Feedback Delay in Precoded Spatial Multiplexing MIMO Systems

Abstract—Precoded spatial multiplexing multiple-input multiple-output (MIMO) systems using limited feedback have been extensively studied to reduce the feedback information based on the notion of delay-free feedback channel. In order to reduce performance degradation of precoded spatial multiplexing MIMO systems due to delay in the feedback channel, in this paper, we take into account the time varying nature of the channel, and consider the feedback delay problem. We propose the use of a linear predictor at the receiver to provide the precoder at the transmitter with predicted channel state information, and hence, mitigate the effect of feedback delay. The predictor is implemented using Kalman filter. The performance of this method is evaluated using computer simulation, and the achieved results demonstrate improved bit error rate performance in frequency selective Rayleigh fading channels.

I. INTRODUCTION

Spatial multiplexing is a well-known technique, in which the data bit stream is demultiplexed into multiple substreams that are sent over different antennas. Spatial multiplexing allows MIMO wireless systems to obtain high spectral efficiency. However, spatial multiplexing is vulnerable to rank deficiencies in the MIMO channel matrix. Linear precoding is a technique employed to combat rank deficiency problems and reduce the probability of error [1], [2]. However, precoding requires complete channel knowledge at the transmitter, and in systems that use frequency division duplexing this information is not available at the transmitter without some sort of feedback. Furthermore, for a time varying channel this information must be continuously updated, otherwise outdated channel information will be used, which results in performance degradation.

The design of an efficient feedback scheme that provides reliable channel state information (CSI) to the transmitter necessitates firstly minimizing the amount of information to be fed back to the transmitter through the feedback channel, and secondly solving the feedback delay problem. The first issue has been extensively studied in [3]-[6], where the precoding matrix is chosen at the receiver from a precoding codebook (a set of matrices), known in advance at the transmitter and the receiver, and conveyed to the transmitter over a limited feedback channel using a limited number of bits. The codebook design is described in [5], [6]. In this work, however, the precoding codebooks proposed in [6] are used. As with the second issue of feedback delay for limited feedback spatial multiplexing MIMO systems, to the best of the authors' knowledge, this has not been previously investigated, instead the channel has been assumed to be perfectly known at the receiver and the feedback channel delay is zero. Therefore, the focus of this paper is on mitigating the feedback delay problem by using a linear channel predictor at the receiver.

In the proposed receiver structure, we extended the selection criterion presented in [3]-[5] to choose the precoder matrix as a function of the predicted channel state, instead of using the current channel state. Linear zero-forcing (ZF) and minimum mean square error (MMSE) decoders are used with minimum singular value (SC-MSV) and mean squared error (SC-MSE) selection criterion respectively, and the predictor is implemented using a Kalman filter.

The rest of this paper is organized as follows: The system model is presented in section II. Numerical results are presented in Section III and concluding remarks are given in Section IV.

II. SYSTEM MODEL

A precoded spatial multiplexing system is shown in Fig. 1. We consider a system with \( N_t \) transmit and \( N_r \) receive antennas. The input bit stream is modulated and then demultiplexed into \( M \) substreams, where the number of substreams \( M = \min(N_t, N_r) \). Let the vector \( s(n) = [s_1(n), s_2(n), ..., s_M(n)]^T \) denote the \( M \times 1 \) transmitted symbol vector, where \( T \) denotes transpose operation, and \( n \) is the time index. We assume that \( E[s_s s_H] = \frac{\varepsilon_s}{M} I_M \) in order to constrain the transmitted power, where \( \varepsilon_s \) denotes the transmit energy. \((\cdot)^H\) refers to matrix conjugate transposition, and \( I_M \) is the \( M \times M \) identity matrix. The symbol vector \( s(n) \) is multiplied by the \( N_t \times M \) precoder matrix \( F(n) \) generating a length \( N_t \) vector \( x(n) = F(n) s(n) \), where \( F(n) \in \mathcal{U}(N_t, M) \) the set of \( N_t \times M \) complex unitary matrices. \( F(n) \) is selected at the receiver from a finite set of possible precoding matrices \( \mathcal{F} = \{F_1, F_2, ..., F_N\} \), represented by a limited number of bits \( B (B = \log_2(N)) \) and conveyed to the transmitter through a limited feedback channel. In this work we consider a burst-mode communication system where the transmitted data is divided into frames, each of which contains multiple symbols. We also assume that the channel remains unchanged in the frame, but it varies from frame to frame. In the published works on limited feedback in spatial multiplexing MIMO systems [3]-[6] the precoder matrix is chosen at the receiver.
from a finite length codebook $\mathcal{F}$ using the current channel state $H(n)$, and perfect channel knowledge at the receiver is assumed. In this work, however, we consider a more practical time varying channel. A Kalman filter is used to estimate the channel at the receiver and predict the future state of the channel which is used to design the precoder matrix. We have modeled the channel as a first order autoregressive (AR) process. The state space equations are expressed as:

$$h(n+1) = A(n)h(n)$$  \hspace{1cm} (1)

$$y(n) = C(n)h(n) + v(n)$$  \hspace{1cm} (2)

where $h(n)$ represents the $N_tN_r \times 1$ channel taps vector, $A(n)$ is a known $N_tN_r \times N_tN_r$ matrix that denotes the time varying transition matrix, and $C(n)$ is a known $N_r \times N_tN_r$ measurement matrix. The $N_r \times 1$ vector $v(n)$ is the measurement noise, which is modeled as a zero-mean white noise process, whose correlation matrix is $\Phi_v(n)$. A first order AR model provides an adequate model for time varying channels [7]. Consequently, $A(n)$ is a diagonal matrix of autoregressive model factor $\alpha = E[h_{ij}(n+1)h_{ij}^*(n)]$, where $E(.)$ represents expectation. According to Jakes model

$$E[h_{ij}(n+1)h_{ij}^*(n)] = J_0(2\pi f_d T_s)$$  \hspace{1cm} (3)

where $J_0(.)$ denotes the zeroth order Bessel function of the first kind, and $f_d$ and $T_s$ are the Doppler frequency and the symbol duration respectively. The measurement matrix $C(n)$ for a spatial multiplexing system with four transmit antennas and two receive antennas, $C(n)$ is given as:

$$C(n) = \begin{bmatrix}
  x_1(n) & 0 & 0 & 0 \\
  0 & x_1(n) & 0 & 0 \\
  x_2(n) & 0 & x_2(n) & 0 \\
  0 & x_2(n) & 0 & x_3(n) \\
  x_3(n) & 0 & 0 & x_3(n) \\
  0 & x_3(n) & 0 & x_4(n) \\
  x_4(n) & 0 & 0 & 0 \\
  0 & x_4(n) & 0 & 0
\end{bmatrix}^T$$  \hspace{1cm} (4)

where $x_j(n)$ is the transmitted symbol from antenna $j$ at time $n$.

The Kalman filter equations are given by [8], where the prediction part is:

$$\tilde{h}(n+1/n) = A(n)\tilde{h}(n/n)$$  \hspace{1cm} (5)

$$P(n+1/n) = A(n)P(n/n)A^H(n)$$  \hspace{1cm} (6)

$$\alpha(n) = y(n) - C(n)\tilde{h}(n+1/n)$$  \hspace{1cm} (7)

$$K(n) = P(n+1/n)C^H(n) \left[ C(n)P(n+1/n)C^H(n) + \Phi_v(n) \right]^{-1}$$  \hspace{1cm} (8)

And the update part is:

$$\tilde{h}(n+1/n+1) = \tilde{h}(n+1/n) + K(n)\alpha(n)$$  \hspace{1cm} (9)

$$P(n+1/n+1) = [I - K(n)C(n)]P(n+1/n)$$  \hspace{1cm} (10)

where $K(n)$ is the Kalman gain, $P(n)$ is the correlation matrix of the error, and $\alpha(n)$ is the innovations vector.

Similar to [3]-[6], we assume that the receiver is capable of feeding back a finite number of bits to the transmitter through a zero error feedback channel. Moreover, we consider the feedback delay due to signal processing delay at both the receiver and transmitter, and the transmission delay. To overcome the effect of the feedback delay on system performance we select the precoder matrix $F(n)$ from the codebook $\mathcal{F}$ as a function of the predicted channel state, which is then fed back to the transmitter.

The precoder $F(n)$ is designed to optimize some criterion, various selecting criteria have been proposed in [5], [6] using linear receivers. In this paper we consider the minimum singular value criterion (SC-MSV) with a zero forcing (ZF) linear receiver, and the mean squared error criterion (SC-MMSE) with minimum mean square error (MMSE) linear receiver proposed in [5].

For Minimum Singular value criteria select $F$ such that:

$$F(n+1) = \arg \max_{F \in \mathcal{F}} \lambda_{min}(H_p F)$$  \hspace{1cm} (11)
where \( \lambda_{\text{min}} \) is the minimum singular value of the effective channel matrix \( \mathbf{H}_p \mathbf{F} \).

