

# Effects of Isospin Asymmetry and In-Medium Corrections on Balance Energy

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## Abstract

The effects of an isospin asymmetry and in-medium corrections to the nucleon collision cross section on the balance energy are explored. The BUU model for intermediate energy heavy-ion collisions is used with isospin-dependent mean fields to calculate the balance energies of  $^{58}\text{Fe} + ^{58}\text{Fe}$  and  $^{58}\text{Ni} + ^{58}\text{Ni}$  for a range of impact parameters. We find that we are able to reproduce the impact parameter dependence of the balance energy, and the sign (but not the magnitude) of the shift in balance energy as a function of isospin asymmetry.

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## I. INTRODUCTION

The bulk properties of nuclear matter under extreme conditions are of interest to both nuclear physics and astrophysics. To generate these conditions, we collide nuclei in search for hints on the nature of the nucleus-nucleus interaction. Theoretical studies of these events allow us to make a connection between models and experiment through observables such as the balance energy [1–3]. Balance energy, or the beam energy at which transverse, in-plane directed flow [4] changes sign as a function of beam energy, is sensitive to the forces underlying the dynamics of these collisions. Thus its study can yield valuable information about the complex interplay of the repulsive nature of the hard scattering nucleon cross section, the repulsion and attraction of the nuclear mean field, and the repulsive contribution of the Coulomb force.

The BUU (Boltzmann-Uehling-Uhlenbeck) [13–18] transport model is successful in reproducing and predicting experimental magnitudes and trends in collective single-particle observables [19–21], such as the balance energy, and we use it here. BUU is a theory for the single-particle phase space distribution of baryons (plus resonances and mesons). In its most popular numerical realization, the model propagates test particles with Hamilton’s equations through the influence of experimentally observed nucleon collision cross sections [22], the Coulomb field, and a nuclear mean field based on a Skyrme-type interaction [23] which typically depends upon local density and momentum.

However, the inter-nucleon potential is not symmetric with respect to the isospin of the nucleon. Protons and neutrons experience different forces due to their constituents. This means that an asymmetry term exists in the nuclear mean-field. In addition, the nucleon cross section is modified from its vacuum values in nuclear matter [24–26].

Work by Müller and Serot [5] has shown that the liquid-gas phase transition may in fact be of second rather than first order. Their work, based upon a thermodynamic approach, demonstrated that local chemical instabilities are responsible for the transition when an asymmetric potential is used. This contrasts with the common view that mechanical instabilities are responsible for the onset of fragmentation in excited, diffuse systems. Studies using the percolation model [6] also have been used to explore the isospin degree of freedom in the nuclear fragmentation transition [7,8]. The new experimental facilities coming on-line which will probe heavy-ion dynamics near the drip lines affords an opportunity to explore these results.

In stellar evolution the softening of the compressibility of nuclear matter as neutron ratios deviate from 1/2 is critical for generating a supernova explosion for massive stars ( $12 M_{\odot}$  to  $15 M_{\odot}$ ) [9]. Core collapse cannot sustain a shock if the nuclear Equation of State (EoS) retains its equilibrium stiffness; the result is at least a delay of such important process as nucleosynthesis.

Neutron stars cool by emitting neutrinos. After an initial fast-cooling phase, they enter a more sedate cooling epoch producing neutrinos via the modified URCA process:  $(n, p) + p + e^{-} \rightarrow (n, p) + n + \nu_e$ ,  $(n, p) + n \rightarrow (n, p) + p + e^{-} + \bar{\nu}_e$ . However, if the proton density exceeds a critical value between 11% and 15%, which is dependent upon the symmetry energy, the direct URCA process can occur and become the dominant cooling mechanism [10]:  $n \rightarrow p + e^{-} + \bar{\nu}_e$ ,  $p + e^{-} \rightarrow n + \nu_e$ .

The exploration of the isospin degree of freedom in heavy ion collision thus carries the

promise that it will help determine the answers to these questions of astrophysical relevance [11].

