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On a multivariate gamma distribution

Edward Furman*

Department of Mathematics and Statistics, York University, Toronto, Ontario M3J 1P3, Canada

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Abstract

A multivariate probability model possessing a dependence structure that is reflected in its variance–covariance structure and gamma distributed univariate margins is introduced and studied. In particular, the higher order moments and cumulants, Chebyshev-type inequalities and multivariate probability density functions are derived. The model suggested herein is believed to be capable of describing dependent insurance losses.

1. Introduction

Gamma distributions play a prominent role in actuarial science. This can be explained by e.g. the fact that most total insurance claim distributions have roughly the same shape as gamma distributions: skewed to the right, non-negatively supported and unimodal. In addition, gamma distributions are well studied and analytically tractable. As a result, there are numerous examples of applying gamma approximations for modeling insurance portfolios (cf., e.g., Hürlimann (2001); Melnick and Tenenbein (2000) and Rioux and Klugman (2004)). Also, Herzog (1999) and Hossack et al. (1983) note that gamma distributions provide a convenient model for the average rate of claims filed by various policyholders of an insurance company. Bowers et al. (1997) use translated gamma distributions as a model for the aggregate insurance claims.

In the 'traditional' risk theory, the individual losses in a portfolio are assumed to be independent, although in the majority of cases that assumption does not comply with reality. To close the gap, one must determine the appropriate multivariate dependent probability model that provides a satisfactory fit for a real life multi-line insurance business. On the basis of the high popularity of the univariate gamma distributions, it is fairly natural to attempt modeling portfolios of insurance losses using dependent multivariate probability models with gamma distributed univariate margins.

It should be noted that in addition to the aforementioned 'classical' actuarial applications, multivariate models with univariate gamma margins have been recently utilized in financial risk measurement. Namely, Furman and Landsman (2005) examine the tail conditional expectation risk measure (TCE) in the case of a multivariate gamma portfolio of risks. The authors develop explicit formulas for both the TCE and the risk capital allocation based on it in the context of a multivariate model possessing dependent gamma margins (cf. Hürlimann (2004) and Furman and Landsman (2007) for related generalizations).

* Tel.: +1 416 736 2100; fax: +1 416 738 5757.
E-mail address: efurman@mathstat.yorku.ca.

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Several multivariate extensions of the univariate gamma distributions exist in the literature (cf. Kotz et al. (2000)), pages 431–484 and in particular Mathai and Moschopoulos (1992)). In this note we explore the multivariate reduction technique (cf. Section 2) for introducing and investigating a new multivariate gamma probability model. Some seemingly useful properties of the multivariate gamma distributions suggested herein are developed in Section 3. The corresponding multivariate probability density functions (pdf's) are derived in Section 4. Section 5 concludes the paper.

2. Multivariate reduction

We now consider the multivariate reduction technique in a fairly general form. Let $\mathbf{Y} = (Y_0, Y_1, \ldots, Y_n)^T$ be an $(n+1)$-variate random vector with mutually independent corresponding cumulative distribution functions (cdf's) $F_i(y; \xi_i)$, $i = 0, 1, \ldots, n$, and let $\mathbf{X} = (X_1, X_2, \ldots, X_n)^T$ be another, say, *resulting*, random vector. Denote by T a functional mapping from $\mathbf{R}^{n+1}$ to $\mathbf{R}^n$, such that

$$\mathbf{X} = T(\mathbf{Y}). \tag{2.1}$$

Definition 2.1. The random vector $\mathbf{X} \in \mathbf{R}^n$ is said to possess the cdf $F(\mathbf{x}; \boldsymbol{\xi}^*)$ parameterized by the vector $\boldsymbol{\xi}^* = (\xi_1, \xi_2, \ldots, \xi_n)^T$, such that $\xi_j = \eta_j(\xi_0, \xi_1, \ldots, \xi_n)$ for specific functions η_j, $j = 1, 2, \ldots, n$.

