RANDOM MATRIX BASED INPUT SHAPING CONTROL OF UNCERTAIN PARALLEL MANIPULATORS

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ABSTRACT

Input shaping control techniques have proven to be very effective in improving performance and lowering actuation requirements for Linear Time-Invariant (LTI) systems. However, deployments tend to be very sensitive to variation of the system natural frequencies and damping ratios. This limits the application of input shaping schemes in the nonlinear systems (e.g., robotic manipulators) or systems with uncertain parameters. The main focus of this paper is to address the parametric uncertainty in the system and the design of an input shaping control based upon the estimated parameters. The variations induced by system non-linearity are tackled by the method of the linearization. The uncertainty is characterized by treating the linearized system mass matrix as a random matrix. This provides the probability density function of the eigenvalues of the underdamped system that are then used to design the input shaping control for the uncertain system. We believe that such a characterization (and desensitization to production variability) is especially important for successful deployment of newer generations of 3D printed robotic systems. This is verified via a Monte Carlo simulation — the statistics of the percent residual vibration verifies the superiority of the input shaping control scheme based on the estimated parameter values (compared to the one based on the nominal parameters).

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INTRODUCTION

Over the past few decades, input shaping (IS) technique has shown some success in the control of the high-speed dynamical systems with high precision applications. A main challenge in the control of these systems is the controversy of the speed and system residual vibration. In a closed-loop control system, to achieve a fast response, it is desired to tune the controller gains such that the system remains underdamped while its stability is ensured. The payoff of the short rise time is the excess overshoot and long settling time of the residual vibration that must be eliminated or reduced due to the high required precision.

The IS method dates back to the work by Singer and Seering [1] in 1990. As the name suggests, it shapes the input command using a series of the impulses such that the superposition of the responses corresponding to each impulse yields zero (or near zero) vibration. Refer to [2, 3] for a detailed discussion of the IS technique.

Despite of its effectiveness in linear time invariant (LTI) systems, the standard IS method is highly sensitive to the variation of the system natural frequencies (ωi’s) and damping ratios (ζi’s), the parameters based on which the input shaper is designed. This limits the application of the standard IS method in nonlinear or uncertain systems where ωi and ζi are time varying or are not exactly known. Robust IS technique is one of the most common strategies to overcome the variation of the system natural frequencies. In addition to the zero residual vibration amplitude,
it introduces sensitivity constraints that requires the derivatives of the (residual) vibration amplitude respect to the modeling natural frequency to be equal to zero (ZVD) [1, 4]. A measure of the robustness called specified insensitivity (SI) was also proposed by Singhose et al. [5]. In their work, a frequency sampling method [6] was used for (approximately) satisfying the derivative constraints. It was shown that the rise time increases with the degree of the robustness of the input shaper. In addition to the robust IS method, there exist different other treatments to the parameter variation problem including adaptive [7,8] and nonlinear [9,10] input shapers that are often computationally demanding. Linearization method is a common way to tackle the system nonlinearity in IS control designs. For example, Singh et al. [11] implemented the IS method to control flexible/rigid link robots in which the natural frequencies and damping ratios were obtained from the linearized system. In a more recent study, Kozak et al. [12] examined different input shapers designed based on the linearized dynamics of the multi-degrees-of-freedom (multi-DOF) parallel manipulators. Different input shaping strategies were also examined by Narayanan [13] for reduction of the residual vibration in the end effector (EE) of the 2-RRR and 3-RRR manipulators.

However, methods to address the variability of the system parameters for the uncertain dynamical systems (especially in the context of IS control design) appears to not have been explored. One may solely use the robust IS schemes, however, as discussed above, they suffer from the increased rise time and the optimized design can only be achieved using the information obtained from the distribution of the uncertain parameters. The appropriate choice of the modeling frequency and the upper and lower bounds of the variations (for robust IS control designs) are some of the useful information that can be obtained from the random parameter probability density functions (pdf’s).