Recently, it was experimentally shown [12] that the balance energy for symmetric systems of equal mass has different values for different isospin asymmetry of the colliding nuclei. In this paper we therefore study the isospin-dependence of the balance energy, attempt to isolate its origin, and hope to find sensitivity to the isospin-dependence of the nuclear mean field.

## II. MODEL INPUT

The BUU transport equation evolves in time a single nucleon phase-space distribution under the influences of nucleon-nucleon collisions, the Coulomb field, and a nuclear mean field. In this study, we choose two mean fields: One recently used by Bao-An Li, *et al.* [29,30] (also see Ref. [31–33]):

$$U = A \left( \frac{\rho}{\rho_0} \right) + B \left( \frac{\rho}{\rho_0} \right)^\sigma + C \tau_z \left( \frac{\rho_n - \rho_p}{\rho_0} \right), \quad (2.1)$$

where  $\rho_0$  is the normal nuclear density,  $\rho_n$  is the neutron density,  $\rho_p$  is the proton density, and  $\tau_z$  is the isospin factor which is 1 for neutrons and  $-1$  for protons. Coefficients  $A$ ,  $B$  and  $\sigma$  are typically chosen to match the ground state properties of symmetric nuclear matter such as the saturation density and saturation binding energy. The compressibility is a free parameter, and in this study we chose it to be 200 MeV. This gives  $A = -109$  MeV,  $B = 82$  MeV, and  $\sigma = \frac{7}{6}$ . In keeping with Bao-An Li’s work,  $C = 32$  MeV.

The other mean field is derived from a Hamiltonian due to Sobotka [34]:

$$\mathcal{H} = \mathcal{K}\mathcal{E} + \frac{4a}{\rho_0}(\rho_n^2 + b\rho_n\rho_p + \rho_p^2) + \frac{4c}{\rho_0}(\rho_0^2\rho_p + \rho_n\rho_p^2). \quad (2.2)$$

Here the coefficients  $a = -3.66$  MeV,  $b = 15.0$  and  $c = 23.4$  MeV and  $\mathcal{K}\mathcal{E}$  is the kinetic energy term. Neutrons are acted upon by:

$$U_n = \frac{\partial \mathcal{H}}{\partial \rho_n} = 8a \frac{\rho_n}{\rho_0} + 4ab \frac{\rho_p}{\rho_0} + 8c \frac{\rho_n \rho_p}{\rho_0^2} + 4c \frac{\rho_p^2}{\rho_0^2}, \quad (2.3)$$

whereas protons are affected by:

$$U_p = \frac{\partial \mathcal{H}}{\partial \rho_p} = 8a \frac{\rho_p}{\rho_0} + 4ab \frac{\rho_n}{\rho_0} + 8c \frac{\rho_n \rho_p}{\rho_0^2} + 4c \frac{\rho_n^2}{\rho_0^2}. \quad (2.4)$$

In symmetric matter, this mean field reduces to the often-called “Stiff” equation of state (compressibility 380 MeV). It should be noted that there is no transformation of the mean field of Sobotka that will yield an equivalent “Soft” equation of state. Thus direct comparisons between the two formulations are impossible. One may simply alter  $A$ ,  $B$  and  $\sigma$  in Equation 2.1 to give the appropriate compressibility in symmetric matter.

With the isospin asymmetry

$$\delta = \frac{\rho_n - \rho_p}{\rho_0} \quad (2.5)$$

and the reduced density

$$\bar{\rho} = \frac{\rho}{\rho_0} \quad (2.6)$$

Equations 2.1 and 2.3 can be written as:

$$U = A\bar{\rho} + B\bar{\rho}^\sigma + C\tau_z\delta \quad (2.7)$$

and

$$U_n = \bar{\rho}(4a + 2ab) + \delta(4a - 2ab) - c\delta^2 + 3c\bar{\rho}^2 - 2c\delta\bar{\rho}, \quad (2.8)$$

for the neutron potential, and

$$U_p = \bar{\rho}(4a + 2ab) + \delta(2ab - 4a) - c\delta^2 + 3c\bar{\rho}^2 + 2c\delta\bar{\rho}, \quad (2.9)$$

where  $a$ ,  $b$  and  $c$  are as defined above. Figure 1 (left side) shows Equation 2.7 as neutron excess and normalized nuclear density are varied. The right side of the same figure shows Equation 2.8 as neutron excess and normalized nuclear density are varied. One can see that Equation 2.8 is more attractive to neutrons in the midst of proton-rich matter for all densities than Equation 2.7.

