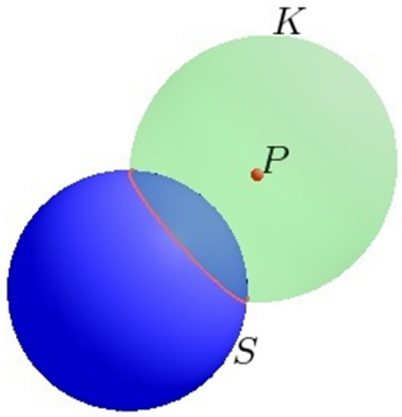
Foundations of Geometric Algebra Computing

26.10.2012

Dr.-Ing. Dietmar Hildenbrand
LOEWE Priority Program Cocoon
Technische Universität Darmstadt



Achtung Änderung!

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- Die Übung findet Montags jeweils 11:40 bis 13:10 statt !!!

Overview

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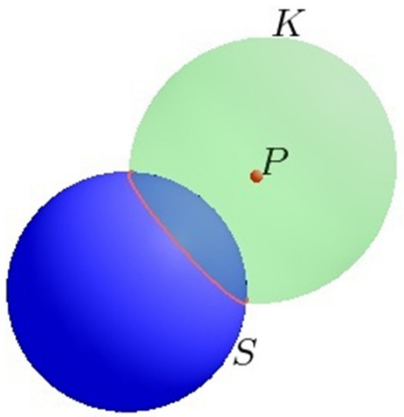
- Foundations of Geometric Algebra (GA)
- GA Applications
- One GA example
- GA Computing
  - precompiler for standard programming languages
- Molecular dynamics application



COCOON

History of Geometric Algebra&Calculus

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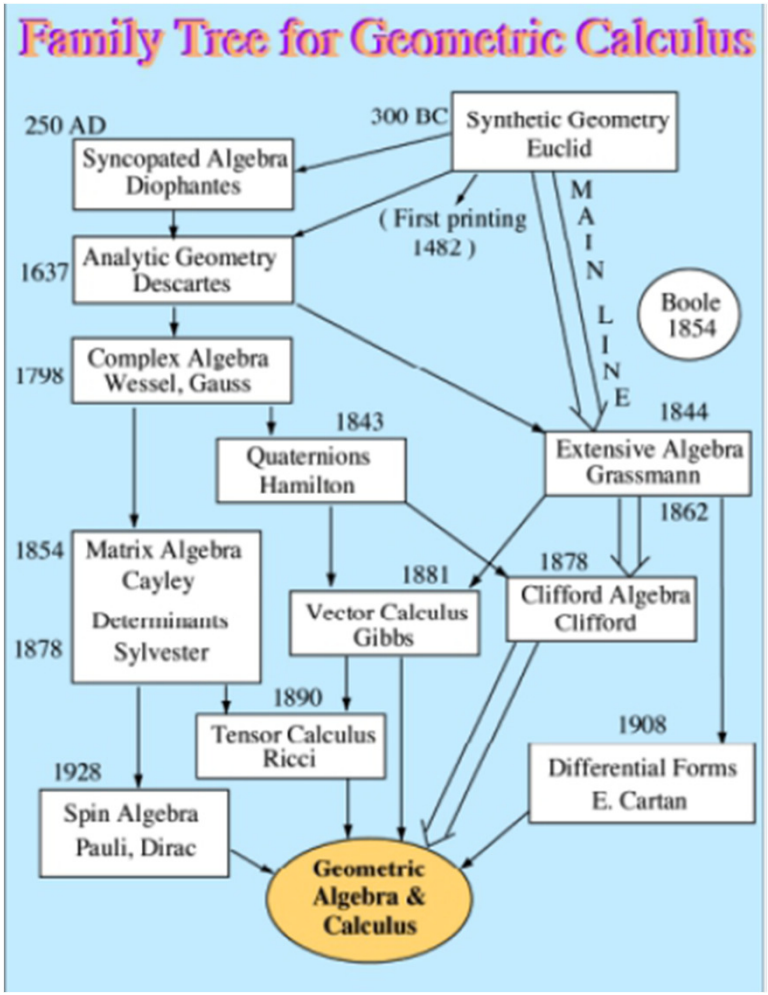

- [David Hestenes 2001]

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History of Geometric Algebra&Calculus

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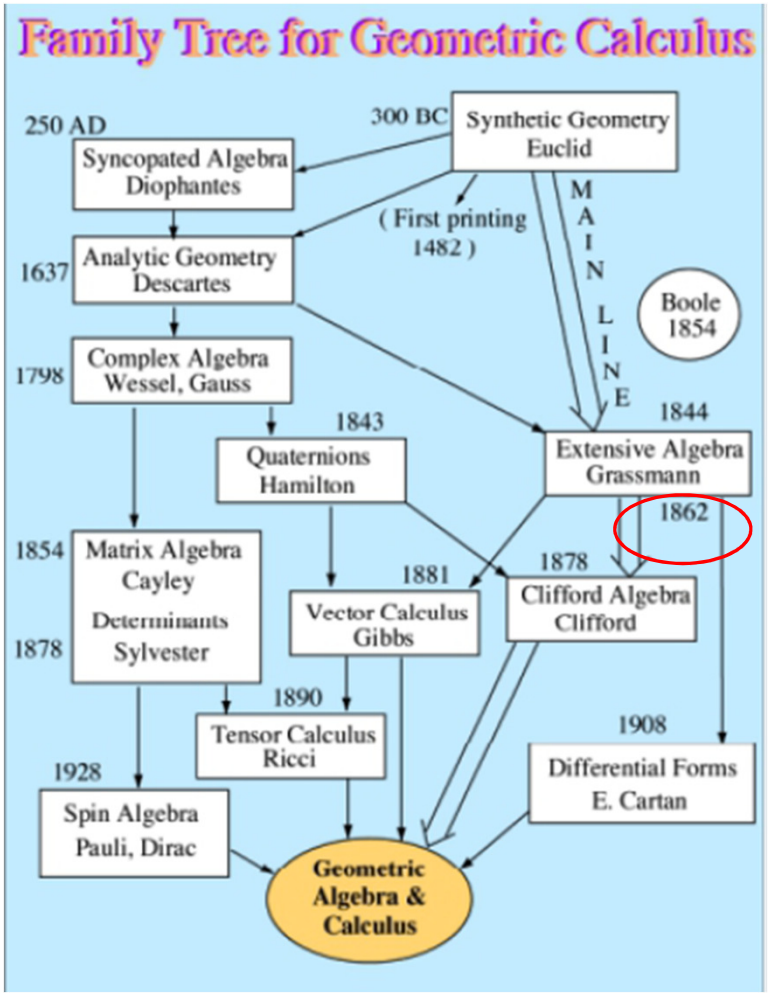

150 years

- [David Hestenes 2001]

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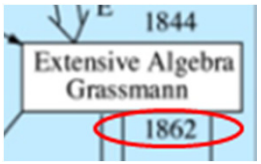
Hermann G. Grassmann



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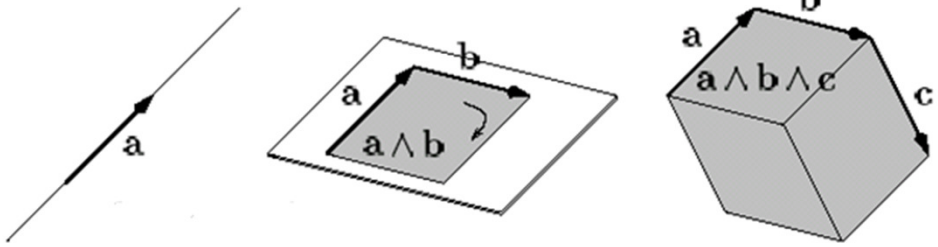
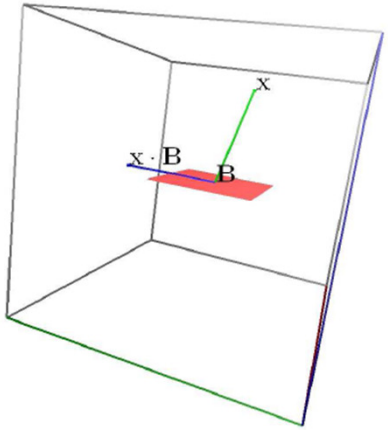
- Outer Product



vector bivector trivector …

- Inner Product



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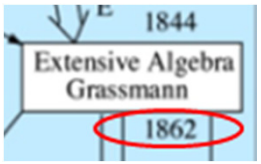

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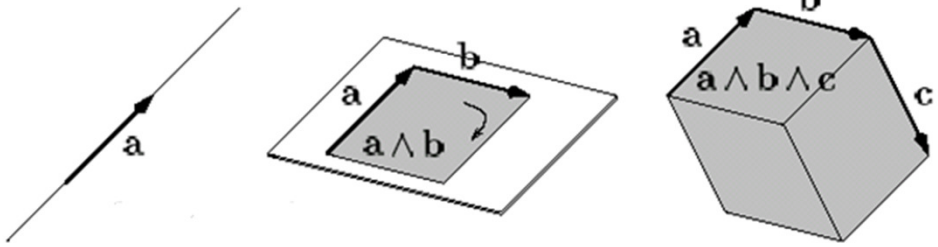

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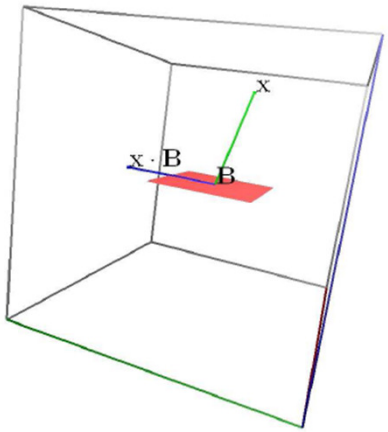

cross product and scalar product are special cases of these general products

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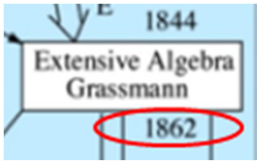

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- Preface of „*Ausdehnungslehre*“ (Extensive Algebra) of 1862:

"I remain completely confident that the labor I have expended on the science presented here and which has demanded a signicant part of my life, as well as the most strenuous application of my powers, will not be lost. It is true that I am aware that the form which I have given the science is imperfect and must be imperfect. But I know and feel obliged to state (though I run the risk of seeming arrogant) that even if this work should again remain unused for another seventeen years or even longer, without entering into the actual development of science, still that time will come when it will be brought forth from the dust of oblivion and when ideas now dormant will bring forth fruit."
and he went on to say:
"there will come a time when these ideas, perhaps in a new form, will arise anew and will enter into a living communication with contemporary developments."