Hence, in this paper we will examine the extension of IS control techniques to include uncertainty characterization. In particular, an IS control scheme will be designed for a parallel manipulator (PM) whose mass parameters are uncertain. Such situations can easily occur in real-world settings as: (i) time-varying loads are being transported; or (ii) low-cost techniques (such as desktop 3D printing) are employed for production. To characterize the uncertainty, the linearized system mass matrix is considered to be random and a probabilistic model is constructed using random matrix theory. This approach provides a closed form pdf of the (undamped) eigenvalues of the random system that are used to design an IS control scheme for the uncertain PM.

The rest of this paper is organized as follows. In the next section, we review the dynamics of a kinematically redundant planar PM (3-(P)RRR) [14] and derive the linearized closed-loop system equations of motion (EoM). Next, we construct the random matrix based probability model for linearized system with random mass matrix that, in turn, facilitates the characterization of the pdf of the random system eigenvalues. Afterward, the design of IS control scheme for the uncertain 3-(P)RRR PM, based on the mathematical tools provided in the preceding section, is discussed. Moreover, using a Monte Carlo analysis, the performance of the designed IS control scheme is evaluated and compared to the results obtained from an IS control method based on the nominal parameter values. Finally a brief discussion and directions for future work are presented in the last section.

KINEMATICALLY REDUNDANT 3-(P)RRR planar PM

Parallel platforms are well suited for the high speed and precise manipulations and machining due to the higher stiffness and lower inertia compared to their serial counterparts. In this work, we consider a 4-DOF kinematically redundant planar PM (3-(P)RRR) to implement the IS control method for fast and accurate manipulations. The kinematics and dynamics of the 3-(P)RRR PM [14], shown in Fig. 1, is reviewed in this section and its corresponding linearized closed-loop EoM are derived to be used in the implementation of the IS control strategies in the subsequent sections.

The loop-closure constraints for ith serial limb of the PM can be written as

$$f_i = \begin{bmatrix} x_{G_i} + l_{i1}\cos(\theta_i) + l_{i2}\cos(\theta_i + \psi_i) - x_E - r_i\cos(\phi_E + \beta_i) \\ y_{G_i} + l_{i1}\sin(\theta_i) + l_{i2}\sin(\theta_i + \psi_i) - y_E - r_i\sin(\phi_E + \beta_i) \end{bmatrix} = \mathbf{0}$$

(1)

The general EoM of the system shown in Fig. 1 can be written in a matrix form as

$$M(q)\ddot{q} + h(q, \dot{q}) + \Phi_q^T\lambda = \tau$$

(2)

where \( q = [\delta, \theta_1, \psi_1, \theta_2, \psi_2, \theta_3, \psi_3, x_E, y_E, \phi_E]^T \), \( \lambda \) is the vector of the Lagrange multipliers corresponding to the kinematic constraints and \( M(q) \), the inertial mass matrix of the system, \( h(q, \dot{q}) \), the Coriolis and centripetal force vector, and \( \tau \), the input vector, are given by

$$M(q) = \text{diag}(M_1, M_2, M_3, M_4)$$

$$h(q, \dot{q}) = \begin{bmatrix} h_{11}^T, h_{12}^T, h_{13}^T, h_{14}^T \end{bmatrix}^T$$

$$\tau = [\tau_1^T, \tau_2^T, \tau_3^T, \tau_4^T]^T$$

(3)-(5)

in which \( M_i, h_i \) and \( \tau_i \) (\( i = 1, \ldots, 4 \)) are the mass matrix, Coriolis and centripetal force vector and external force/moment vector.
of the ith limb and moving platform \((i = 4)\), respectively. Note that the gravitational forces are ignored assuming that the manipulator is planar. Now, differentiating \( f = [f_1, f_2, f_3]^T \) \((f_i, (i = 1, \ldots, 3)\) are given in Eq. (1)) with respect to the time yields

\[
\frac{\partial f}{\partial q} \dot{q} = 0 \quad \text{and} \quad \Phi_q = \frac{\partial f}{\partial q}
\] (6)

To eliminate the Lagrangian multipliers (projection into motion space), Eq. (2) is pre-multiplied by \( R^T \) where \( R \) is the null space of the \( \Phi_q \), i.e. \( \Phi_q R = 0 \). Then,

\[
R^T M(q) \ddot{q} + R^T h(q, \dot{q}) = R^T \tau
\] (7)