Other parameterization of nuclear mean field potential also exist, [35] but were not considered here.

The nucleon-nucleon cross sections are parameterization from the Particle Data Group [22] with medium modification implemented according to the density dependent prescription:

$$\sigma_{\text{NN}} = \sigma_{\text{NN}}^{\text{free}} (1 + \alpha \bar{\rho}) \quad (2.10)$$

where  $\alpha$  is the logarithmic derivative of the in-medium cross section with respect to the density, taken at  $\rho = 0$ ,

$$\alpha = \rho_0 \frac{\partial}{\partial \rho} (\ln \sigma_{\text{NN}})|_{\rho=0} \quad (2.11)$$

It is the lowest coefficient of a Taylor-expansion of the cross-section in powers of the density [25]. This is a parameterization of the Pauli-blocking of intermediate states is motivated by Brückner  $G$ -matrix theory [24]. One can show [26–28] that values of  $\alpha$  in the range between  $-0.4$  and  $-0.2$  yield the best agreement of the simple parameterization used here and the more involved  $G$ -matrix calculation using a realistic nucleon-nucleon interaction. In addition, it has been shown that an in-medium reduction of  $\alpha \approx -0.2$  provides the best reproduction of the flow signals of symmetric collisions for a broad range of energies and projectile-target combinations [36,25,3].

### III. ELEMENTARY CONSIDERATIONS

Before we proceed to show our numerical results, it is appropriate to attempt to understand the reasons why we would expect a difference in the balance energy for symmetric

systems of equal mass, but different isospin. There are three isospin effects that we may consider: the average effective nucleon-nucleon cross section, the difference in the Coulomb potential, and the isospin dependence of the mean field.

In order to understand the sign of the different effects, we first remind ourselves that the balance energy marks the beam energy at which the repulsive and attractive interactions are roughly equal. At beam energies lower than the balance energy, the attractive interaction dominates, and at energies higher than the balance energy, repulsive interactions determine the flow.

The difference in the Coulomb interaction is very simple. Fe has 26 protons and Ni 28. This means that the ratio of the Coulomb forces is  $(26/28)^2 = 0.862$ . Since the Coulomb force adds repulsion, we thus expect that the balance energy for nickel should be lower than for iron.

The difference in the isospin dependent mean field has the opposite effect. The isospin asymmetry,  $\delta$ , is three times bigger for iron than it is for nickel:

$$\delta(^{58}\text{Fe}) = 0.103, \quad \delta(^{58}\text{Ni}) = 0.034 \quad (3.1)$$

A larger isospin asymmetry results in more repulsion, compare Fig. 1. Thus we expect that the isospin dependent mean field alone would favor a lower balance energy for the  $^{58}\text{Fe}$  system than for the  $^{58}\text{Ni}$  system.

The numbers of protons and neutrons in the colliding system also have an influence on the average nucleon-nucleon cross section, because the neutron-proton cross section is approximately a factor of 3 bigger than the proton-proton cross section in the energy regime of interest here. We can then calculate the average nucleon-nucleon cross section from the number of collisions between nucleons of equal and opposite isospin,

$$\begin{aligned} \bar{\sigma} &= \frac{N_{np} \sigma_{np} + (N_{nn} + N_{pp}) \sigma_{pp}}{N_{np} + N_{nn} + N_{pp}} \\ &= \frac{3 N_{np} + N_{nn} + N_{pp}}{N_{np} + N_{nn} + N_{pp}} \sigma_{pp} \end{aligned} \quad (3.2)$$

where  $N_{np}$  is the number of neutron-proton collisions.

