Discussion on: “The Reliable Stabilization Problem for Discrete Time-Varying Linear Systems”

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The paper by Tianqiu Yu is an interesting contribution to the important area of control design for the stabilization of infinite-dimensional discrete-time linear time-varying system using a multi-controller configuration. There are several very interesting aspects in this paper and a number of these aspects can be considered for further investigations. The potential impact of the achievements in this paper is significant. The objective of this discussion is to help the readers know the framework of the paper better and provide some issues for further study.

1. Discussion of the Background

As the development of $\mathcal{H}_\infty$ control theory (see [1, 2, 3]), a lot of insight can be obtained by considering its time-varying analogue, the nest algebra of causal stable operators on an appropriate complex Hilbert space of input-output signals. In the context of operator theory, causal linear time-varying systems are represented by the nest algebra. The control theory for infinite-dimensional time-varying system was sparked and developed based on the nest algebra approach in 1980s. The paper by Yu is a study under this framework. The approach is purely operator theoretic and does not use any state space realization.

The author has successfully applied the known system theory and operator approach to solve the reliable stabilization problem. Although the paper is systematically organized, it heavily relies on a strong mathematical background, which can make the paper understanding difficult to the reader. For this reason, we would interpret the mathematical background from another perspective.

The system considered in the paper can be seen as a (possibly unbounded) infinite-dimensional lower triangular matrix. Then, the connection of two systems can be seen as the multiplication of the two infinite-dimensional matrixes and the stability of system is equivalent to the boundedness of the matrix. From this viewpoint, we can derive that the systems considered in this paper are more general and complex than the infinite-dimensional linear time-invariant system (lower triangular Toeplitz matrix) and finite-dimensional systems.

This interpretation of dynamical systems has generated significant research interest over the last decade, as witnessed by recent contributions such as [4–8].

The objective of reliable stabilization problem is to design $C_1$ and $C_2$ such that they stabilize $L$ together, in addition, each of them individually stabilizes $L$ when the other fails (Fig. 1).

Yu presents the concept of reliable stabilization from the point of view of mathematics by “Reliable Stabilization Problem” in the paper. In fact, this concept provided by Yu can not ensure the internal stability of the system $\{L, C_1, C_2\}$. Here, we give a more feasible form of reliable stabilization, which can handle the internal stability of $\{L, C_1, C_2\}$.

**Modified Reliable Stabilization Problem:**
Find two controllers $C_1, C_2 \in \mathcal{L}$ such that $G(L, C_1, 0)$, $G(L, 0, C_2)$, $G(L, C_1, C_2) \in \mathcal{M}(S)$, where $G(L, C_1, C_2)$ is the transfer operator from $\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ to $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$.

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L and M(S) are defined as the same as these in the paper by Yu.

In contrast to the infinite-dimensional linear time-varying case, these two definitions are equivalent in the time-invariant case and finite-dimensional linear time-varying case. So it is again shown that the system considered by the author is more complex and useful than time-invariant system and finite-dimensional linear time-varying systems.

2. Main Ideas and Results

This paper provides some sufficient conditions for the existence of reliable controller pairs for the discrete-time LTV system and also includes a method for designing the reliable controller pairs for strongly stabilizable systems, together with a simple example to illustrate the design method shown in Theorem 3.4. Although these problems have been solved by the author, there are still some interesting issues to be addressed in our opinion.

First, how about the necessary conditions for the existence of reliable controller pairs? One of the main results in the paper is that the strong stabilizability is a sufficient condition for the existence of reliable solution (Theorem 3.1), is it a necessary one? The answer is “yes”. Liu and I have demonstrated this in [9]. Moreover, the designing methods brought out in the paper by Yu just characterize some of the reliably stabilizing controllers, but not all. That is, the parametrization of such stabilizing controllers has not been obtained. Finally, this result does not yield directly to practical methods or algorithms.

3. Further Study

Reliable stabilization is the problem of stabilizing a system through a multi-controller configuration. This is the first paper about multi-controller configuration in the framework of nest algebra. From this point of view, some other stabilization problem using multi-controller can be further considered under this background, such as the stabilizing controller with an internal loop (see [10, 11]) and the tracking problem using two-controllers.

A fundamental aspect of reliable stabilization configuration is that the system would operate well even when one of the two controllers is broken. Furthermore, another advantage of this configuration is that it may admit better optimization index than that of the standard feedback system (see [12]). Reliable decentralized integral-action controller design is to guarantee stability and satisfy performance criteria based on a given $H_{\infty}$ norm bound. In the framework of the paper, the performance can be given in terms of the operator induced norm.

The issues pointed out above could be considered for further investigation. Anyway we agree to the fact that the paper by Yu may lead to more interesting results and much deeper mathematics and algorithms in this field.

References

Discussion on: “The Reliable Stabilization Problem”

1. Discussion

The control theory for infinite-dimensional time-varying system was sparked and developed based on the nest algebra approach in 1980s. As the development of $H^{\infty}$ control theory, a lot of insight can be obtained by considering its time-varying analogue, the nest algebra of causal stable operators on an appropriate complex Hilbert space of input-output signals. In the context, causal linear time-varying systems are represented by the nest algebra.

In our case the nest is discrete. The linear system is a (possibly unbounded) infinite-dimensional lower triangular matrix with respect to the standard basis.

Definition 2.1: The closed loop system determined by the plant $L$ and the compensator $C$ is stable if all the entries of $H(L, C) = \begin{bmatrix} (I + CL)^{-1} & C(I + LC)^{-1} \\ L(I + CL)^{-1} & -(I + LC)^{-1} \end{bmatrix}$ are stable systems on $\mathcal{H}$.

In this paper, we consider the reliable stabilization problem for discrete time-varying linear systems using a two-controller configuration in the framework of nest algebras. We establish some sufficient conditions for the existence of solutions of the reliable stabilization problem and obtain the parametrization of the reliable controller pairs for strongly stabilizable linear systems.

1. In this note, we present the concept of reliable stabilization for discrete time-varying linear systems using a two-controller configuration. The two-controller feedback configuration $\mathcal{N}(L, C_1, C_2)$ which is shown in Figure 3.1, is used to consider the reliable stabilization problem here.

In this note, we present the concept of reliable stabilization for discrete time-varying linear systems using a two-controller configuration. In fact, this concept provided can not ensure the internal stability of the system $[L, C_1, C_2]$. In [1], Yufeng Lu and Liu Liu give a more feasible form of reliable stabilization.

Modified Reliable Stabilization Problem: Given a plant $L \in \mathcal{L}$, find if possible a pair of controllers $C_1$ and $C_2$ such that $G(L, C_1, 0), G(L, 0, C_2), G(L, C_1, C_2) \in \mathcal{M}(S)$, where $G(L, C_1, C_2)$ is the transfer operator from $\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ to $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ where $\mathcal{M}(S)$ denotes the set of all matrices with elements in $S$.

In this paper, some sufficient conditions for the existence of solutions of the reliable stabilization problem, and the parametrization of the reliable controller pairs for strongly stabilizable linear systems are also hold.

Final Comments by the Author

T. Yu

In this paper, we consider the reliable stabilization problem for discrete time-varying linear systems using a two-controller configuration in the framework of nest algebras. We establish some sufficient conditions for the existence of solutions of the reliable stabilization problem, and obtain the parametrization of the reliable controller pairs for strongly stabilizable linear systems.