The state vector \( q \) can be decomposed into dependent \((q_D)\) and independent \((z)\) coordinates as follows.

\[
q = K \begin{bmatrix} q_D \\ z \end{bmatrix}
\] (8)

where \( z = [x_E, y_E, \phi_E, \delta]^T \), \( q_D = [\theta_1, \psi_1, \theta_2, \psi_2, \theta_3, \psi_3]^T \) and \( K = [\frac{\partial q}{\partial q_D}, \frac{\partial q}{\partial z}] \) is the constant permutation matrix. Then,

\[
\dot{q} = K \begin{bmatrix} \dot{q}_D \\ \dot{z} \end{bmatrix}
\] (9)

After some algebra, it can be shown that

\[
\dot{q} = R \ddot{z} \Rightarrow \ddot{q} = R \ddot{z} + \dot{R} \dddot{z}
\] (10)

where

\[
R = K \begin{bmatrix} -\Phi_q^{-1} \dot{q} \\ \phi \end{bmatrix}, \quad \Phi_{qD} = \frac{\partial f}{\partial q_D} \quad \text{and} \quad \Phi_z = \frac{\partial f}{\partial z}
\] (11)

in which \( I_{4 \times 4} \) is the identity matrix. Writing \( h(q, \dot{q}) = C(q, \dot{q}) \dot{q} \) in Eq. (7) and substituting the relations given in Eq. (10) yields

\[
\dot{M}(q) \ddot{z} + \dot{C}(q, \dot{q}) \dot{z} = \dot{E}(q) \tau_a
\] (12)

where \( \dot{M} = R^T M(q) R \), \( \dot{C} = R^T M(q) \dot{R} + R^T C(q, \dot{q}) R \), \( \tau_a = [\tau_{\theta_1}, \tau_{\theta_2}, \tau_{\theta_3}, \tau_{\phi}] \) is the vector of the actuator forces/torques and \( \dot{E} \) is the corresponding transformation matrix. For more details on the dynamic and kinematic formulation of the 3-(P)RRR, the readers are referred to Refs. [14, 15].

Eq. (12) represents the nonlinear (unconstrained) EoM of the system in minimal (independent) coordinates. For our ultimate goal, i.e., implementing the IS control scheme, Eq. (12) needs to be linearized. Before proceeding to the linearization process, we first consider a simple feedback PD control law, defined by Eq. (13), based on which the actuator torques are determined.

\[
\tau_a = (\dot{E})^+ \begin{bmatrix} K_D \dot{e} + K_P e \end{bmatrix}
\] (13)

where \((.)^+\) is the Moore–Penrose pseudo-inverse operator that is the conventional matrix inverse in the cases of square matrices, \( e = z_{des} - z \) is the trajectory error that implies \( \dot{e} = \dot{z}_{des} - \dot{z} \), and \( K_D \) and \( K_P \) are the appropriate differential and proportional gain matrices, respectively. Substituting Eq. (13) in Eq. (12) and assuming the Coriolis forces to be negligible (for simplicity in the IS control design), the closed-loop system EoM can be written as

\[
g(z, \dot{z}, \ddot{z}) = 0
\] (14)

where

\[
g_{4 \times 1} = \dot{M}(q(z)) \ddot{z} - K_D \dot{e} - K_P e
\] (15)

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Note that the Coriolis force vector can not be canceled by adding the equivalent terms to the PD control law expression because it is not exactly known in the uncertain real systems.