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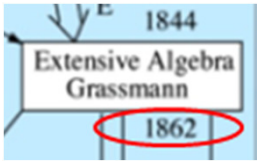

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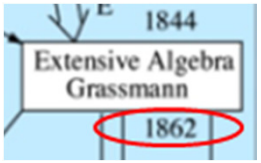

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William K. Clifford

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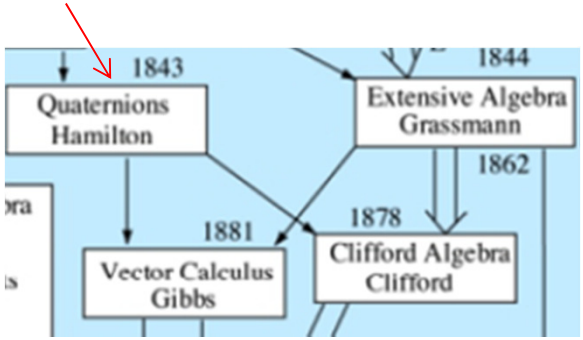
- Geometric Product
- For vectors

$$uv = u \wedge v + u \cdot v.$$

- Quaternions of Hamilton



- Normally called Clifford algebra in honor of Clifford
- He called it geometric algebra

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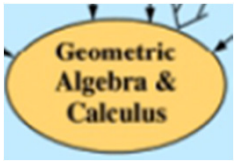
David Hestenes



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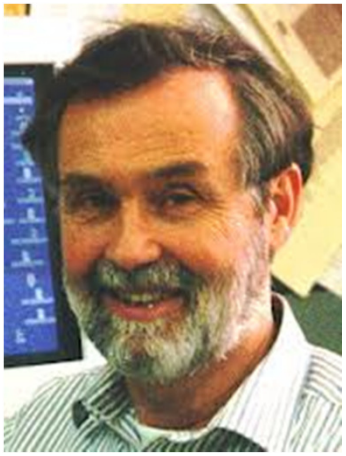
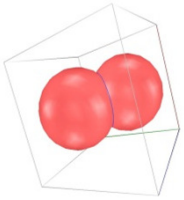
- realized geometric algebra as a general language for physics
  („New Foundations of Classical Mechanics“, …)
- developed calculus
  ("Clifford Algebra to Geometric Calculus:
  A Unified Language for
  Mathematics and Physics")
- developed the Conformal Geometric Algebra





- Geometric Algebra <-> Clifford Algebra?

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Goal of geometric algebra

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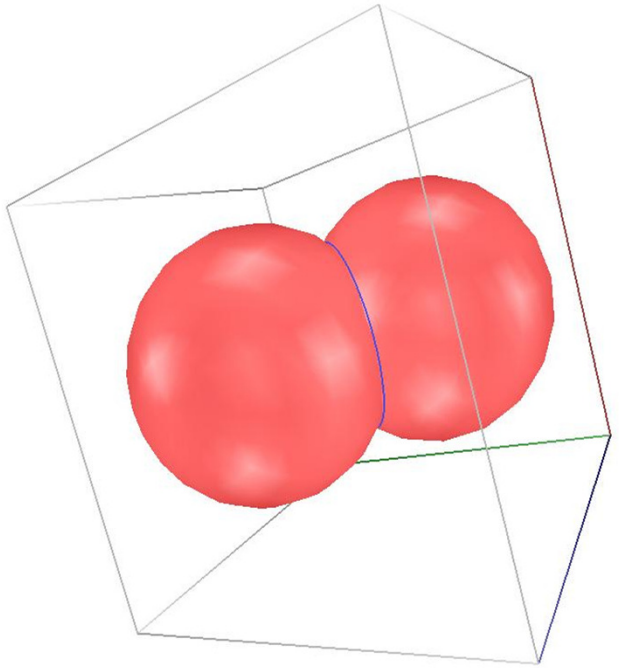
- Mathematical language close to the geometric intuition



Conformal Geometric Algebra (CGA)

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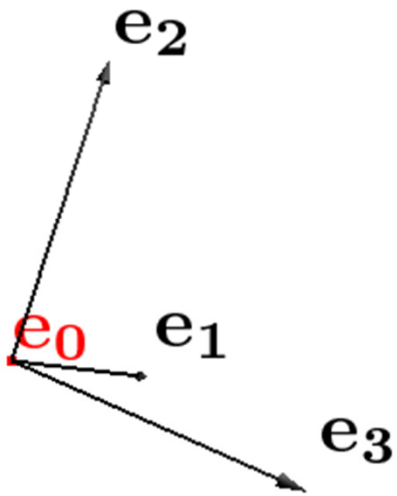
- 5 basis vectors:

  - e_1, e_2, e_3
  - e_0 : origin
  - e_∞ : point at infinity



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Basis elements of CGA

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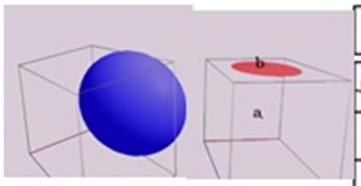

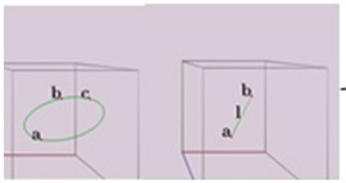

Grade	Term	Blades	No.
0	Scalar	1	1
1	Vector	$e_1, e_2, e_3, e_\infty, e_0$	5
2	Bivector	$e_1 \wedge e_2$, $e_1 \wedge e_3$, $e_1 \wedge e_\infty$, $e_1 \wedge e_0$, $e_2 \wedge e_3$, $e_2 \wedge e_\infty$, $e_2 \wedge e_0$, $e_3 \wedge e_\infty$, $e_3 \wedge e_0$, $e_\infty \wedge e_0$	10
3	Trivector	$e_1 \wedge e_2 \wedge e_3$, $e_1 \wedge e_2 \wedge e_\infty$, $e_1 \wedge e_2 \wedge e_0$, $e_1 \wedge e_3 \wedge e_\infty$, $e_1 \wedge e_3 \wedge e_0$, $e_1 \wedge e_\infty \wedge e_0$, $e_2 \wedge e_3 \wedge e_\infty$, $e_2 \wedge e_3 \wedge e_0$, $e_2 \wedge e_\infty \wedge e_0$, $e_3 \wedge e_\infty \wedge e_0$	10
4	Quadvector	$e_1 \wedge e_2 \wedge e_3 \wedge e_\infty$, $e_1 \wedge e_2 \wedge e_3 \wedge e_0$, $e_1 \wedge e_2 \wedge e_\infty \wedge e_0$, $e_1 \wedge e_3 \wedge e_\infty \wedge e_0$, $e_2 \wedge e_3 \wedge e_\infty \wedge e_0$	5
5	Pseudoscalar	$e_1 \wedge e_2 \wedge e_3 \wedge e_\infty \wedge e_0$	1

i, j, k

$$\begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

...

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Plane and sphere as vector expression

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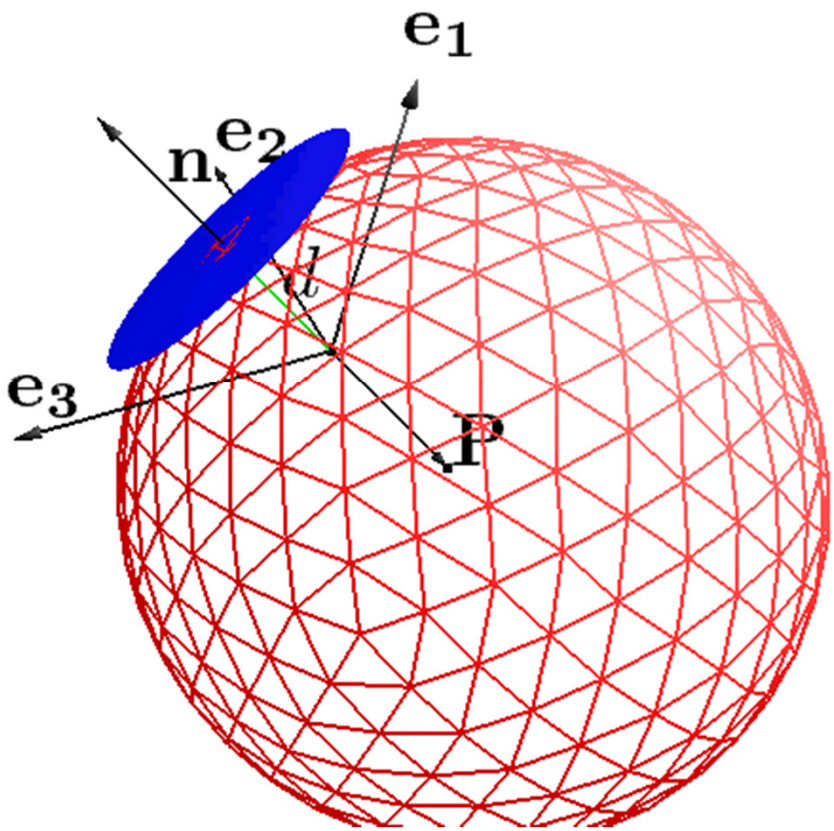
- **sphere**

$$S = P - \frac{1}{2}r^2 e_\infty$$

- **plane**

$$\pi = \mathbf{n} + de_\infty$$



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Intersection of geometric objects

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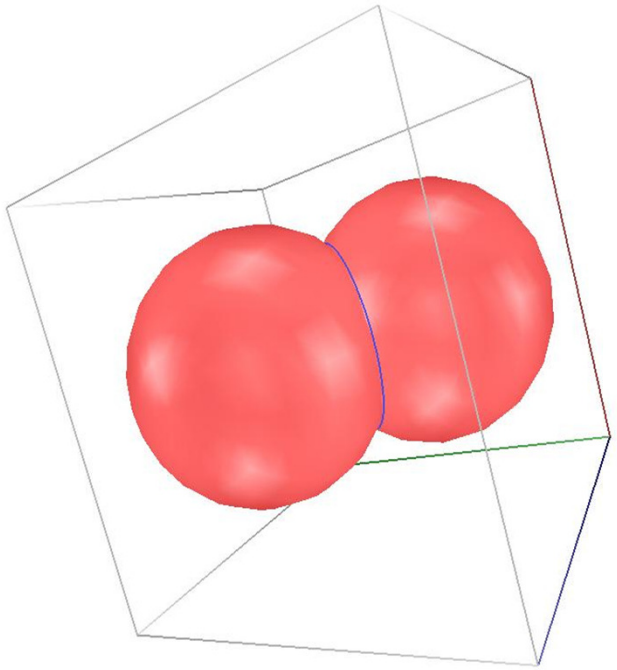
- a,b are sphere
- $a \wedge b$
  - describes the intersection of the spheres
  - represents a circle