Some useful examples of mapping (2.1) are $\mathbf{X} = \min(\mathbf{Y})$, $\mathbf{X} = \max(\mathbf{Y})$ and $\mathbf{X} = \mathbf{e}^T\mathbf{Y}$, for the unit vector $\mathbf{e} = (1, 1, \ldots, 1)^T$. In this note we consider linear forms of the mapping only. Then Eq. (2.1) can be reformulated as

$$\mathbf{X} = A\mathbf{Y}, \tag{2.2}$$

where $A \in Mat_{n\times(n+1)}$ is an $n \times (n+1)$ matrix. Note that A defines the form of the multivariate distribution $F(\mathbf{x}; \boldsymbol{\xi}^*)$ obtained by the method. Taking, for instance, $n+1$ inverse Gaussian random variables (rv's) $Y_i \backsim IG(c_i\mu, c_i^2\lambda)$, $c_i \in \mathbf{R}_+$, $i = 0, 1, \ldots, n$ and

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 1 & 0 & \cdots & 0 \\ 1 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & 0 & \cdots & 1 \end{pmatrix} \tag{2.3}$$

results in an extension of the bivariate inverse Gaussian distribution of Chhikara and Folks (1989) (cf. Furman and Landsman (2007) for other useful forms of matrix A; and Hürlimann (2007) for the analysis of the dependence structure implied by the multivariate reduction method).

In the following section we utilize Definition 2.1 to develop a multivariate gamma probability model.

3. A multivariate gamma

Let $\mathbf{Y} = (Y_0, Y_1, \ldots, Y_n)^T$ be an $(n+1)$-variate random vector and $Y_i \backsim Ga(\gamma_i, \alpha_i)$, $i = 0, 1, \ldots, n$, be mutually independent gamma random variables possessing the pdf's

$$f_{Y_i}(y) = \mathrm{e}^{-\alpha_i y}\frac{y^{\gamma_i - 1}{\alpha_i}^{\gamma_i}}{\Gamma(\gamma_i)}, \quad y > 0, \tag{3.1}$$

where $\gamma_i > 0$ and $\alpha_i > 0$ are the shape and rate parameters, respectively. Also, let

$$A = \begin{pmatrix} \alpha_0/\alpha_1 & 1 & 0 & 0 & \cdots & 0 \\ \alpha_0/\alpha_2 & \alpha_1/\alpha_2 & 1 & 0 & \cdots & 0 \\ \alpha_0/\alpha_3 & \alpha_1/\alpha_3 & \alpha_2/\alpha_3 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \alpha_0/\alpha_n & \alpha_1/\alpha_n & \alpha_2/\alpha_n & \alpha_3/\alpha_n & \cdots & 1 \end{pmatrix}. \tag{3.2}$$

Definition 3.1. The joint distribution of $\mathbf{X} = A\mathbf{Y}$ denoted by $\mathbf{X} \backsim MG(\overline{\boldsymbol{\gamma}}, \boldsymbol{\alpha})$ where $\overline{\boldsymbol{\gamma}} = (\gamma_0 + \gamma_1, \gamma_0 + \gamma_1 + \gamma_2, \ldots, \gamma_0 + \cdots + \gamma_n)^T$ and $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_n)^T$ are the n-variate vectors of the shape and rate parameters, respectively, is referred to as the multivariate ladder-type gamma distribution. (This name immediately comes into mind due to the special form of matrix A.)

We further enumerate some elementary properties of the above defined model. Recalling that the moment generating function (mgf) of Y_i is formulated as

$$M_{Y_i}(t) = \left(1 - \frac{t}{\alpha_i}\right)^{-\gamma_i}, \tag{3.3}$$

the mgf of $\mathbf{X}$ becomes, for $\mathbf{t} = (t_1, t_2, \ldots, t_n)^T$,

$$\begin{aligned} M_{\mathbf{X}}(\mathbf{t}) &= \mathbf{E}[\mathrm{e}^{\mathbf{X}^T\mathbf{t}}] = M_{Y_0}\left(\sum_{j=1}^{n} \frac{\alpha_0}{\alpha_j} t_j\right) \prod_{i=1}^{n} M_{Y_i}\left(\sum_{j=i}^{n} \frac{\alpha_i}{\alpha_j} t_j\right) \\ &= \left(1 - \sum_{j=1}^{n} \frac{t_j}{\alpha_j}\right)^{-\gamma_0} \prod_{i=1}^{n} \left(1 - \sum_{j=i}^{n} \frac{t_j}{\alpha_j}\right)^{-\gamma_i} \end{aligned} \tag{3.4}$$

which exists for

$$\left| \sum_{j=i}^{n} \frac{t_j}{\alpha_j} \right| < 1,$$

for all $i = 1, 2, \ldots, n$.