Now the system can be linearized about an operating point that is an equilibrium point. The net force in an equilibrium point is zero if the generalized velocity and acceleration are zero [12]. Following the method described in [12], we define \( \eta_{12\times1} = [z^T \ 0^T \ 0^T]^T \) and the equilibrium point is then \( \eta_0 = [z^0 \ 0^T \ 0^T]^T \). Using the Taylor’s theorem (ignoring second and higher order terms), the behavior of the system described by Eq. (14) about \( \eta_0 \) can be approximated by the following linear equations.

\[
g(\eta) = g(\eta_0) + J(\eta_0)\Delta\eta = 0 \tag{16}
\]

where \( \Delta\eta = (\eta - \eta_0) \) and \( J_{i,j} = \frac{\partial g_i}{\partial \eta_j} \) is the Jacobian matrix of the \( g \). Using Eq. (15), \( J(\eta_0) \) can be calculated as

\[
J(\eta_0) = \begin{bmatrix}
\frac{\partial g_1}{\partial \eta_1} & \cdots & \frac{\partial g_1}{\partial \eta_4} \\
\vdots & \ddots & \vdots \\
\frac{\partial g_5}{\partial \eta_5} & \cdots & \frac{\partial g_5}{\partial \eta_8} \\
\vdots & \cdots & \vdots \\
\frac{\partial g_9}{\partial \eta_9} & \cdots & \frac{\partial g_9}{\partial \eta_{12}} \\
\end{bmatrix}
\tag{17}
\]

Given \( g(\eta_0) = 0 \) (based on the definition of the equilibrium point) and re-writing \( \Delta\eta = [\Delta z^T \ \Delta \xi^T \ \Delta \xi^T]^T \), Eq. (16) can be written as

\[
\dot{M}(z_0)\Delta\xi + K_D\Delta\xi + K_P\Delta\xi = 0 \tag{18}
\]

The natural frequencies (\( \omega_i \)’s) of the linear system described by Eq. (18) can be derived from the standard eigenvalue problem given by

\[
(S - \lambda I)\mu = 0 \tag{19}
\]

where \( S = \dot{M}(z_0)^{-1}K_F \) and \( I \) is a \( 4 \times 4 \) identity matrix. Then \( \omega_i = \sqrt{\lambda_i} \) where \( \lambda_i \) are the roots of the characteristic equation \( \det(S - \lambda I) = 0 \). Alternatively, the natural frequencies of the system along with its damping ratios (\( \zeta_i \)’s) can be calculated by solving the following characteristic equation.

\[
\det(\lambda^2\dot{M}(z_0) + \lambda K_D + K_P) = 0 \tag{20}
\]

The roots of Eq. (20) are of the form \( \lambda_{i\pm} = -\zeta_i\omega_i \pm j\omega_i\sqrt{1 - \zeta_i^2} \).

**UNCERTAIN MANIPULATOR — RANDOM MASS MATRIX**

This section provides a random matrix based probability model for the linearized system described by Eq. (18) with a random mass matrix. This is followed from the earlier works in which random matrix theory was employed to model the Jacobian matrix [16, 17] and dynamic system matrices [18] of an uncertain manipulator. A brief review of the procedure is presented here and an appropriate model is proposed to facilitate the derivation of the pdf of random system eigenvalues.

Let \( A \) be an \( n \times n \) random symmetric positive definite matrix, denoted by \( A \in \mathbb{M}_n^+ \) (the bold letters represent random quantities hereafter), whose mean (\( \bar{A} \)) is known. The objective is to find an appropriate matrix-variate pdf of \( A (p_A(A)) \) based on the available information. The only available information are described in Eqs. (21) and (22) as

\[
\int_{\mathbb{M}_n^+} p_A(A) dA = 1 \tag{21}
\]

\[
E[A] = \int_{\mathbb{M}_n^+} A p_A(A) dA = \bar{A} \tag{22}
\]

Eq. (21) is the normalization constraint that must be satisfied by every valid density function. Eq. (22) enforces the expectation of \( A \) to be equal to the given mean value (\( \bar{A} \)). Using maximum entropy principle [19], it can be shown that the random matrix \( A \) turns out to have a Wishart distribution with parameters \( n, p = n + 1 \) and \( \Sigma = \frac{A}{n+1} \), denoted by \( A \sim W(n, p, \Sigma) \) (for detailed derivations refer to [17, 18] and references therein) whose density function is given by [20]

\[
p_A(A) = \left\{ 2^\frac{p(n-1)}{2} \Gamma_n \left(\frac{1}{2}p\right) \det(\Sigma)^\frac{1}{2} \right\}^{-1} \det(A)^\frac{1}{2}(p-1)^\frac{n}{2} \etr(-\frac{1}{2}\Sigma^{-1}A) \tag{23}
\]