Intersection of sphere and line

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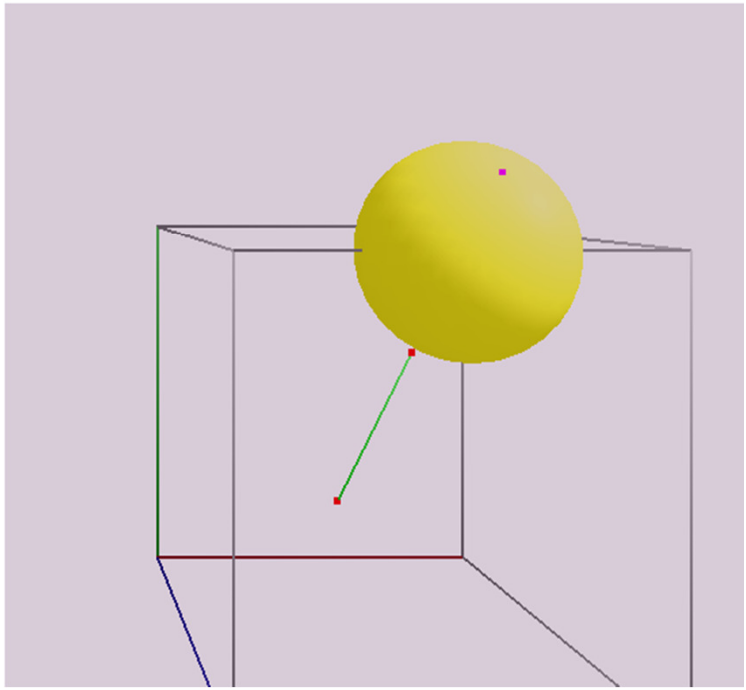
- $Pp = s_1 \wedge l$



Circle and line

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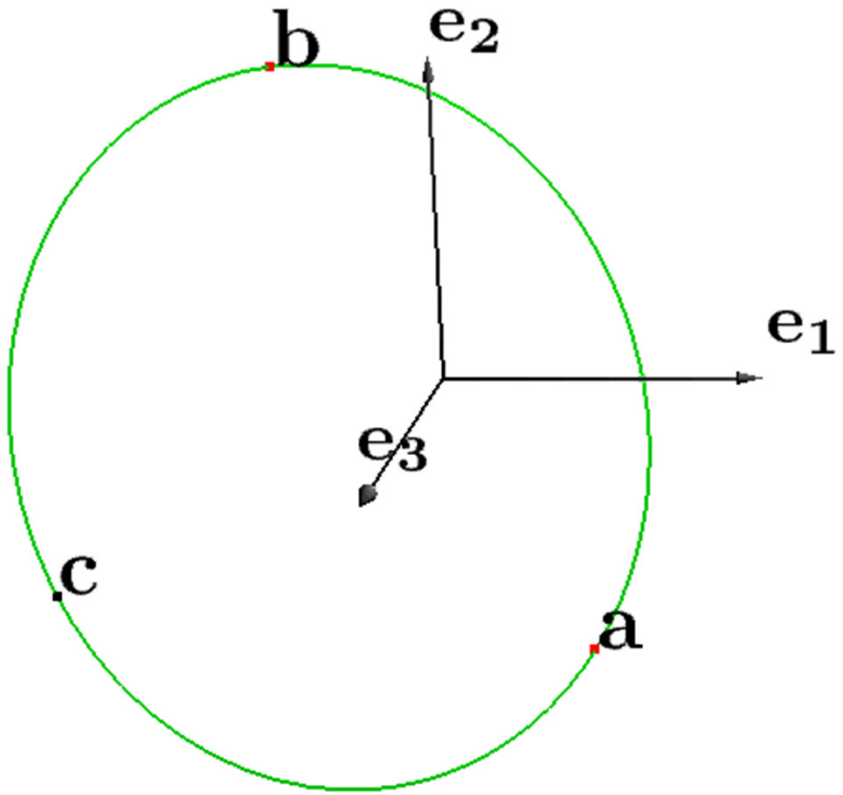
- Circle (a^b^c)
- Circle -> Line

($c \longrightarrow e_\infty$)



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Geometric Operations

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- Rotation about L (through the origin)
- **Rotor**

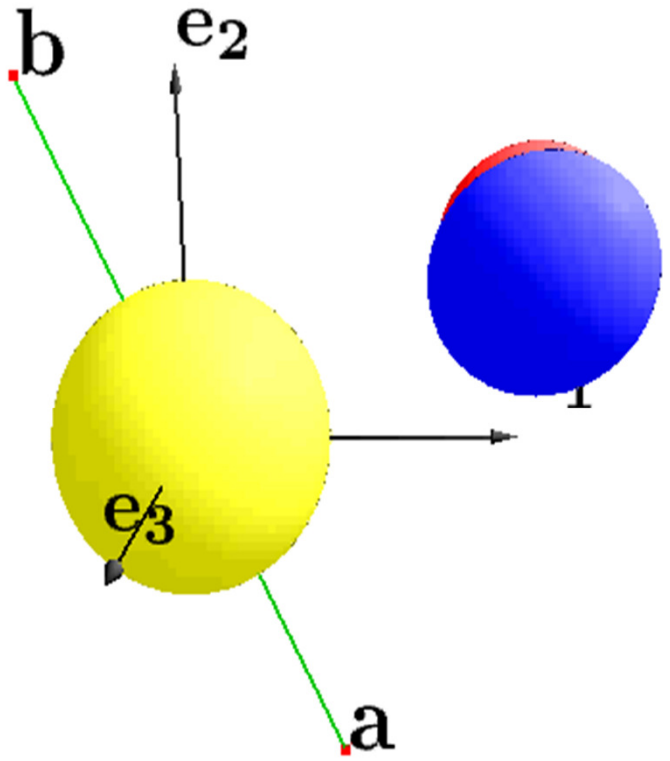
$$e^{-\frac{\phi}{2}\mathbf{l}}$$

= Quaternion

$$Q = \cos(\frac{\phi}{2}) + L\sin(\frac{\phi}{2})$$

- Note.: the line can be arbitrary

-> Rotation about an arbitrary axis(dual quaternion)



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The inner product

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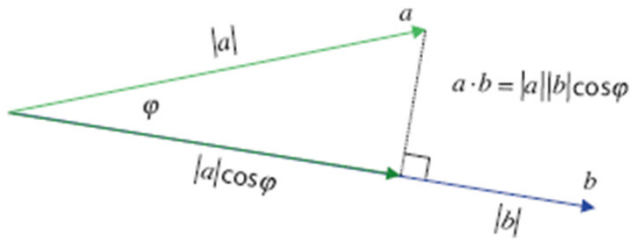
$$a \cdot b$$

- a,b 3D vectors
  -> scalar product

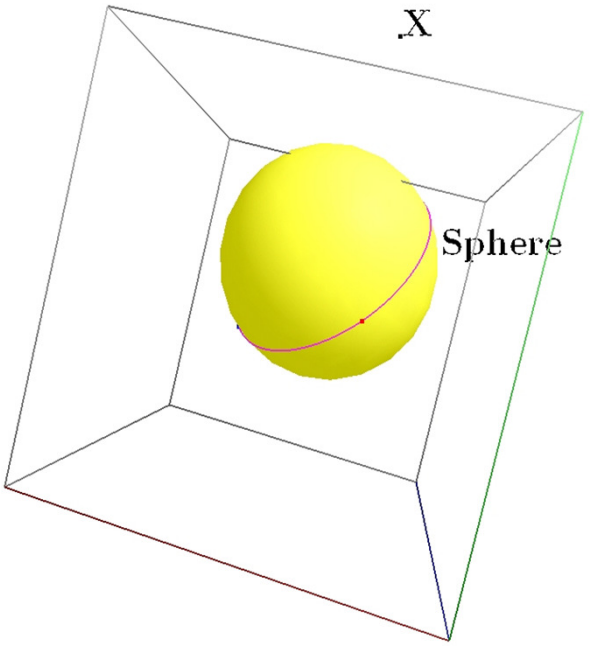


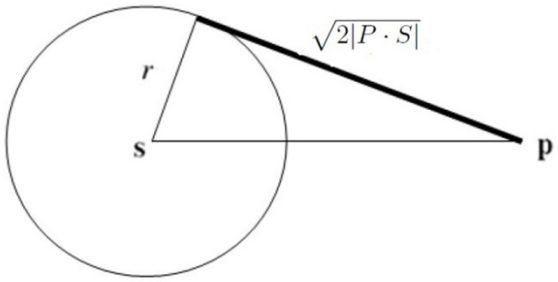
Distances

$a \cdot b$

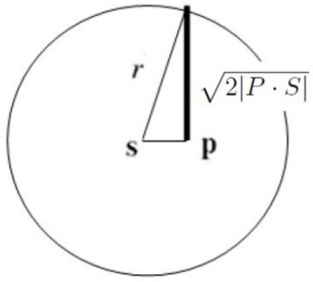
One formula for

- distance of two points
- distance between point and plane
- is a point inside or outside of a sphere? $a^2 = -2S \cdot P$





a)



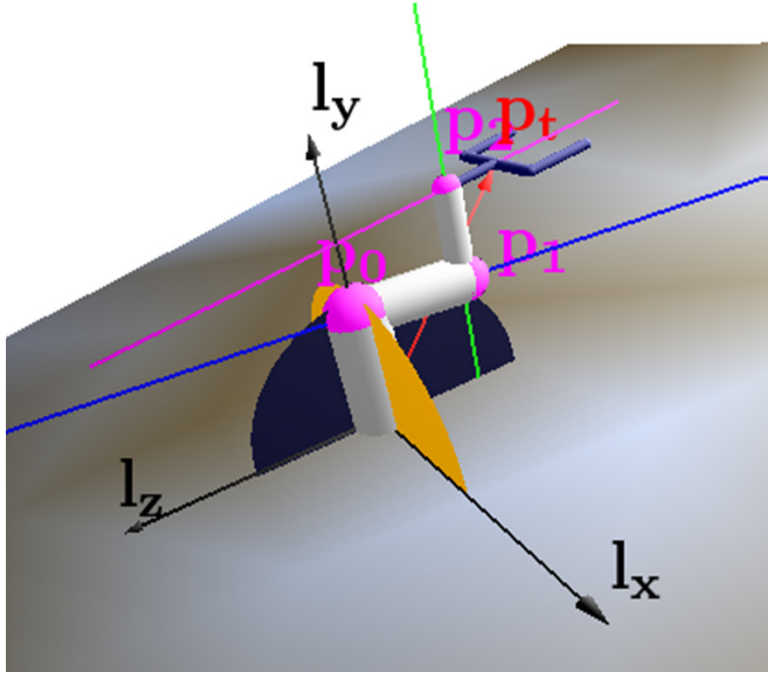
b)

Angles

$$\cos(\theta) = a \cdot b$$

One formula for the angle between

- 2 lines
- 2 planes
- 2 circles
- …



Properties of Conformal Geometric Algebra

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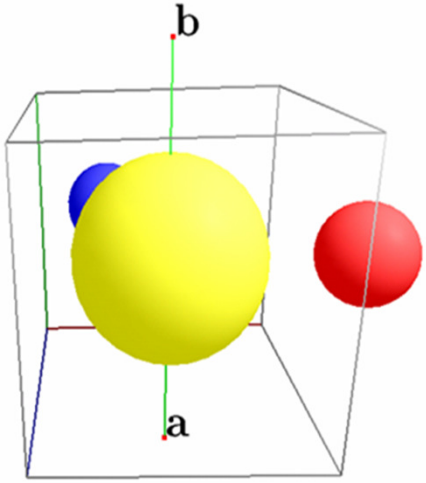
- Easy calculations with geometric objects and transformations
  - Geometric intuitiveness
  - Simplicity
  - Compactness
- Unification of mathematical systems
  - Complex numbers
  - Vector algebra
  - Quaternions
  - Projective geometry
  - Plücker coordinates
  - ….