We make the *Ansatz* that the number of collisions of two types of nucleons is proportional to the number of nucleons of each species, taken to some power,  $\zeta$ ,

$$\begin{aligned} N_{np} &\propto (N_{n,T} N_{p,P})^\zeta + (N_{n,P} N_{p,T})^\zeta \\ N_{nn} &\propto (N_{n,T} N_{n,P})^\zeta \\ N_{pp} &\propto (N_{p,T} N_{p,P})^\zeta \end{aligned} \quad (3.3)$$

where  $N_{n,T}$ ,  $N_{n,P}$  are the numbers of neutrons in target and projectile, and  $N_{p,T}$ ,  $N_{p,P}$  are the numbers of protons in target and projectile. In intermediate energy heavy ion collisions, we find numerically that  $\zeta$  has values roughly in the range between 2/3 and 1.

Inserting this *Ansatz* into our equation for the average cross section yields the result displayed in Figure 2. The difference in the systems with different isospin is small (only approximately 0.5% for  $\zeta=1$ ), but is clearly visible. Since the isospin-averaged cross section is slightly bigger for the  $^{58}\text{Ni}$  system, this results in a higher kinetic pressure, therefore more repulsion, and thus a lower balance energy.

The isospin composition of the projectile and target also has an effect on the Pauli exclusion principle for the final scattering states of the nucleons. If there are more neutrons than protons, then these neutrons necessarily occupy a larger fraction of the available phase space for the scattered nucleons, resulting in a smaller number of Pauli-allowed  $nn$  collisions than  $pp$  ones. However, for the considerations of the isospin dependence of the average in-medium nucleon-nucleon cross section, the interplay between Pauli-principle and isospin content of the nuclei has only a small effect. To convince ourselves of this, we note that the Boltzmann collision integral contains the factor

$$\Pi_{ab} = [1 - f_a(\vec{r}, \vec{p})] \cdot [1 - f_b(\vec{r}, \vec{p})] \quad (3.4)$$

where  $a$  and  $b$  are the isospin indices of the two nucleons that take part in the scattering event. The factor  $\Pi_{ab}$  represents the correction of the free nucleon-nucleon cross section due to the Pauli exclusion principle for the final scattering states. If we average over all possible two-body scattering events, we find for beam energies larger than the Fermi energy:

$$\langle \Pi_{pn} \rangle \approx \frac{1}{2} (\langle \Pi_{pp} \rangle + \langle \Pi_{nn} \rangle) \quad (3.5)$$

The leading correction term to this approximation enters as  $\delta^2$ . Thus the differential effect of the Pauli-exclusion principle on the isospin dependence of the average in-medium cross section is only a small correction to the determination of  $\bar{\sigma}$ .

#### IV. NUMERICAL RESULTS

We preface this section by briefly outlining our procedure to determine the balance energies: We begin by calculating the proton freeze-out phase-space distribution at reduced impact parameters  $\hat{b} = b/b_{max} = 0.275, 0.375, 0.475$  and  $0.575$ , beam energies 50, 60, 70, 80, 90 and 100 MeV/A, using Equations 2.1 and 2.3, and various in-medium corrections  $\alpha$ , Eq. 2.10 This is followed by calculating the flow for each permutation, and then the balance energy for each  $\hat{b}$ , using a  $\chi^2$  minimization procedure.

In Fig. 3, we show the experimental results of Pak *et al.* [12] as the shaded rectangles. The width of these rectangles represent the width of the impact parameter bins used for the integrations of the experimental data. The height of the rectangles represent the error bars in the balance energy (standard deviation of the mean). The darker shaded rectangles represent the experimental results for the  $^{58}\text{Fe} + ^{58}\text{Fe}$  system, and the lighter gray areas those for the  $^{58}\text{Ni} + ^{58}\text{Ni}$ . The thick horizontal lines through the middle of each rectangle represent the quoted experimental values. This experimental information is the same in all three panels of Fig. 3, each time compared to a different calculation.