Also, we can easily show either using the mgf above or the following equation:

$$X_k = \sum_{i=0}^{k} \frac{\alpha_i}{\alpha_k} Y_i, \quad k = 1, 2, \ldots, n$$

that:

(1) The jth marginal distribution is gamma with shape $\overline{\gamma}_j = \gamma_0 + \gamma_1 + \cdots + \gamma_j$ and rate α_j, i.e., $X_j \backsim Ga(\overline{\gamma}_j, \alpha_j)$.

(2) The mathematical expectation of X_j is formulated as

$$\mathbf{E}[X_j] = \overline{\gamma}_j / \alpha_j.$$

(3) The variance of X_j is given by

$$\mathbf{Var}[X_j] = \overline{\gamma}_j / \alpha_j^2.$$

(4) Let $i < j$; then the covariance of (X_i, X_j) is

$$\mathbf{Cov}[X_i, X_j] = \mathbf{Cov}[X_i, X_i] = \mathbf{Var}[X_i] = \overline{\gamma}_i / \alpha_i^2,$$

which leads to the following covariance matrix:

$$\Sigma = \begin{pmatrix} \overline{\gamma}_1/\alpha_1^2 & \overline{\gamma}_1/\alpha_1^2 & \cdots & \overline{\gamma}_1/\alpha_1^2 \\ \overline{\gamma}_1/\alpha_1^2 & \overline{\gamma}_2/\alpha_2^2 & \cdots & \overline{\gamma}_2/\alpha_2^2 \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\gamma}_1/\alpha_1^2 & \overline{\gamma}_2/\alpha_2^2 & \cdots & \overline{\gamma}_n/\alpha_n^2 \end{pmatrix}. \tag{3.5}$$

(5) The correlation for the pair (X_i, X_j), $i < j$, is written as

$$\rho_{ij} = \mathbf{Corr}[X_i, X_j] = \sqrt{\frac{\mathbf{Var}[X_i]}{\mathbf{Var}[X_j]}}.$$

To develop the multiple correlation, we establish an auxiliary result that is formulated in the following lemma.

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Lemma 3.1. *Consider the partitioning*

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix},$$

where $\Sigma_{11} = \overline{\gamma}_1/\alpha_1^2$, *and let* $\mathbf{e} = (1, 1, \ldots, 1)^T$ *be an* $(n-1)$*-variate vector of ones. Then*

$$\mathbf{e}^T \Sigma_{22}^{-1} \mathbf{e} = \left(\frac{\overline{\gamma}_2}{\alpha_2^2}\right)^{-1}. \tag{3.6}$$

Proof. First note that

$$\begin{aligned} |\Sigma| &= |\Sigma_{22}|\,|\Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}| \\ &= |\Sigma_{22}| \left(\frac{\overline{\gamma}_1}{\alpha_1^2} - \frac{\overline{\gamma}_1}{\alpha_1^2}\mathbf{e}^T \Sigma_{22}^{-1}\mathbf{e}\frac{\overline{\gamma}_1}{\alpha_1^2}\right) \\ &= |\Sigma_{22}| \frac{\overline{\gamma}_1}{\alpha_1^2}\left(1 - \mathbf{e}^T \Sigma_{22}^{-1}\mathbf{e}\frac{\overline{\gamma}_1}{\alpha_1^2}\right). \end{aligned} \tag{3.7}$$