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First commercial product

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- since 2007:
  - Real time lighting engine Enlighten
  - Geomerics, Cambridge UK
  - Unreal Game Engine

Our Applications

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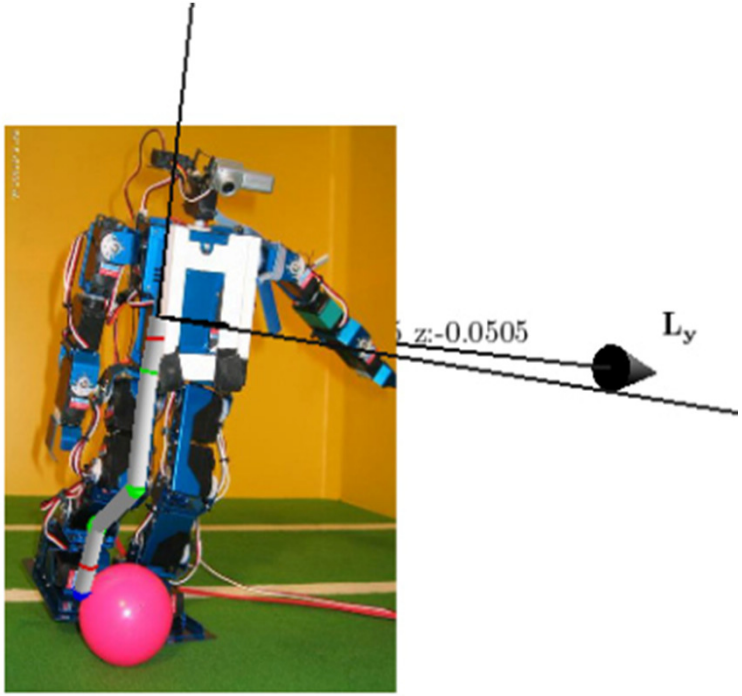
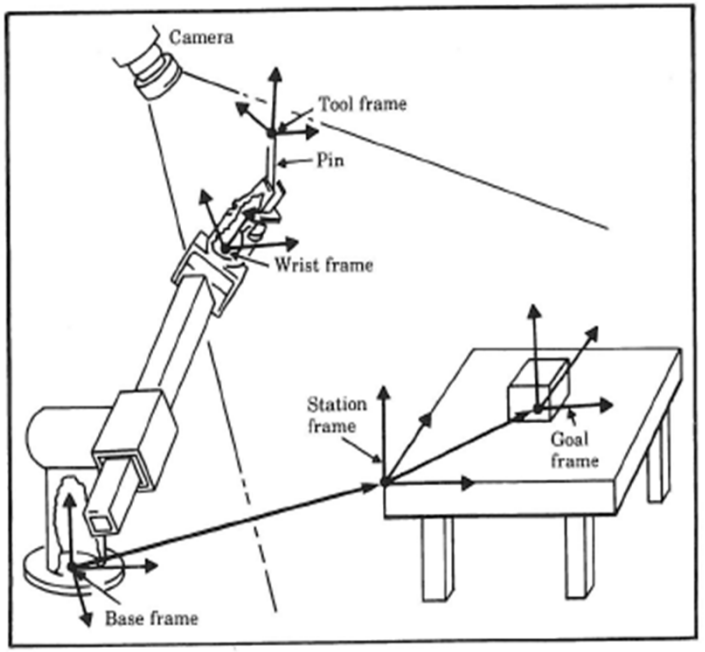
- Robot kinematics





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Application

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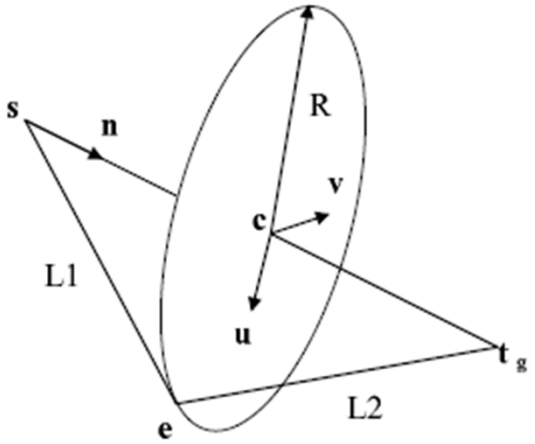
- Computer animation/
  Virtual reality



[Tolani et al. 2000]

Application

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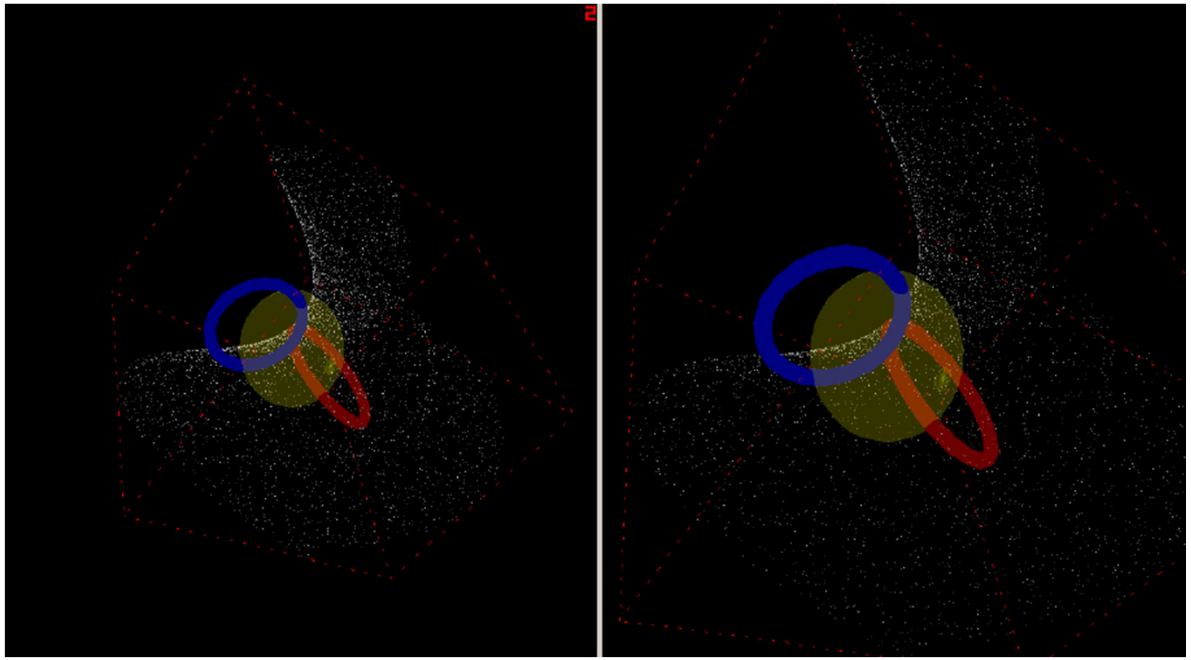
- Approximation of geometric objects to point clouds of laser scans



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Application

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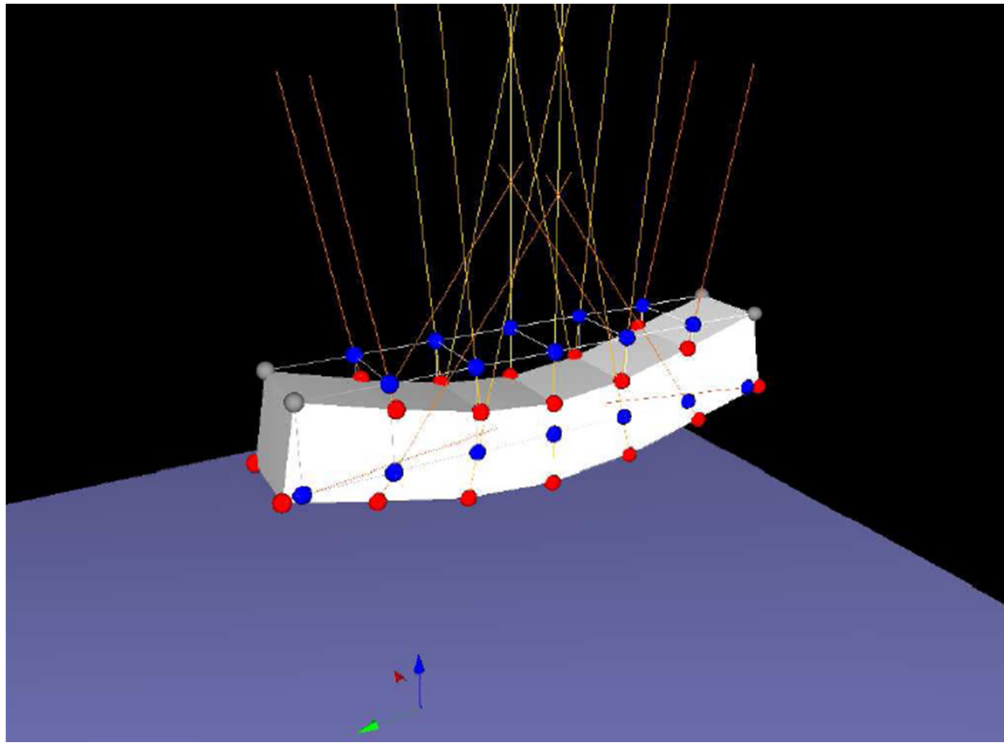
- Finite Element Solver
  - compact expressions for
    - velocities
    - forces
  - combining rotational and linear parts