Before we discuss the ingredients of each calculation, we should point out that the experimentally found balance energies in  $^{58}\text{Fe} + ^{58}\text{Fe}$  system are always higher than those for the  $^{58}\text{Ni} + ^{58}\text{Ni}$  system. This is what we would expect from our elementary considerations of the effects of the Coulomb interaction and the two-body collisions, but contrary to what we expect from the mean field alone. We will return to this point later.

The theoretical results for the  $^{58}\text{Fe} + ^{58}\text{Fe}$  system are indicated by the open plot symbols, and those for  $^{58}\text{Ni} + ^{58}\text{Ni}$  by the filled ones.

In the left panel of Fig. 3, we show the result of a calculation with the mean field interaction of B.A. Li [12], and with  $\alpha = 0$ . Pak *et al.* [12] found that BUU under-predicted the balance energies of  $^{58}\text{Fe} + ^{58}\text{Fe}$  and  $^{58}\text{Ni} + ^{58}\text{Ni}$  collisions. This is consistent with previous work [36,25,3] that has shown the BUU model utilizing free-space scattering cross sections consistently under-predicting the balance energies of various systems. However, the positive results of Pak *et al.* was the correct reproduction of the differential effect in the balance energies – the difference between the balance energies for the two systems has the right sign and approximately the right magnitude. This can be verified by examining the left panel of Fig. 3.

In the central and right panels, we use the isospin dependent mean fields of Li (center, [29,30], Eq. 2.1) and of Sobotka (right, [34], Eq. 2.3), combined with the in-medium corrections to the elementary two-nucleon scattering cross sections. For the central panel, we use  $\alpha = -0.3$ , and for the right panel, we use  $\alpha = -0.2$ , respectively.

Let us first compare the results of the central and the left panel. For both calculations, we use identical isospin-dependent mean fields. The only difference is the change in the scattering in-medium correction. We can make two rather obvious observations:

1. The theoretical calculations are much closer to the data when using the in-medium reduction of the scattering cross section – for all impact parameter intervals. This observation is consistent with previous results that did not look at isospin-dependent effects [36,25,3].
2. Even though the balance energies for the iron system are still slightly higher than for the nickel system, the magnitude of the splitting has been reduced and is now at least a factor of 4 smaller than what is observed in experiment.

The same effects can be observed when using a different iso-spin dependent mean field, compare the right panel of Fig. 3. Here, the differences between the theoretical balance energies are somewhat larger, but still a factor of at least 2 smaller than those found in the data.

We mentioned above that the observed sign of the difference in balance energies as a function of isospin is opposite to what one would expect from the behavior of the isospin-dependence of the mean field. This assertion can be verified in our numerical calculations by switching the isospin-dependent term in the mean field on and off. This comparison has been performed in Fig. 4. Here we use the same results as in the right panel of Fig. 3 (Sobotka mean field,  $\alpha = -0.3$ , represented by the diamonds in Fig. 4) and compare them to a mean field without isospin dependence, but with the same value of the nuclear compressibility (circles). In particular for the larger impact parameters, we can clearly see the expected effect: The isospin-dependent part of the mean field actually *reduces* the difference in the balance energies for the two systems. This is consistent with the results of our elementary considerations. The three-times larger isospin asymmetry in the iron system results in a stronger repulsion due to the isospin dependent mean field and is thus pushing the balance energy in the iron system more down than in the nickel system.