Further, applying the routine Gaussian elimination to matrix (3.5) we arrive at

$$|\Sigma| = \frac{\overline{\gamma}_1}{\alpha_1^2}\left(\frac{\overline{\gamma}_2}{\alpha_2^2} - \frac{\overline{\gamma}_1}{\alpha_1^2}\right)\left(\frac{\overline{\gamma}_3}{\alpha_3^2} - \frac{\overline{\gamma}_2}{\alpha_2^2}\right)\cdots\left(\frac{\overline{\gamma}_n}{\alpha_n^2} - \frac{\overline{\gamma}_{n-1}}{\alpha_{n-1}^2}\right)$$

and

$$|\Sigma_{22}| = \frac{\overline{\gamma}_2}{\alpha_2^2}\left(\frac{\overline{\gamma}_3}{\alpha_3^2} - \frac{\overline{\gamma}_2}{\alpha_2^2}\right)\left(\frac{\overline{\gamma}_4}{\alpha_4^2} - \frac{\overline{\gamma}_3}{\alpha_3^2}\right)\cdots\left(\frac{\overline{\gamma}_n}{\alpha_n^2} - \frac{\overline{\gamma}_{n-1}}{\alpha_{n-1}^2}\right),$$

which along with Eq. (3.7) completes the proof. □

We further derive the multiple correlation of X_1 on $X_2, \ldots, X_n$.

Theorem 3.1. *Let* $\mathbf{X} \backsim MG(\overline{\boldsymbol{\gamma}}, \boldsymbol{\alpha})$ *with* $\overline{\boldsymbol{\gamma}}$ *and* $\boldsymbol{\alpha}$ *given in Definition 3.1; then the multiple correlation of* X_1 *on* $X_2, \ldots, X_n$ *is*

$$\rho^2_{1(2,\ldots,n)} = \frac{\overline{\gamma}_1}{\alpha_1^2}\left(\frac{\overline{\gamma}_2}{\alpha_2^2}\right)^{-1}. \tag{3.8}$$

Proof. Immediately follows from Lemma 3.1. □

The class of the multivariate gamma distributions discussed herein is closed under convolutions with common rate vectors. More precisely, this is formulated in the following theorem.

Theorem 3.2. *Let* $\mathbf{X}_1 \backsim MG(\overline{\boldsymbol{\gamma}}_1, \boldsymbol{\alpha})$ *and* $\mathbf{X}_2 \backsim MG(\overline{\boldsymbol{\gamma}}_2, \boldsymbol{\alpha})$ *be two independent gamma random vectors; then* $\mathbf{X}_1 + \mathbf{X}_2$ *is distributed multivariate ladder-type gamma with the shape and rate vectors given by* $\overline{\boldsymbol{\gamma}}_1 + \overline{\boldsymbol{\gamma}}_2$ *and* $\boldsymbol{\alpha}$*, respectively.*

Proof. Follows from mgf (3.4). □

In many actuarial applications, the higher order moments are useful (cf., e.g. Furman and Landsman (2006)). In the present context, the moments of order m of X_k, $k = 1, 2, \ldots, n$, can be obtained from those of Y_i, $i = 0, 1, \ldots, n$. Note that

$$\frac{\mathrm{d}^m}{\mathrm{d}t^m} M_{Y_i}(t) = \frac{\mathrm{d}^m}{\mathrm{d}t^m}\left(1 - \frac{t}{\alpha_i}\right)^{-\gamma_i} = \frac{\Gamma(\gamma_i + m)}{\Gamma(\gamma_i)\alpha_i^m}\left(1 - \frac{t}{\alpha_i}\right)^{-(\gamma_i + m)},$$

and thus a very simple expression for the mth-order moment of Y_i follows as

$$\mathbf{E}[Y_i] = \frac{\Gamma(\gamma_i + m)}{\Gamma(\gamma_i)\alpha_i^m}. \tag{3.9}$$

The mth-order moment of X_k is then readily obtained as

$$\begin{aligned}\mathbf{E}[X_k^m] &= \mathbf{E}\left[\left(\sum_{i=0}^{k} \frac{\alpha_i}{\alpha_k} Y_i\right)^m\right] = \sum \frac{m!}{r_1! r_2! \cdots r_k!} \prod_{i=0}^{k} \frac{\alpha_i}{\alpha_k} \mathbf{E}[Y_i^{r_i}] \\ &= \sum \frac{m!}{r_1! r_2! \cdots r_k!} \prod_{i=0}^{k} \frac{\alpha_i}{\alpha_k} \left(\frac{\Gamma(\gamma_i + m)}{\Gamma(\gamma_i)\alpha_i^m}\right)^{r_i},\end{aligned} \tag{3.10}$$

where the summation is over all solutions in non-negative integers of the equation $r_1 + r_2 + \cdots + r_k = m$.