To further control the level of the uncertainty, a constraint based on the existence of the inverse moment is defined as [21]

\[
E[\|A^{-1}\|_F^v] < \infty \tag{24}
\]

where \( v \) is the order of the inverse moment. It can be shown that incorporating the constraint given by inequality (24) (in addition to the normalization and mean constraints given by Eqs. (21)
and (22)) in the maximum entropy derivation of \( p_A(A) \) yields a Wishart distribution for random matrix \( A \) with parameters \( n, p = 2v + n + 1 \) and \( \Sigma = \frac{A}{2V+n+1} \) [21, 22]. To facilitate choosing the parameter \( v \), the normalized standard deviation of the random matrix \( A (\sigma_A) \) is defined as [21]

\[
\sigma_A = \left\{ \frac{E[\|A - E[A]\|_F^2]}{E[\|A\|_F^2]} \right\}^{1/2}
\]  

(25)

After some algebra, it can be shown that

\[
\gamma = \frac{1}{\sigma_A} \left( 1 + \frac{\text{tr}(A^2)}{\text{tr}(A^2)} \right) - (n + 1)
\]  

(26)

where \( \gamma = 2v \).

Now, re-writing Eq. (19) for an uncertain system, we have

\[
(S - \lambda I)u = 0
\]  

(27)

where \( S = \hat{M}^{-1}K_p \) is now a random matrix (because \( \hat{M} \) is a random matrix). Given that \( \hat{M} \in M_{n}^+ \) then \( \hat{M}^{-1} \in M_{n}^+ \). Assuming \( K_p = k_p I_n \times n \) where \( k_p > 0 \) is the PD controller proportional gain parameter, then \( S = k_p \Sigma \) is symmetric and positive definite, \( i.e., S \in M_n^+ \). If the mean dynamic system is known, \( i.e., \) if \( \hat{M} \) is given, then by assuming \( S = \hat{M}^{-1}K_p \) and using the approach described above, it can be concluded that

\[
S \sim W(n, 2v + n + 1, \frac{S}{2V+n+1}).
\]

Given that the random matrix \( S \) has a Wishart distribution, closed form pdf of the undamped system eigenvalues can be obtained. There is a vast literature on the distribution of the eigenvalues of the Wishart random matrices (for example, see [23–26]). Here, we use the closed form expression of the pdf of the random Wishart matrices, developed in an excellent work by Zanella et al. [25]. The pdf of the smallest eigenvalue of the random matrix \( A \sim W(n, p, \Sigma) \) with \( \Sigma \neq I \), is given by

\[
p_{\lambda_s}(\lambda_s) = K_{cc} \sum_{k=1}^{n} \sum_{l=1}^{n} (-1)^{k+l} \lambda_s^{p-n+k-1} \exp(-\frac{\lambda_s}{\sigma_l}) \det(\Omega^c_n)
\]  

(28)

where

\[
K_{cc} = \frac{\prod_{i=1}^{n}(i-1)! \det(\Sigma)^{-p}}{\det(V_2(\sigma))}
\]

(29)

\[
K_{uc} = \left[ \prod_{i=1}^{n}(p-i)! \prod_{j=1}^{n}(n-j)! \right]^{-1}
\]

(30)

\[
V_2(\sigma)_{i,j} = -\sigma_j^{-i}
\]

(31)

and \( \sigma = [\sigma_1 \ldots \sigma_n]^T \) where \( \sigma_1 > \sigma_2 > \cdots > \sigma_n \) are the eigenvalues of \( \Sigma \). \( \omega_{i,j}^c \) is the \( p-n+r_{ik} \)th \( i, j \)th element of the matrix \( \Omega^c \), is defined as

\[
\omega_{i,j}^c = (\sigma_{r_{ij}})^{p-n+r_{ik}} \Gamma(p-n+r_{ik}, \frac{\lambda_s}{\sigma_{r_{ij}}})
\]

(33)

where

\[
r_{k,l} = \begin{cases} 
   k, & \text{if } k < l \\
   k + 1, & \text{if } k \geq m
\end{cases}
\]

(34)

and \( \Gamma(a,u) \) is the incomplete Gamma function defined as

\[
\Gamma(a,u) = \int_u^\infty x^{a-1} e^{-x} dx
\]

(35)

The density function given in Eq. (28) is then used to find the best estimate of the smallest natural frequency of the system, to be used in the IS control design.