  [Elmar Brendel et al]



cocoon

Application

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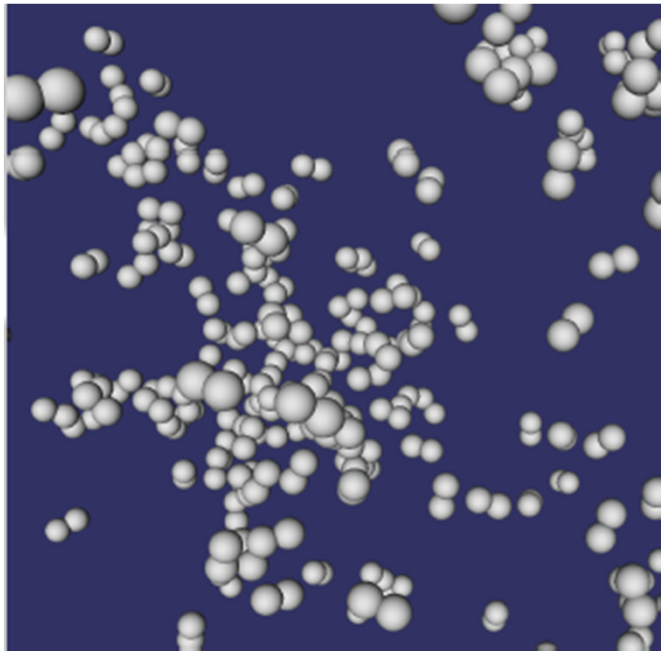
- Molecular dynamics simulation



Application within Cocoon

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- EM simulation

Vector Algebra

$$\nabla \cdot \boldsymbol{E} = \rho$$
$$\nabla \times \boldsymbol{E} = -\partial_t \boldsymbol{B}$$
$$\nabla \cdot \boldsymbol{B} = 0$$
$$\nabla \times \boldsymbol{B} = \boldsymbol{J} + \partial_t \boldsymbol{E}$$

Geometric Algebra

$$\nabla \cdot \boldsymbol{E} = \rho$$
$$\nabla \wedge \boldsymbol{E} = -\partial_t (I\boldsymbol{B})$$
$$\nabla \cdot \boldsymbol{B} = 0$$
$$\nabla \wedge \boldsymbol{B} = I(\boldsymbol{J} + \partial_t \boldsymbol{E})$$

Replace:
scalar product with *inner product*
cross product with *outer product*

Geometric Algebra

$$\nabla \boldsymbol{E} = \rho - \partial_t (I\boldsymbol{B})$$
$$\nabla \boldsymbol{B} = 0 + I(\boldsymbol{J} + \partial_t \boldsymbol{E})$$
$$\Leftrightarrow$$
$$\nabla (I\boldsymbol{B}) = -(\boldsymbol{J} + \partial_t \boldsymbol{E})$$

Geometric Product:
Combine inner and outer product

$$\boldsymbol{a}\,\boldsymbol{b} = \boldsymbol{a} \cdot \boldsymbol{b} + \boldsymbol{a} \wedge \boldsymbol{b}$$

Finally combine them to a **single multivector**:

$$\nabla (\boldsymbol{E} + I\boldsymbol{B}) + \partial_t (\boldsymbol{E} + I\boldsymbol{B}) = \rho - \boldsymbol{J}$$

- One formula for Maxwell equations

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Horizon Example

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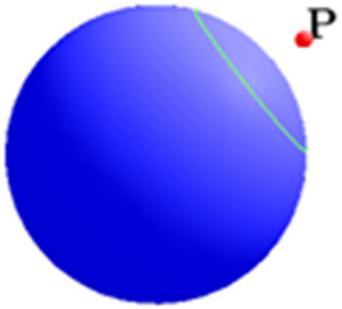
- Compute the horizon circle with
  - observer *P*
  - Earth *S*



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Warum Horizont-Aufgabe?

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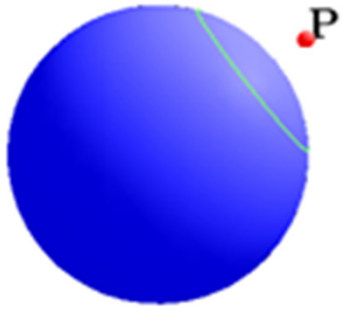
- Nachweis von
  - Nähe der algebraischen Beschreibung zur geometrischen Vorstellung
  - Einfachheit/Kompaktheit der Algorithmen



The Solution

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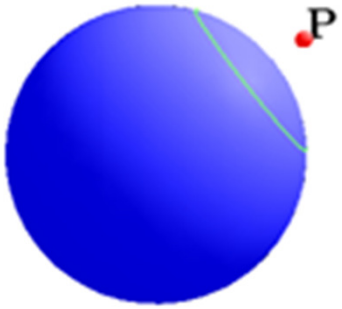
- *P*, *S*, … *C*?



The Solution

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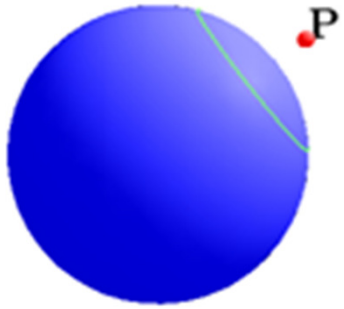
- Which sphere intersects with the sphere *S* at the horizon circle?

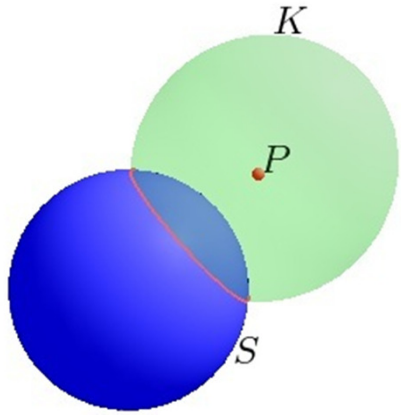


The Solution

TECHNISCHE UNIVERSITÄT DARMSTADT

- Which sphere intersects with the sphere *S* at the horizon circle?

  Sphere *K* with center *P* and radius such that it touches *S*



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The solution

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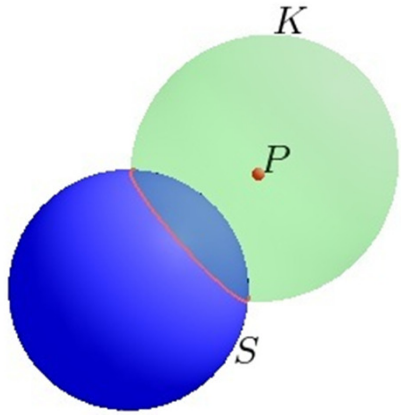
- Which sphere intersects with the sphere *S* at the horizon circle?
  Sphere *K* with center *P* and radius such that it touches *S*

- The radius of *K*?



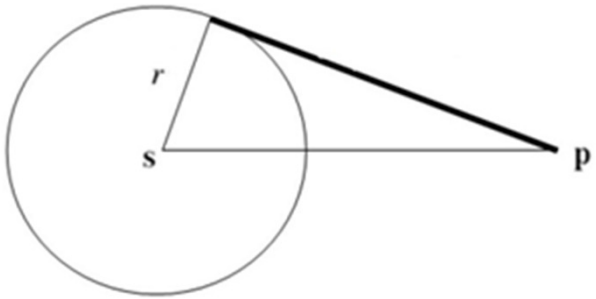
The solution

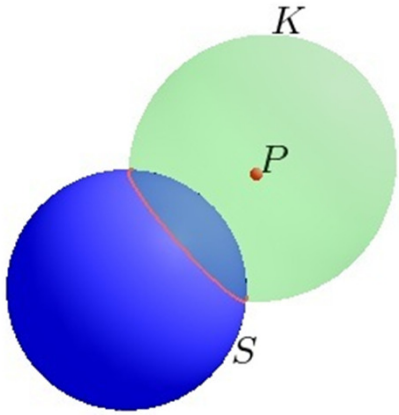
TECHNISCHE UNIVERSITÄT DARMSTADT

- Which sphere intersects with the sphere *S* at the horizon circle?
  Sphere *K* with center *P* and radius such that it touches *S*

- The radius a of *K*?

$$a^2 = -2S \cdot P$$





The solution

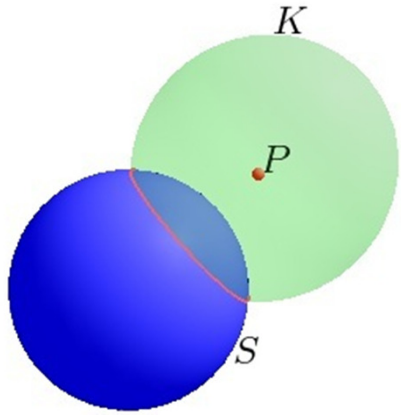
TECHNISCHE UNIVERSITÄT DARMSTADT

- Which sphere intersects with the sphere *S* at the horizon circle?
  Sphere *K* with center *P* and radius such that it touches *S*

- The radius a of *K*?

$$a^2 = -2S \cdot P$$

- How to describe the sphere *K*?



The solution

TECHNISCHE UNIVERSITÄT DARMSTADT

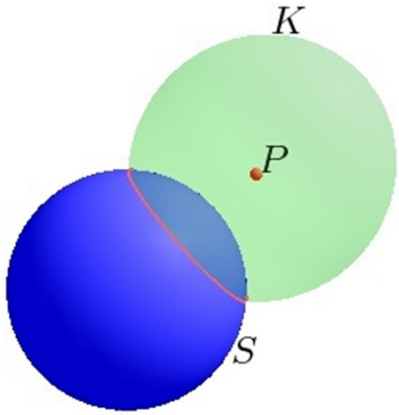
- Which sphere intersects with the sphere *S* at the horizon circle?
  Sphere *K* with center *P* and radius such that it touches *S*

- The radius a of *K*?

$$a^2 = -2S \cdot P$$

- How to describe the sphere *K*?

$$K = P - \frac{1}{2}a^2 e_\infty = P + (S \cdot P)e_\infty$$



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The solution

TECHNISCHE UNIVERSITÄT DARMSTADT

- Which sphere intersects with the sphere *S* at the horizon circle?
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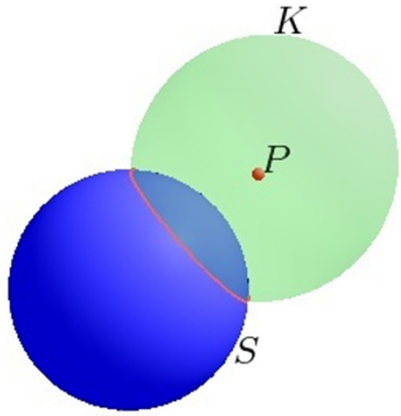
$$a^2 = -2S \cdot P$$

- How to describe the sphere *K*?

$$K = P - \frac{1}{2}a^2 e_\infty = P + (S \cdot P)e_\infty$$

- The formula for the horizon circle *C*?



