.7</td>
<td>0.6</td>
<td>$5</td>
<td>$2</td>
<td>2.6</td>
<td>1.5</td>
<td>83.4</td>
<td>53</td>
</tr>
<tr>
<td>Propeller</td>
<td>0.85</td>
<td>$50</td>
<td>$1000</td>
<td>$0.5</td>
<td>$1500</td>
<td>0.5</td>
<td>0.4</td>
<td>$2.5</td>
<td>$1</td>
<td>4.8</td>
<td>2</td>
<td>125.1</td>
<td>67.3</td>
</tr>
<tr>
<td>Avionics</td>
<td>0.87</td>
<td>$200</td>
<td>$4000</td>
<td>$2</td>
<td>$6000</td>
<td>0.9</td>
<td>0.8</td>
<td>$10</td>
<td>$4</td>
<td>1.8</td>
<td>1.2</td>
<td>70</td>
<td>47.3</td>
</tr>
</tbody>
</table>

Table 3. Supplier decision variables and the outcomes

<table>
<thead>
<tr>
<th>Compensation Model</th>
<th>Engine (\lambda) (per year)</th>
<th>Engine (\mu) (per year)</th>
<th>Propeller (\lambda) (per year)</th>
<th>Propeller (\mu) (per year)</th>
<th>Avionics (\lambda) (per year)</th>
<th>Avionics (\mu) (per year)</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Component</th>
<th>Engine</th>
<th>Propeller</th>
<th>Avionics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compensation Model</td>
<td>(P_B)</td>
<td>(P_{DT})</td>
<td>(P_B)</td>
</tr>
<tr>
<td>(\lambda) per year</td>
<td>1.83</td>
<td>1.81</td>
<td>3</td>
</tr>
<tr>
<td>(\mu) per year</td>
<td>73.32</td>
<td>73.62</td>
<td>109.58</td>
</tr>
<tr>
<td>(S)</td>
<td>5</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>(E(B))</td>
<td>2.9</td>
<td>2.2</td>
<td>5.7</td>
</tr>
<tr>
<td>(E(DT))</td>
<td>2555.1</td>
<td>2141</td>
<td>3236.8</td>
</tr>
</tbody>
</table>
The multi objective genetic algorithm toolbox (Sastry, 2007) is applied for solving the model (equations 14-16). For each scenario, the number of backorders and the total downtime of systems are found as two important outcome measures for the customer. Results in Table 3 provide evidence that defining the compensation model according to the time average of backorders helps the customer to reach lower average number of backorders and average total down time of systems, simultaneously. Moreover, compensating the supplier partially for his total costs (including the cost for improving the repair and failure rates) motivates the supplier to improve all parts of the system, especially failure and repair rates.

7. Conclusion
The average number of backorders has been used in the earlier performance-based compensation models as a negative incentive, which reduces the number of backorders without influencing the length of backorder. In this paper, we evaluate the effectiveness of the time average of backorders as an effective incentive under performance contracting. Our results indicate that defining the compensation model according to the time average of backorders leads to lower average number of backorders and average total downtime of systems, simultaneously. Moreover, the findings show that reimbursing the supplier’s costs for improving the component reliability and the repair facility efficiency will incentivize the supplier to improve the failure and repair rates as two effective parameters that increase system availability. In this study, we use a linear compensation model. According to agency model literature, the optimal compensation model in contract is linear only under very special assumptions. Analyzing the contract terms and the outcomes under different types of compensation models is a potential area for future work.

References