## V. CONCLUDING REMARKS

We see an improvement in the performance of the BUU model's prediction of balance energies as a function of impact parameter for collisions of  $^{58}\text{Fe} + ^{58}\text{Fe}$  and  $^{58}\text{Ni} + ^{58}\text{Ni}$  by including both an asymmetry energy term in the mean field and in-medium reduction of the nucleon cross section. We observe similar performance among the different formulations used for the mean field: One formulation recently used by Bao-An Li [29,30] and one due to Sobotka [34]. However, the mean field of Bao-An Li, Equation 2.1, requires  $\alpha = -0.3$  in Equation 2.10, whereas the Sobotka formulation, Equation 2.3, requires  $\alpha = -0.2$  for substantial improvement. Efforts to distinguish between these two mean fields will likely come from heavy ion calculations in conjunction with experiments near the drip lines. In the systems studied in the experiments by Pak *et al*, the range in isospin asymmetry was not sufficiently large to constrain our parameter space any further.

We have shown the different contributions of the Coulomb force, the NN-scattering, and the isospin-dependent mean field on the isospin dependence of the balance energy. Both our elementary considerations and our numerical results support the conclusion that the sign and the bulk of the isospin-difference in the balance energies is caused by the difference in the Coulomb interaction and the isospin dependence in the effective nucleon-nucleon two-body scattering.

While we can understand the absolute magnitude of the balance energies and the sign of the isospin-dependent difference between the two systems, our theoretical calculations do not yield the correct absolute magnitude of the difference. One can speculate on the origin of this important disagreement. A lack in our understanding of the isospin dependence of the nuclear mean field or the isospin-dependent in-medium modification of the two-body scattering cross section are the leading candidates. However, there is strong reason to suggest that the higher beam intensities of the radioactive beam facilities currently in planning or under construction will allow us to make progress in our understanding of this problem by enabling us to explore a larger isospin asymmetry in heavy ion reactions.

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FIGURES

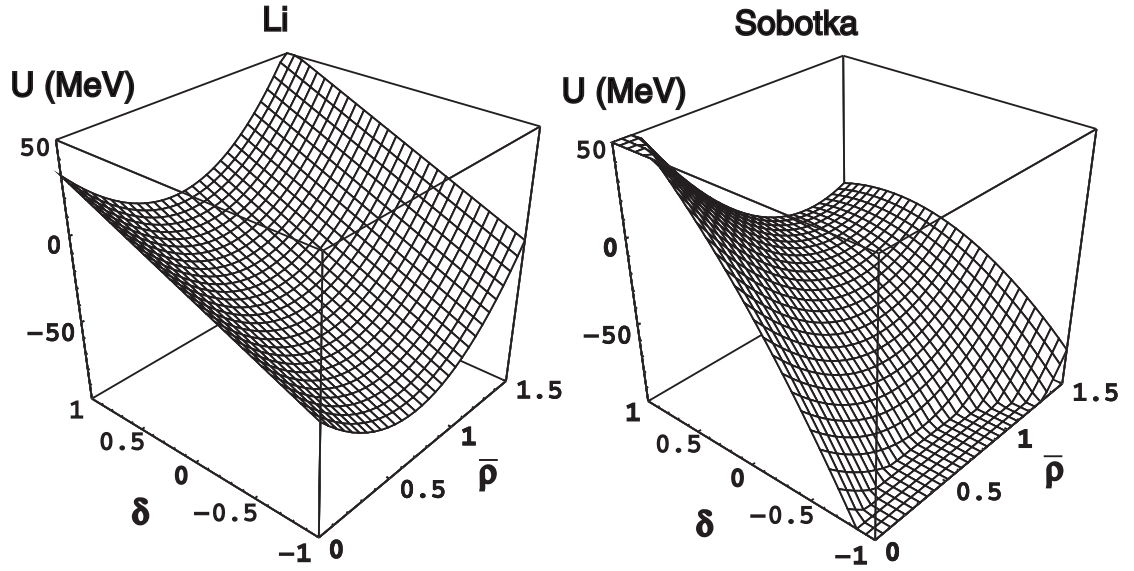


FIG. 1. Mean field potential as a function of reduced density,  $\hat{\rho} = \rho/\rho_0$ , and isospin asymmetry,  $\delta$ , as used by Li et al. (left) [29,30] and Sobotka (right) [34].