The solution

TECHNISCHE UNIVERSITÄT DARMSTADT

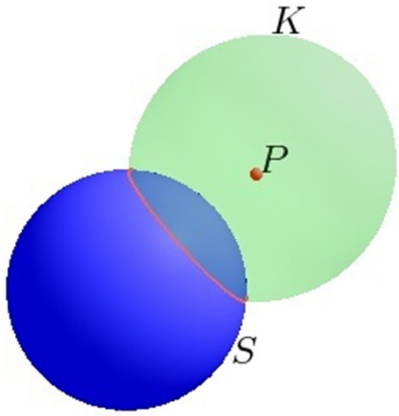
- Which sphere intersects with the sphere *S* at the horizon circle?
  Sphere *K* with center *P* and radius such that it touches *S*

- The radius a of *K*?

$$a^2 = -2S \cdot P$$

- How to describe the sphere *K*?

$$K = P - \frac{1}{2}a^2 e_\infty = P + (S \cdot P)e_\infty$$



- The formula for the horizon circle *C*?

$$C = S \wedge K$$

$$C = S \wedge (P + (S \cdot P)e_\infty).$$

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Properties of Geometric Algebra

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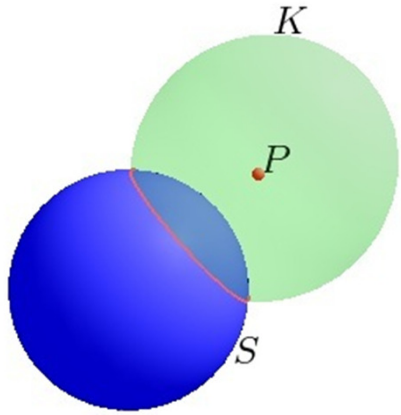
- geometrically intuitive?
- simple?
- compact?
- role of coordinates?

$$C = S \wedge (P + (S \cdot P)e_\infty).$$



How to support the widespread use of geometric algebra?

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- What form of geometric algebra do we need today?

"there will come a time when these ideas, perhaps in a new form, will arise anew and will enter into a living communication with contemporary developments.“ [Grassmann 1862]

- Making it accessible for as many people as possible and their applications
- -> provide a suitable computing technology

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Geometric Algebra Computing

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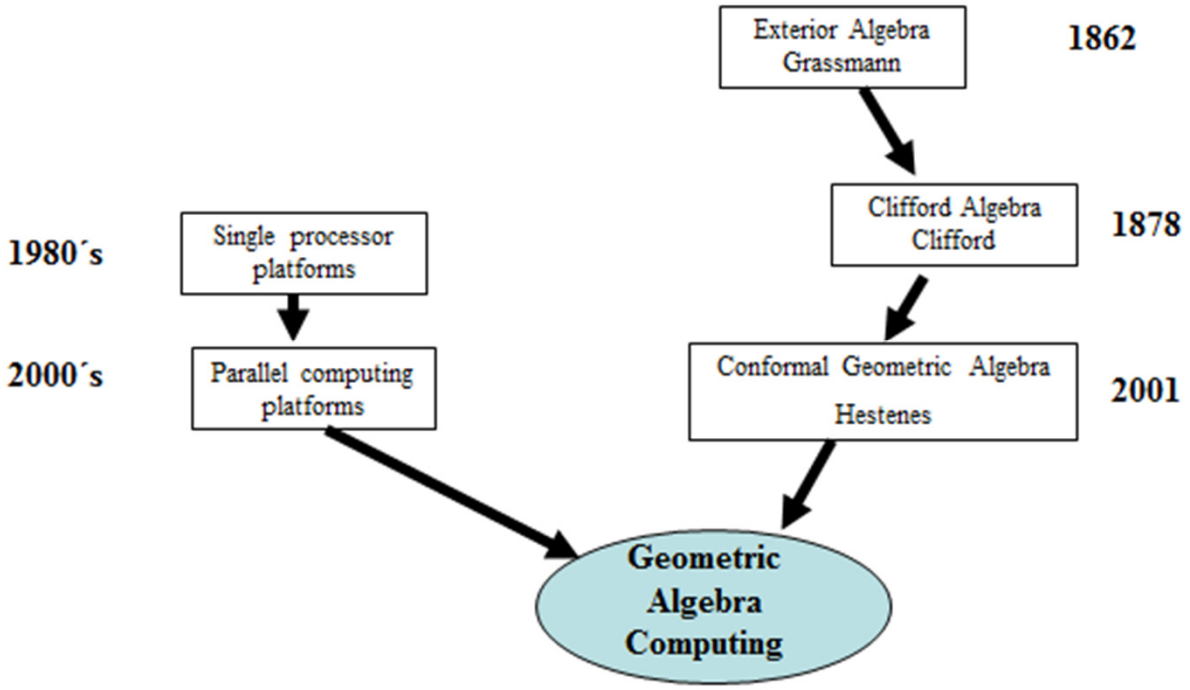
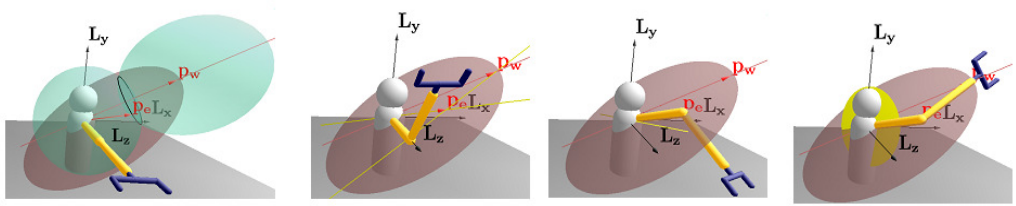


Goals of Geometric Algebra Computing

- **G**eometric **a**lgebra **al**gorithms ***op***timizer (www.gaalop.de)

- Easy development process
  - integration in standard programming languages
  - Intuitive and compact descriptions based on geometric algebra
  - leading to
    - reduction in development time
    - better maintainability
- High performance and robustness of the implementation

Proof-of-Concept Application



- Visual and interactive development
- Compact algorithms (about 30%)
- First implementation:
  - 50 times slower than conventional implementation
- Eurographics 2006:
  - Paper " Competitive runtime performance for inverse kinematics algorithms using conformal geometric algebra " by Dietmar Hildenbrand, Daniel Fontijne, Yusheng Wang, Marc Alexa and Leo Dorst
  - First geometric algebra implementation that was faster than the conventional implementation

Geometric Algebra Computing Architecture

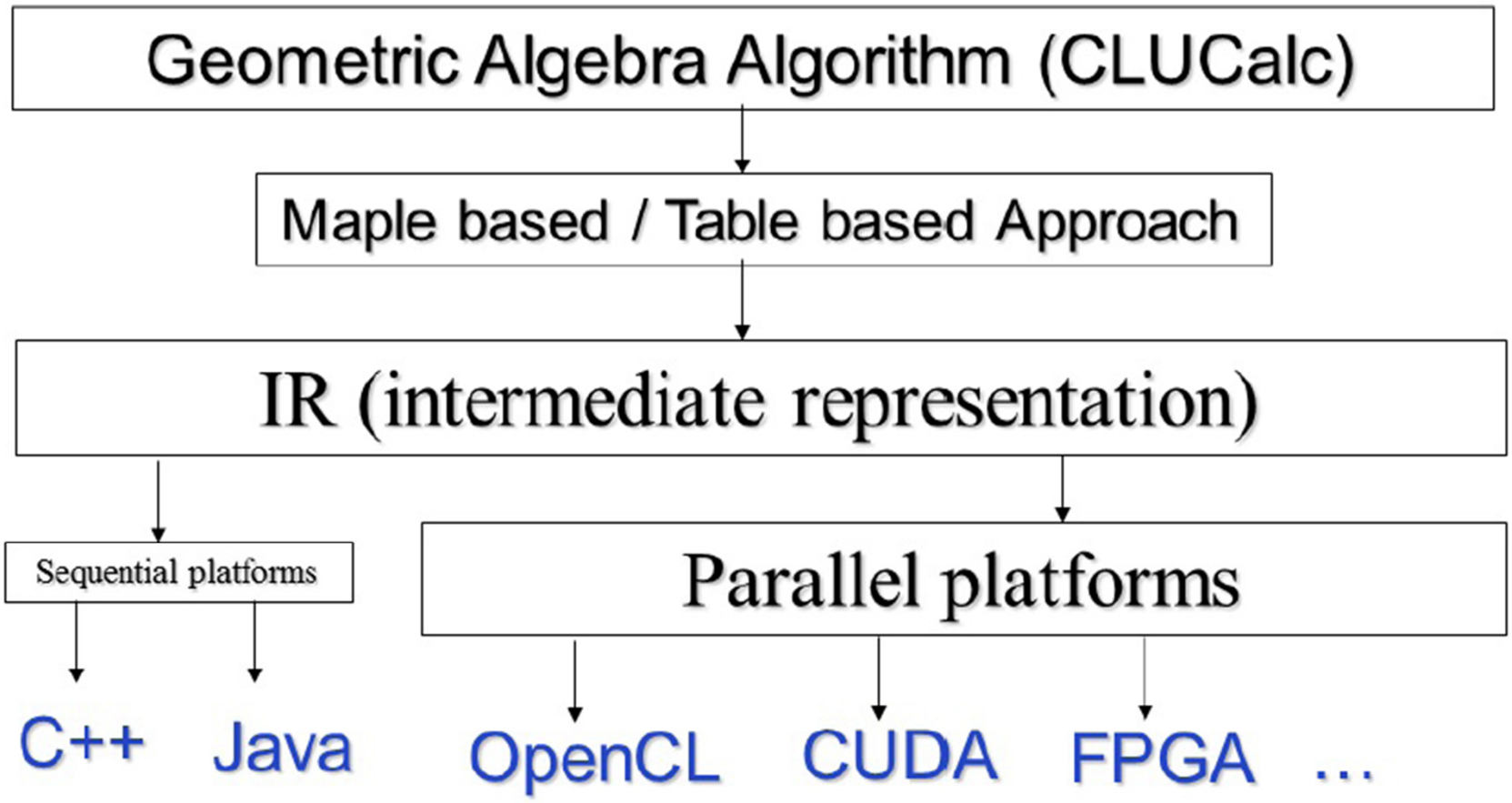


Table based Compilation Example

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- Geometric product multiplication table in 3D GA:

		b		$\mathbf{b_1}$	$\mathbf{b_2}$	$\mathbf{b_3}$				
			E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8
a			1	e_1	e_2	e_3	e_{12}	e_{23}	e_{13}	e_{123}
	E_1	1	0	0	0	0	0	0	0	0
$\mathbf{a_1}$	E_2	e_1	0	E_1	E_5	E_7	0	0	0	0
$\mathbf{a_2}$	E_3	e_2	0	$-E_5$	E_1	E_6	0	0	0	0
$\mathbf{a_3}$	E_4	e_3	0	$-E_7$	$-E_6$	E_1	0	0	0	0
	E_5	e_{12}	0	0	0	0	0	0	0	0
	E_6	e_{23}	0	0	0	0	0	0	0	0
	E_7	e_{13}	0	0	0	0	0	0	0	0
	E_8	e_{123}	0	0	0	0	0	0	0	0