**INPUT SHAPING CONTROL**

This section discusses the IS control of the described redundant PM to reduce the residual vibration in the EE. As the IS technique is based upon the system natural frequencies and damping ratios, it is first necessary to study the variation of these parameters over the workspace of the PM. The natural frequency and damping ratio of the nonlinear closed-loop system, corresponding to the dominant pole of the locally linearized system (described by Eq. (18)) are plotted in Fig. 2 for a subset of the manipulator workspace \( (x_E \in [0.25, 0.45] \text{ m}, y_E \in [0.25, 0.45] \text{ m}, \phi_E = 0 \text{ and } \delta = 0.05 \text{ m}) \).

Because the main concern of this paper is to investigate the parameter variations due to the parametric uncertainty, we define
FIGURE 2: Variation of (a) natural frequency and, (b) damping ratio corresponding to the dominant pole over the workspace

a simple task in which the manipulator behavior is almost linear. This can cover a broad range of application (especially by an appropriate motion planning for a given task). From Fig. 2, we consider the following desired trajectory for the EE position and orientation.

Initial EE pose: \( x_E = 0.3, \ y_E = 0.29, \ \phi_E = 0 \)

Desired EE pose: \( x_E = 0.4, \ y_E = 0.29, \ \phi_E = 0 \)

The initial and desired length of the (redundant) prismatic actuator of the limb 1 is set to be \( \delta = 0.05 \) m. The response of the nominal system is shown in Fig. 3 corresponding to the solution of the nonlinear and linearized (about desired EE pose) EoM’s ((Eq. (14)) and (18), respectively). It can be seen from Fig. 3 that the nonlinear system can be well approximated by the equivalent linearized system for the defined task. This enables us to perform the Monte Carlo analysis directly based on the developed random matrix formulation that will be described shortly. Hereafter, all the reported responses correspond to the equivalent linearized system.

The main objective is to design an IS control scheme to reduce the residual vibration of the system response, mainly the residual vibration in \( x_E \). First, we implement the IS technique for the deterministic manipulator with given nominal parameters. Standard input shapers are characterized by the impulse amplitudes \( (A_i)'s \) and the time instances \( (T_i)'s \) in which the impulses are to be applied. The parameters of a two-impulse standard input shaper \( (A_1, A_2, T_1 \text{ and } T_2) \) for a system with natural frequency \( \omega \) and damping ratio \( \zeta \) (corresponding to dominant pole) are given by [1]

\[
A_1 = \frac{1}{K + 1}, \quad A_2 = \frac{K}{K + 1} \quad \text{where} \quad K = e^{-\frac{-\zeta \pi}{\sqrt{1 - \zeta^2}}} \quad (36)
\]

\[
T_1 = 0, \quad T_2 = \frac{\pi}{\omega \sqrt{1 - \zeta^2}} \quad (37)
\]

The responses corresponding to the unshaped and shaped inputs are shown in Fig. 4. The effectiveness of the input shaper in reduction of the EE residual vibration can be seen from the results plotted in Fig. 4. To quantify the efficiency of the input shaper, the percent residual vibration (PRV) is defined as [27]

\[
PRV = e^{-\zeta \omega \mu_m \sqrt{C^2 + S^2}} \quad (38)
\]

where

\[
C = \sum_{i=1}^{m} A_i e^{-\zeta \omega t_i} \cos(\omega \sqrt{1 - \zeta^2} t_i)
\]

\[
S = \sum_{i=1}^{m} A_i e^{-\zeta \omega t_i} \sin(\omega \sqrt{1 - \zeta^2} t_i)
\]

in which \( m \) is the number of impulses. Indeed, PRV measures the ratio of the dissipation envelope magnitude when the shaper is used respect to the one without shaper, upon the completion of the command.