The cumulants of X_k are available from the following cumulant generating function (cgf) of $\mathbf{X}$, which is written as

$$K_{\mathbf{X}}(\mathbf{t}) = -\gamma_0 \ln\left(1 - \sum_{j=1}^{n} \frac{t_j}{\alpha_j}\right) - \sum_{i=1}^{n} \gamma_i \ln\left(1 - \sum_{j=i}^{n} \frac{t_j}{\alpha_j}\right). \tag{3.11}$$

Thus, we formulate the mth-order cumulant of X_k as

$$K_m = \frac{(m-1)!\overline{\gamma}_k}{\alpha_k^m},$$

and the (m_1, m_2)th product cumulant of X_k and X_l as

$$K_{m_1,m_2} = \frac{(m_1 + m_2 - 1)!\overline{\gamma}_{\min(k,l)}}{\alpha_k^{m_1}\alpha_l^{m_2}}.$$

3.1. Chebyshev-type inequalities

As an illustration of the previous results, we further derive some Chebyshev-type inequalities in the case of the proposed multivariate gamma distribution.

Theorem 3.3. *Let ε_j and X_j, $j = 1, 2, \ldots, n$, be some arbitrary positive constants and the components of the multivariate ladder-type gamma, respectively. Then*

$$P(X_1 \le \varepsilon_1, \ldots, X_n \le \varepsilon_n) \ge 1 - \sum_{j=1}^{n} \frac{\overline{\gamma}_j}{\alpha_j \epsilon_j} \tag{3.12}$$

and

$$P(X_1 \ge \varepsilon_1, \ldots, X_n \ge \varepsilon_n) \le \left(\sum_{j=1}^{n} \frac{\overline{\gamma}_j}{\alpha_j}\right) \Big/ \left(\sum_{j=1}^{n} \varepsilon_j\right). \tag{3.13}$$

Proof. The first part follows straightforwardly from the following well known inequality:

$$P(X > \varepsilon) \le \frac{\mathbf{E}[X]}{\varepsilon},$$

on recalling that

$$\mathbf{E}\left[X_j\right] = \frac{\overline{\gamma}_j}{\alpha_j}.$$

Indeed,

$$P\left(\bigcap_{j=1}^{n}\{X_j \le \varepsilon_j\}\right) \ge 1 - \left(\sum_{j=1}^{n} P\left(\{X_j > \varepsilon_j\}\right)\right)$$
$$\ge 1 - \sum_{j=1}^{n}\frac{\mathbf{E}[X_j]}{\varepsilon_j}.$$

Inequality (3.13) follows from noting that

$$P\left(X_1 \ge \varepsilon_1, \ldots, X_n \ge \varepsilon_n\right) \le P\left(\sum_{i=1}^{n} X_i \ge \sum_{i=1}^{n}\varepsilon_i\right),$$

which completes the proof. □

4. Multivariate densities

In this section we derive the joint pdf of $\mathbf{X} \backsim MG(\overline{\boldsymbol{\gamma}}, \boldsymbol{\alpha})$. We first note that due to the independence of $Y_0, Y_1, \ldots, Y_n$, the following holds for the joint pdf of $\mathbf{Y}$:

$$f_{\mathbf{Y}}(y_0, y_1, \ldots, y_n) = \prod_{i=0}^{n} \mathrm{e}^{-\alpha_i y_i}\frac{y_i^{\gamma_i-1}\alpha_i^{\gamma_i}}{\Gamma(\gamma_i)}.$$