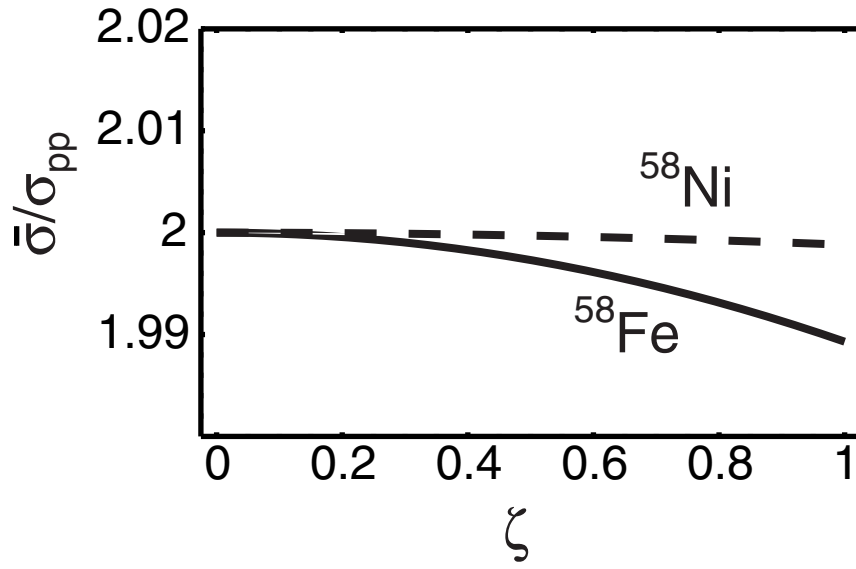


FIG. 2. Dependence of the average nucleon-nucleon cross section as a function of the parameter  $\zeta$  for the two different isospin systems.