Now, let us assume the mass parameters of the 3-(P)RRR PM (e.g., mass of the links, center of gravities, moments of the inertia etc.) are uncertain but the kinematic parameters (e.g.,
FIGURE 3: Response of the system to a single step input; Solid red lines show the response of the nonlinear system and dotted blue lines show the response of the equivalent linearized (about desired EE pose) system.

FIGURE 4: Unshaped (dashed blue line) and shaped (solid red line) responses of the deterministic system.
length of the links, position of the bases, geometry of the triangular platform etc.) are known. Hence, only the mass matrix is random. Using the probabilistic formulation developed in preceding section, the pdf of the smallest eigenvalue of the linearized system is given by Eq. (28) in terms of the mean mass matrix that is constructed by substituting the nominal parameter values. Given the density function of the system (smallest) eigenvalue, the eigenvalue that maximizes the density function is chosen to determine the modeling natural frequency in input shaper design, i.e.,

$$\hat{\lambda}_s = \arg\max_{\lambda_s} p_{\lambda_s}(\lambda_s)$$

(39)

$$\hat{\omega} = \sqrt{\hat{\lambda}_s}$$

(40)

where $p_{\lambda_s}(\lambda_s)$ is given by Eq. (28). The damping ratio is considered identical to the damping ratio of the nominal system.

To evaluate the performance of the IS control scheme based on the estimated natural frequency, a Monte Carlo analysis is performed as follows. Because the mass matrix, due to the linearization, is time invariant, then the random matrix $S$ is also time invariant. Hence, we can directly generate samples of $S$ from the Wishart distribution. Here, we first construct the mean of the $S$ matrix as $S = \hat{M}^{-1}K_P$. Given $S$ and choosing the dispersion parameter $\sigma_S$, samples of the $S$ can be generated from the Wishart distribution. Dispersion parameter represents the level of the uncertainty and can be determined based on prior knowledge of the system, experience or from the experiment. Here we assumed $\sigma_S = 0.25$. For each sample of the random matrix $S$, a realization of the system natural frequency is obtained and using Eq. (38) the PRV’s corresponding to the input shaper based on (i) nominal modeling frequency and, (ii) estimated modeling frequency are calculated. We perform this procedure for 1000 realizations of the system random matrix $S$ to provide sufficient number of PRV samples for statistical analysis. Fig. 5 shows the histograms of the PRV’s corresponding to the input shapers based on the nominal and estimated natural frequencies.

Results depicted in Fig. 5 verify the superiority of the input shaper based on the estimated natural frequency. The mean of the PRV realizations for input shaper based on the estimated natural frequency is 0.123. This value is 0.205 for the input shaper designed using the nominal natural frequency. Moreover, the PRV standard deviations are 0.089 and 0.122, respectively.

**DISCUSSION**

The input shaping control of a kinematically redundant parallel manipulator with uncertain parameters was addressed in this paper. The method of linearization was employed to overcome the time variation of the system parameters (natural frequencies and damping ratios). The linearized system mass matrix was assumed to be random and a random matrix based approach was proposed to characterize the uncertainty in the system. This resulted in a closed form pdf of the (undamped) system eigenvalues. The distribution of the smallest eigenvalue was utilized to obtain the estimate of the modeling natural frequency. In a Monte Carlo analysis, comparing the performance of the input shaping control schemes based on the nominal and estimated natural frequencies, it was shown that the latter is more effective (in a statistical sense) in reducing the EE residual vibration.

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The treatment to the nonlinearity of the robotic manipulators may depend on the problem at hand (kinematic architecture of the manipulator, defined task etc.). In this work a simple task was considered for the numerical simulations such that the behavior of the system was well approximated by the linearized system. Although this can cover a broad cases of the applications (specifically by appropriate motion planning), however, more complex systems/tasks are required to be examined to investigate the possible new challenges. This will hopefully be the subject of our future studies.

ACKNOWLEDGMENT

This work was partially supported by the National Science Foundation awards IIS-1319084 and CNS-1314484.

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