Further, as long as $y_1 = x_1 - \frac{\alpha_0}{\alpha_1}y_0$, and noticing that $y_k = x_k - \frac{\alpha_{k-1}}{\alpha_k}x_{k-1}$ for all $k = 2, 3, \ldots, n$, we obtain the multivariate pdf of $\mathbf{X}^* = (Y_0, X_1, \ldots, X_n)^T$ as

$$f_{\mathbf{X}^*}(y_0, x_1, \ldots, x_n) = \mathrm{e}^{-\alpha_n x_n}\prod_{j=0}^{n}\frac{\alpha_j^{\gamma_j}}{\Gamma(\gamma_j)}\prod_{j=2}^{n}\left(x_j - \frac{\alpha_{j-1}}{\alpha_j}x_{j-1}\right)^{\gamma_j-1} y_0^{\gamma_0-1}\left(x_1 - \frac{\alpha_0}{\alpha_1}y_0\right)^{\gamma_1-1},$$

which after integrating out y_0 reduces to the multivariate pdf of $\mathbf{X}$. Note that

$$0 < y_0 < \min\left(\frac{\alpha_1}{\alpha_0}x_1, \frac{\alpha_2}{\alpha_0}x_2, \ldots, \frac{\alpha_n}{\alpha_0}x_n\right),$$

and thus, for $x^* = \min\left(\frac{\alpha_1}{\alpha_0}x_1, \frac{\alpha_2}{\alpha_0}x_2, \ldots, \frac{\alpha_n}{\alpha_0}x_n\right)$, we have that

$$f_{\mathbf{X}}(x_1, x_2, \ldots, x_n) = \mathrm{e}^{-\alpha_n x_n}\prod_{j=0}^{n}\frac{\alpha_j^{\gamma_j}}{\Gamma(\gamma_j)}\prod_{j=2}^{n}\left(x_j - \frac{\alpha_{j-1}}{\alpha_j}x_{j-1}\right)^{\gamma_j-1}\int_0^{x^*} y_0^{\gamma_0-1}\left(x_1 - \frac{\alpha_0}{\alpha_1}y_0\right)^{\gamma_1-1}\mathrm{d}y_0,$$

where $\gamma_j > 0, \alpha_j > 0, x_j > \frac{\alpha_{j-1}}{\alpha_j}x_{j-1}$, $j = 2, 3, \ldots, n$, and $x_n < \infty$.

We note that the pdf above is in general complicated to calculate; however letting $\gamma_0 = 1$ and assuming all rate parameters to be a constant, say α, we arrive at the following easy to handle expression:

$$f_{\mathbf{X}}(x_1, x_2, \ldots, x_n) = \mathrm{e}^{-\alpha_n x_n}\frac{x_1^{\gamma_1}\alpha^{\overline{\gamma}_j}}{\Gamma(\gamma_1+1)}\prod_{j=2}^{n}\frac{\left(x_j - x_{j-1}\right)^{\gamma_j-1}}{\Gamma(\gamma_j)}.$$

Figs. 1 and 2 provide some contour plots conclude by providing some contour plots of variously dependent bivariate gamma distributions. It can be easily observed that the higher correlation coefficients imply higher risk inherent in the model.

5. Conclusions

The class of univariate gamma distributions is of high significance in numerous fields of actuarial science. However, the popularity of the multivariate gamma probability models is much lower. In this note a seemingly useful multivariate probability model possessing a dependence structure and gamma distributed univariate margins has been