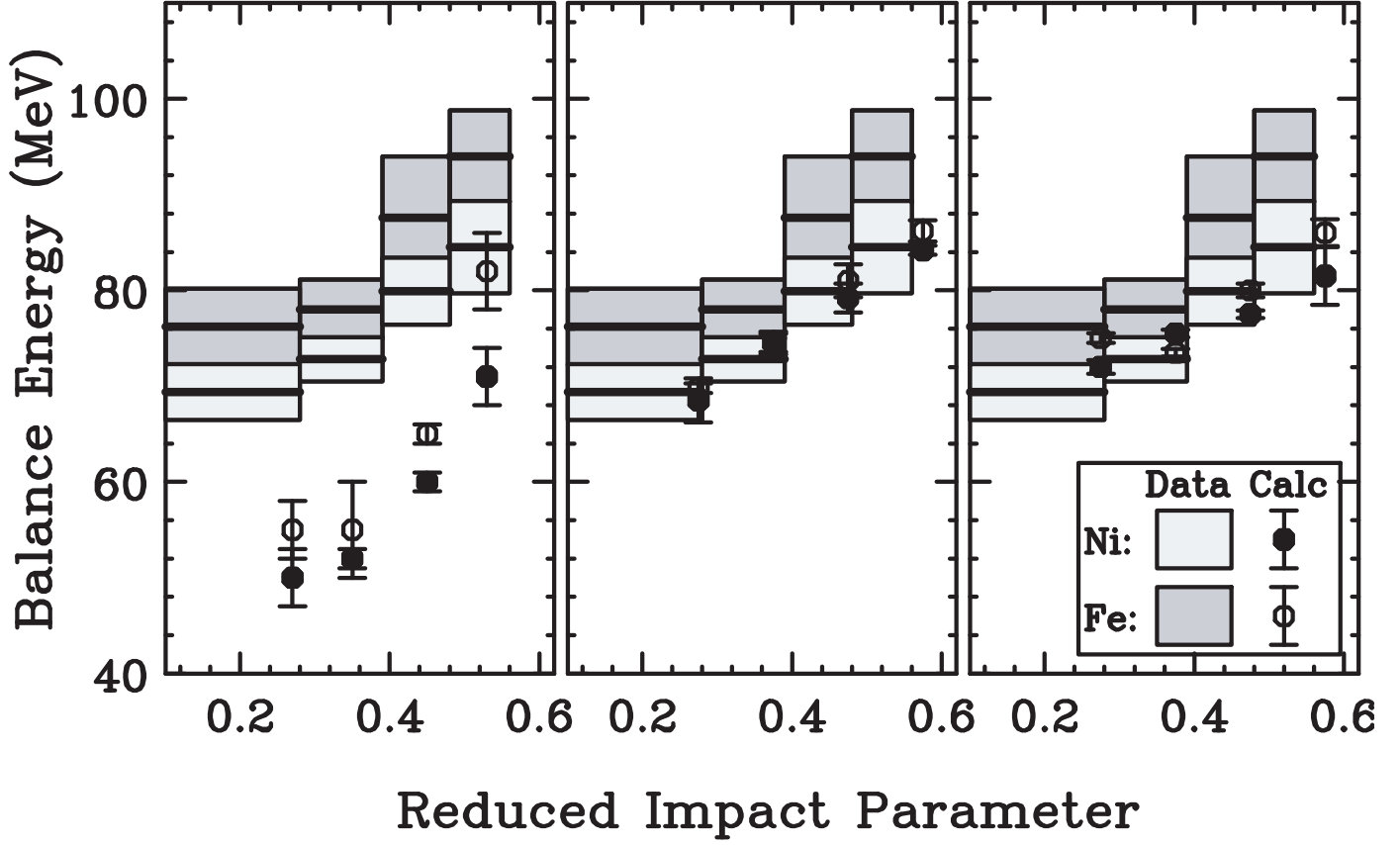


FIG. 3. Impact parameter dependence of the balance energy for the systems  $^{58}\text{Ni} + ^{58}\text{Ni}$  (light shaded rectangles: data; open circles: calculations) and  $^{58}\text{Fe} + ^{58}\text{Fe}$  (dark shaded rectangles: data; filled circles: calculations). Left panel: results of B.A. Li with free nucleon-nucleon cross sections; middle panel: mean field of B.A. Li and  $\alpha = -0.3$ ; right panel: mean field of Sobotka and  $\alpha = -0.2$ .

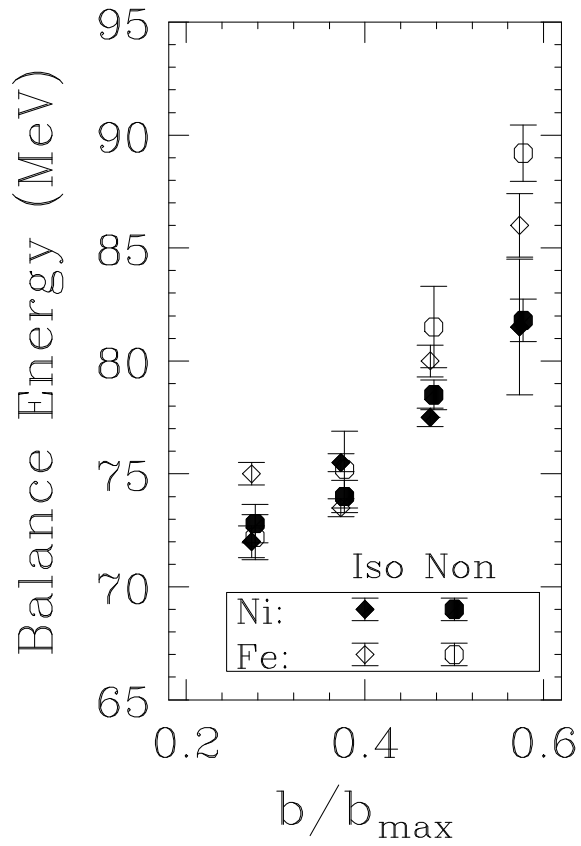


FIG. 4. Comparison of the effects of the isospin-dependent mean field (diamonds) relative to the corresponding mean field without isospin dependence (circles).