- GA algorithm:

```
a=a1*e1+a2*e2+a3*e3;
b=b1*e1+b2*e2+b3*e3;
?c=a*b;
```

resulting C code:

```
c[1]=a1*b1+a2*b2+a3*b3;
c[5]=a1*b2-a2*b1;
c[6]=a2*b3-a3*b2;
c[7]=a1*b3-a3*b1;
```


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Table based Compilation Example

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			E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8
a			1	e_1	e_2	e_3	e_{12}	e_{23}	e_{13}	e_{123}
	E_1	1	0	0	0	0	0	0	0	0
$\mathbf{a_1}$	E_2	e_1	0	E_1	E_5	E_7	0	0	0	0
$\mathbf{a_2}$	E_3	e_2	0	$-E_5$	E_1	E_6	0	0	0	0
$\mathbf{a_3}$	E_4	e_3	0	$-E_7$	$-E_6$	E_1	0	0	0	0
	E_5	e_{12}	0	0	0	0	0	0	0	0
	E_6	e_{23}	0	0	0	0	0	0	0	0
	E_7	e_{13}	0	0	0	0	0	0	0	0
	E_8	e_{123}	0	0	0	0	0	0	0	0

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```


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Horizon Example in Gaalop

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- based on the CLUCalc language (www.clucalc.info)

```
P = VecN3(px,py,pz);          // view point
M = e0;                       // center point of earth set to origin
S = M-0.5*r*r*einf;           // sphere representing earth
K = P+(P.S)*einf;             // sphere around P
?C=S^K;                       // intersection circle
```


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Horizon Example in Gaalop

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- 32-dim multivector array with optimized coefficient computations

```
C[8] = 0.5f * px * r * r;   // e1^einf
C[9] = -px;                 // e1^e0
C[11] = 0.5f * py * r * r;  // e2^einf
C[12] = -py;                // e2^e0
C[13] = 0.5f * pz * r * r;  // e3^einf
C[14] = -pz;                // e3^e0
C[15] = -r * r;             // einf^e0
```

- … to be further optimized from the storage point of view

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Horizon Example in Gaalop

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- 32-dim multivector array with optimized coefficient computations

```
C[8] = 0.5f * px * r * r;   // e1^einf
C[9] = - px;                // e1^e0
C[11] = 0.5f * py * r * r;  // e2^einf
C[12] = - py;               // e2^e0
C[13] = 0.5f * pz * r * r;  // e3^einf
C[14] = - pz;               // e3^e0
C[15] = - r * r;            // einf^e0
```

- … to be further optimized from the storage point of view

This kind of very easy arithmetic expressions lead to high runtime performance and robustness

Gaalop Precompiler

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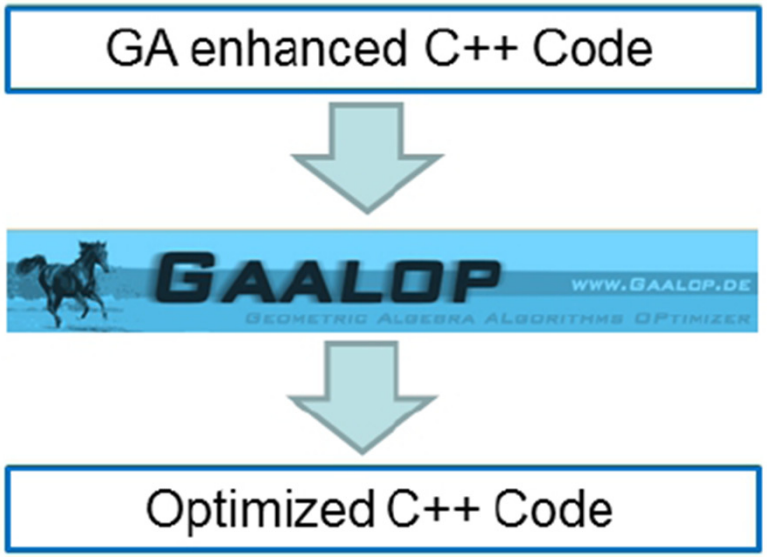
```
#pragma gpc begin
  ...
  Import of multivectors (if needed)
  ...
  #pragma clucalc begin
    ...
    Geometric Algebra code based on CLUCalc
    ...
  #pragma clucalc end
  ...
  Export of multivectors (if needed)
  ...
#pragma gpc end
```



Gaalop Precompiler

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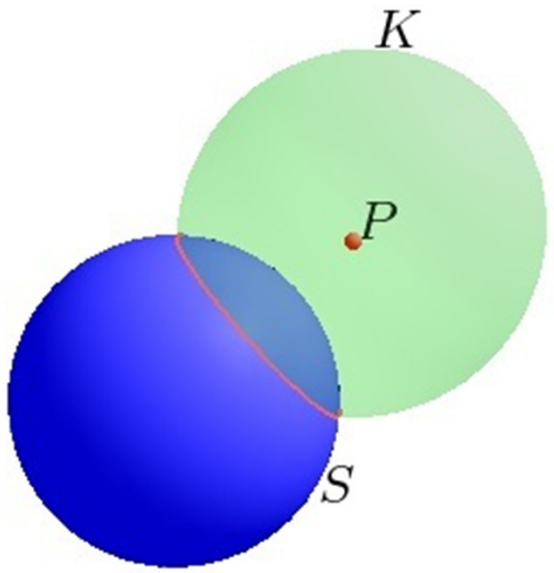
- Visualization of the horizon example

```
void horizon() {
  #pragma clucalc begin
     :Black;
     :P = VecN3(1,1,0);
      r = 1;
     :Blue;
     :S = e0-0.5*r*r*einf;
     :Color(0,1,0,0.2);
     :K = P+(P.S)*einf;
     :Red;
     :C = S^K;
  #pragma clucalc end
}
```



Gaalop Precompiler

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- Horizon computations in C++

```
#include <iostream>
#include <gp.h>

int main() {
  #pragma gpc begin
    #pragma clucalc begin
      P = VecN3(1,1,0);
      r = 1;
      S = e0-0.5*r*r*einf;
      C = S^(P+(P.S)*einf);
      ?homogeneousCenter = C*einf*C;
      ?scale = -homogeneousCenter.einf;
      ?EuclideanCenter = homogeneousCenter / scale;
    #pragma clucalc end
    std::cout << mv_get_bladecoeff(homogeneousCenter,e0) << std::endl;
    std::cout << mv_get_bladecoeff(EuclideanCenter,e0) << std::endl;
    std::cout << mv_get_bladecoeff(EuclideanCenter,e1)
    << "," << mv_get_bladecoeff(EuclideanCenter,e2)
    << "," << mv_get_bladecoeff(EuclideanCenter,e3);
  #pragma gpc end
  return 0;
}
```


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Gaalop Precompiler

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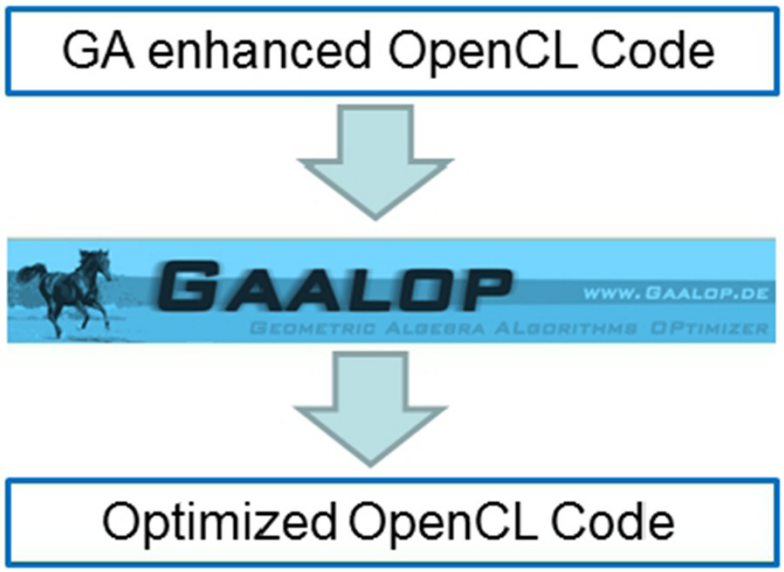
- Gaalop GPC for OpenCL



Gaalop Precompiler

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- Horizon example
- in OpenCL

```
__kernel void horizonKernel(__global
float* circleCenters, __global const float* points,
const unsigned int num_points)
{
  const int id = get_global_id(0);
  #pragma gpc begin
    P = VecN3(points[id],
              points[id+num_points],
              points[id+2*num_points]);
    #pragma clucalc begin
      r = 1;
      S = e0-0.5*r*r*einf;
      C = S^(P+(P.S)*einf);

      ?homogeneousCenter = C*einf*C;
      ?scale = -homogeneousCenter.einf;
      ?EuclideanCenter = homogeneousCenter / scale;
    #pragma clucalc end
    circleCenters = mv_to_stridedarray(EuclideanCenter,
                                       id, num_points, e1,e2,e3);
  #pragma gpc end

}
```


Gaalop GCD for OpenCL …

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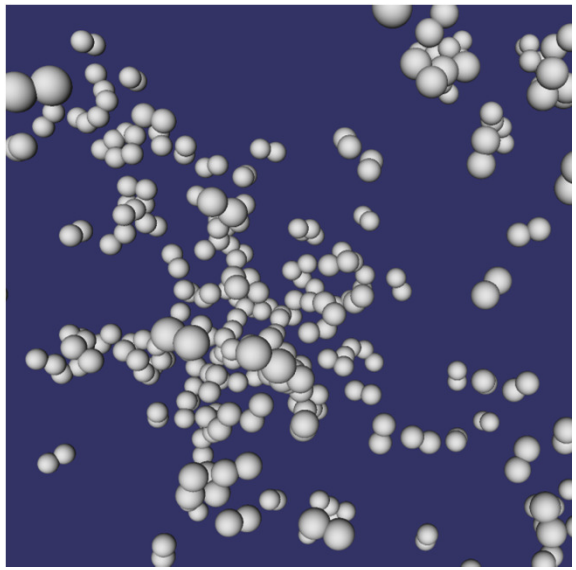
- … available for Windows and Linux (www.gaalop.de).