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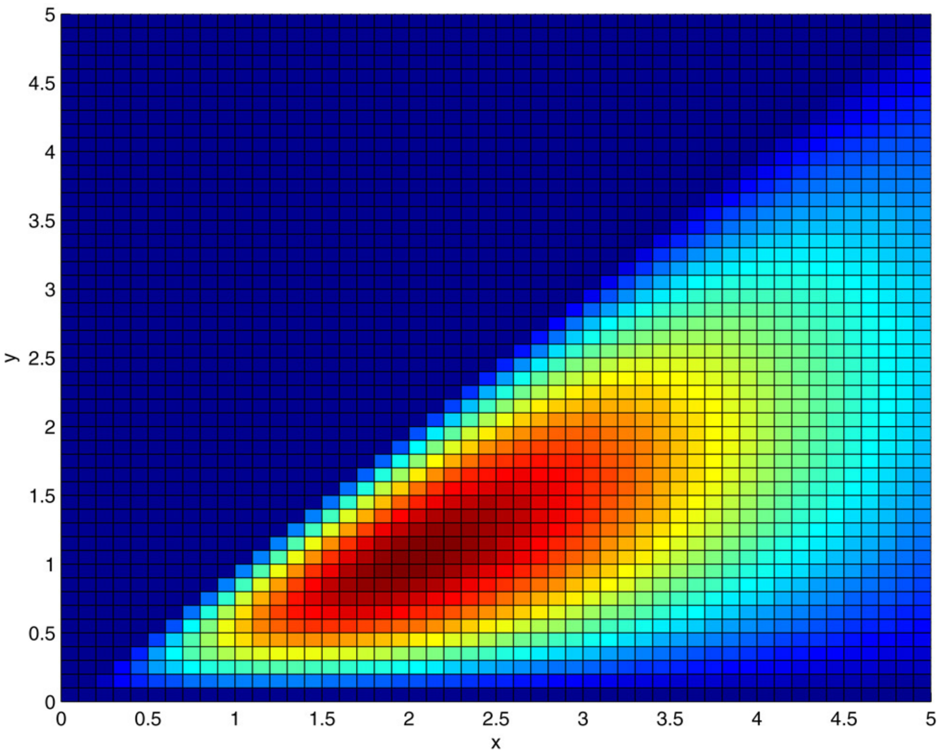

Fig. 1. Bivariate gamma distribution with $\rho = 0.63$.

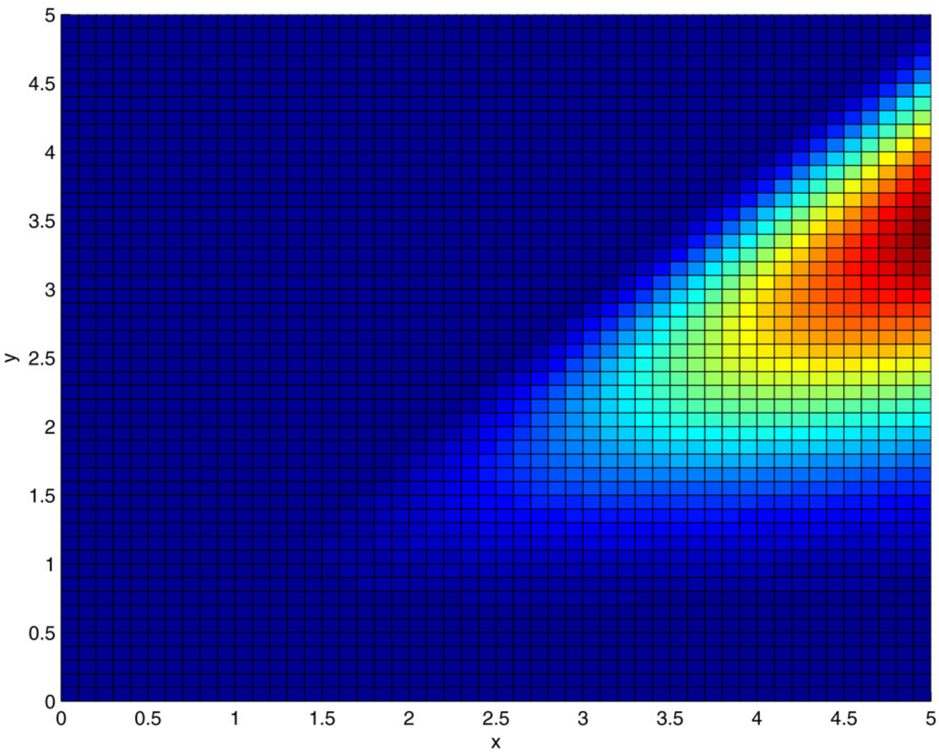

Fig. 2. Bivariate gamma distribution with $\rho = 0.8$.

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developed and studied. In particular, the higher order moments, Chebyshev-type inequalities and multivariate densities have been derived. The class of multivariate distributions suggested herein possesses a dependence structure that is reflected in its variance covariance structure, and it is aimed at modeling dependent insurance losses.

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