- Application example: Molecular dynamics …



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Results

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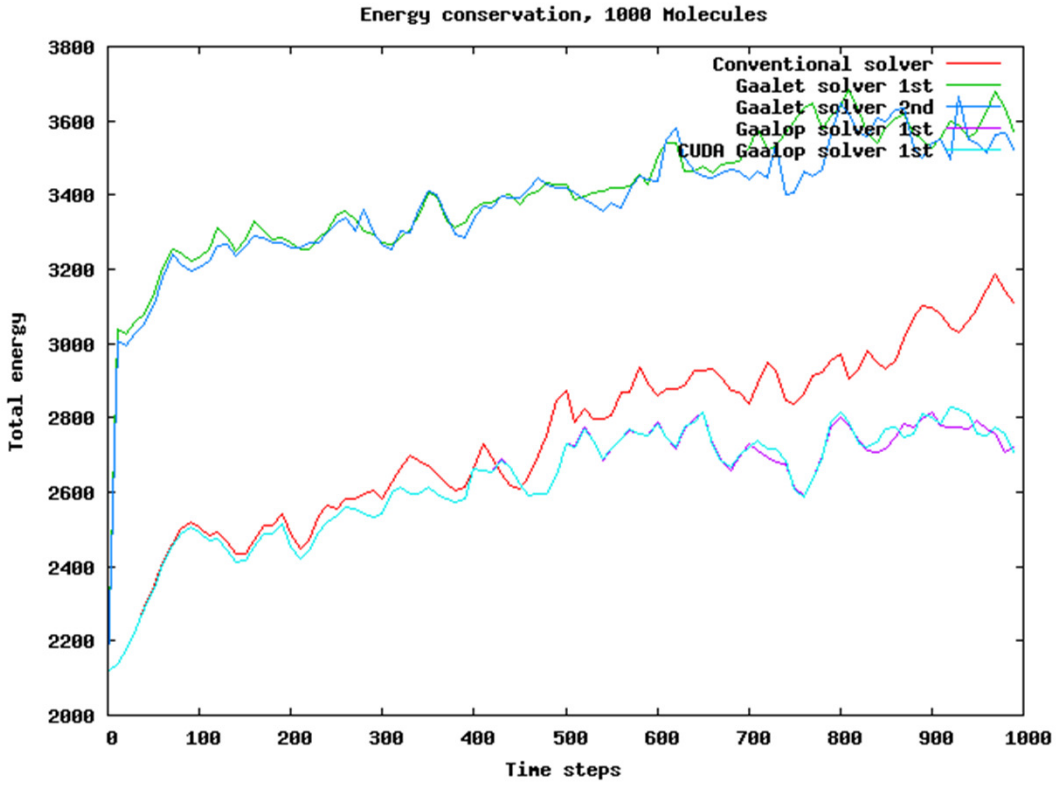
- Better runtime performance
- Better Numerical Stability (Energy Conservation)



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Conclusion

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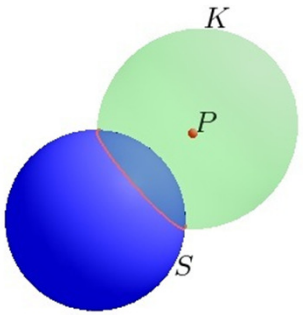
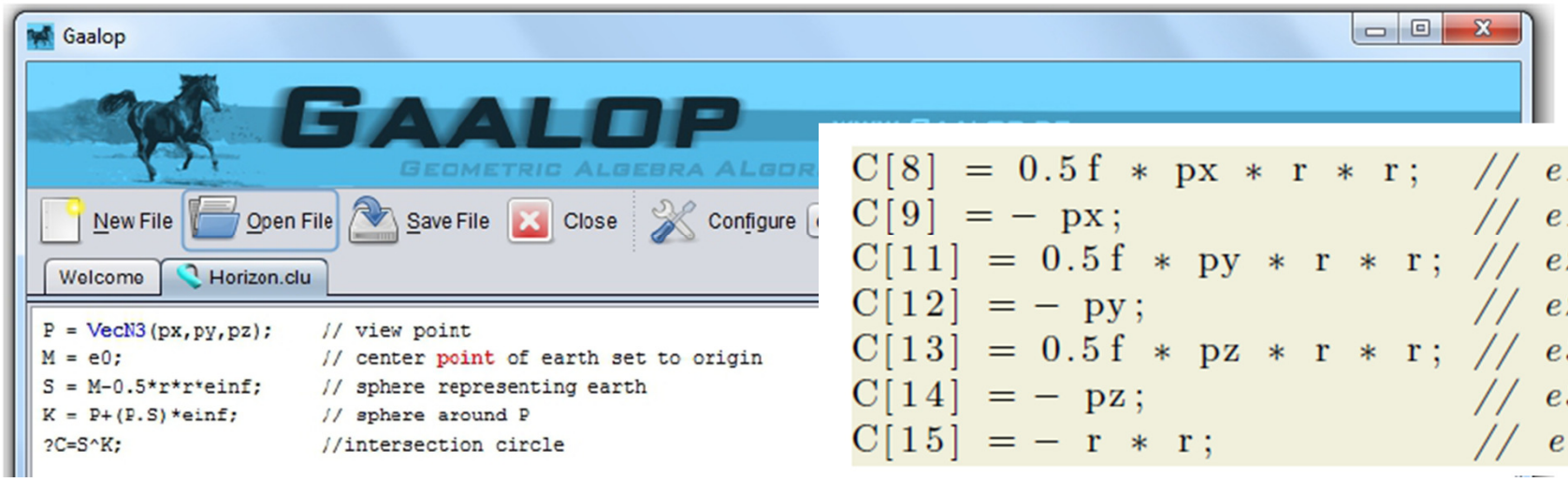
- Geometrically intuitive



- Fast and robust implementations




```
C[8] = 0.5f * px * r * r;   // e1^einf
C[9] = - px;                // e1^e0
C[11] = 0.5f * py * r * r;  // e2^einf
C[12] = - py;               // e2^e0
C[13] = 0.5f * pz * r * r;  // e3^einf
C[14] = - pz;               // e3^e0
C[15] = - r * r;            // einf^e0
```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Reference

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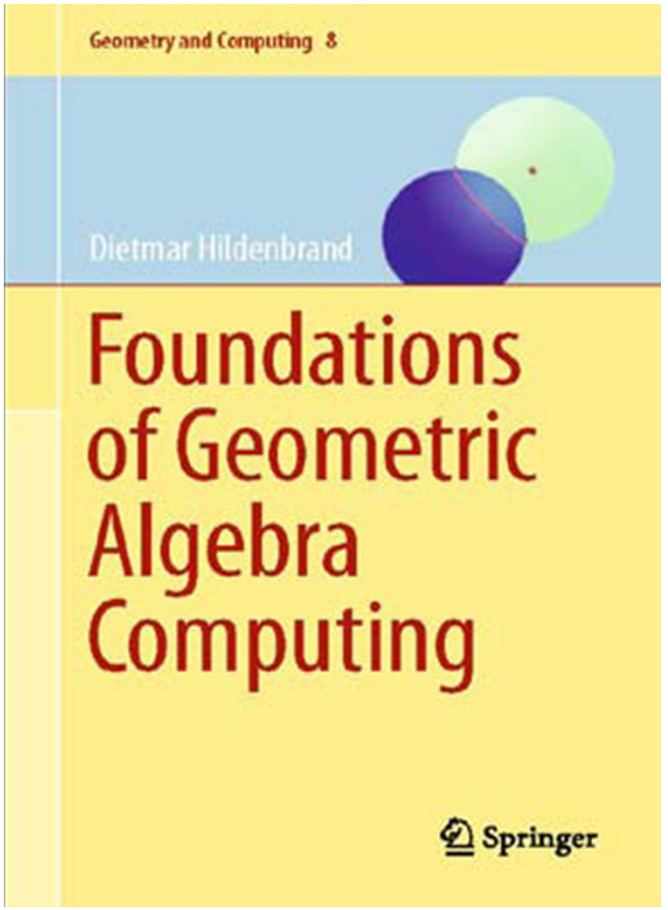
- „Foundations of Geometric Algebra Computing“
- Dietmar Hildenbrand
- Springer, Nov. 2012



- Article in Elektronik-Praxis:

http://www.elektronikpraxis.vogel.de/grundlagenwissen/articles/376044/

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Übungsaufgaben

Übungsthemen

TECHNISCHE UNIVERSITÄT DARMSTADT

- Mögliche Aufgaben:
  - Kegelschnitte und Quadriken in GA: Kapitel 6d/8d von Christian + 9D von Julio
  - Paper „Coordinate Free Perspective Projection of Points in the conformal Model“ von Stephen Mann
  - Beamforming-Paper
  - Selig-Paper zur Dynamik mit 8D
  - …

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Übungsthemen

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- Mögliche Aufgaben:
  - Bsp.-Algorithmen für compass-ruler algebra mit Vergleichen zu herkömmlichen Algorithmen
  - Raytracer (Basis: OpenCL-Raytracer)
  - Game-Engine-Algorithmen mit Test und Vergleich
  - EM-Simulation mit OpenCL
  - Parallele Pi-Suche mit OpenCL
  - eigene Ideen?

  - Gaalop-Themen …

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Gaalop-Erweiterungen I

TECHNISCHE UNIVERSITÄT DARMSTADT

- Precompiler-Erweiterung
  - Visualisierung konformer Objekte
  - Visualisierung für 6d/8d
  - OpenMP / Performance?
  - CluCalc-Visualisierungs-Integration (in C- und Java-Version) in Gaalop?
  - Precompiler für Mathematica?
  - Precompiler für Java?
  - Gaalop -> LaTeX in lineare Algebra-Form (Skalar/Kreuzprodukte etc.)

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Gaalop-Erweiterungen II

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- Precompiler-Erweiterung
  - Gaalop for OpenACC
    - Cmake-Anpassung
    - OpenACC-Backend

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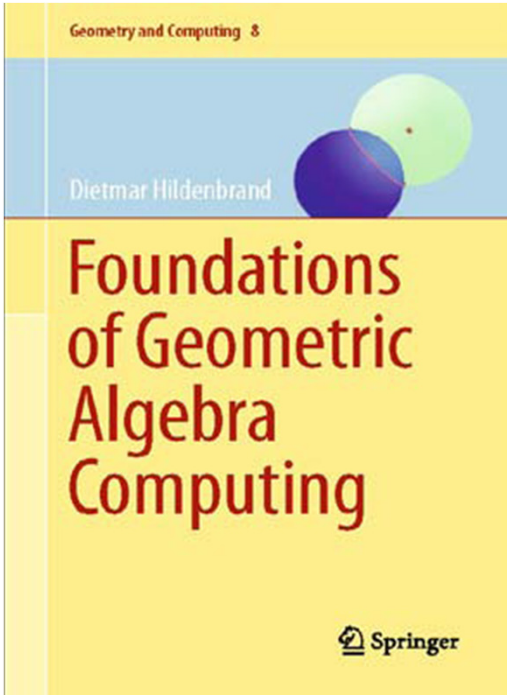
Thanks a lot

Google Dietmar Hildenbrand



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