For (SC-MSE) \( \mathbf{F} \) is chosen according to:

\[
\mathbf{F}(n+1) = \arg \min_{\mathbf{F}} \mathcal{M}(\text{MSE}(\mathbf{F}))
\]

(12)

And the mean squared error (MSE) for linear MMSE receiver is expressed as:

\[
\text{MSE}(\mathbf{F}) = \frac{\varepsilon_s}{M} (\mathbf{I}_M + \frac{\varepsilon_s}{MN_0} \mathbf{F}^H \mathbf{H}_p^H \mathbf{H}_p \mathbf{F})^{-1}
\]

(13)

where \( \mathbf{H}_p \) is the predicted channel matrix, and \( \mathcal{M}(\cdot) \) is either trace \((\text{tr})\) or determinant \((\text{det})\).

The received signal vector is assumed to be added with a noise vector \( \mathbf{n}(n) \) whose entries have an independent and identically distributed \((\text{i.i.d.})\) complex Gaussian distribution with zero mean and variance \( N_0 \). Then the signal seen at the receiver can be written as:

\[
\mathbf{r}(n) = \mathbf{H}(n)\mathbf{x}(n) + \mathbf{n}(n)
\]

(14)

Using the CSI obtained from the training symbols at the beginning of the frame and \( \mathbf{r}(n) \), the linear decoders decode the vector \( \mathbf{s}(n) = \mathbf{Q}(\mathbf{G}(n)\mathbf{r}(n)) \). For a zero forcing (ZF) linear decoder, the linear transform \( \mathbf{G}(n) \) is given as:

\[
\mathbf{G}(n) = (\mathbf{H} \mathbf{F})^+ = [\mathbf{F}^H \mathbf{H}^H + \frac{M N_0}{\varepsilon_s} \mathbf{I}_M]^{-1} \mathbf{F}^H \mathbf{H}^H
\]

(15)

When a minimum mean square error (MMSE) linear decoder is used \( \mathbf{G}(n) \) is given as:

\[
\mathbf{G}(n) = [\mathbf{F}^H \mathbf{H}^H + \frac{M N_0}{\varepsilon_s} \mathbf{I}_M]^{-1} \mathbf{F}^H \mathbf{H}^H
\]

(16)

where \( \mathbf{H} \) is the estimated channel matrix, \((\cdot)^+\) is the matrix pseudo-inverse, and \((\cdot)^{-1}\) denotes the matrix inverse.

III. Simulation Results

In this section, we provide computer simulation results to show the performance of the proposed method for different system configurations \((N_t, N_r)\). We consider the following situations: Firstly, the channel state information (CSI) is assumed to be perfectly known and there is zero delay in the feedback channel, which represents the ideal case; secondly, the channels are estimated at the receiver using a Kalman filter and the precoder matrix \( \mathbf{F}(n) \) is designed at the receiver as a function of the estimated channel \( \mathbf{H}(n) \), which is then fed back to the transmitter in the presence of feedback delay; finally, the channels are estimated at the receiver using a Kalman filter and the predicted future channel state is conveyed to the transmitter by selecting the precoder matrix from the codebook using the predicted channel state \( \mathbf{H}_p \). The codebooks used in the simulation are proposed in [6] and listed in [9].

A 16-QAM (Quadrature-Amplitude-Modulation) was used to simulate \( M \) substreams precoding on a \( N_t \times N_r \) wireless system. We consider a system with carrier frequency 2 GHz, and a symbol duration of \( T_s = 2.5 \mu s \). The maximum Doppler frequency is \( f_d = 200 \) Hz. Thus the normalized Doppler frequency is \( f_d T_s = 5 \cdot 10^{-4} \).

Experiment 1: The purpose of this experiment is to compare the BER performance of precoded spatial multiplexing MIMO system with perfect channel knowledge denoted by 'CSI', conventional case when there is a feedback delay denoted by 'Conv.' , and for Kalman filter based channel prediction denoted by 'Pred.' The simulation results in Fig. 2 show the BER versus the SNR for a \( 4 \times 2 \) system using two substreams and two bits feedback. ZF receiver employing a minimum singular value selection criterion (SC-MSE) was used for this scenario. It can be seen that using channel prediction improves the system performance. It is also observed that at BER of \( 10^{-3} \), the channel prediction scheme achieves \( \sim 1 \) dB improvement over the conventional case. The performance improvement by the prediction scheme is due to mitigating the effect of delay in the feedback channel. However, it is still inferior to the unrealistic case of perfect CSI, which serve as the benchmark performance.

Experiment 2: The same scenario for case 1 is simulated; however, in this case we investigated the effect of increasing the number of feedback bits on the BER performance using channel prediction. The simulation results are presented in Fig. 3. We observe that by increasing the number of feedback bits, (from 2 to 6 bits), \( \sim 3 \) dB improvement was achieved in BER performance. Also, it can be noted that using 6 bits feedback performs approximately the same as the optimal precoding case (infinite number of feedback bits) for high SNRs. This demonstrates that the system performance significantly improves as the number of feedback bits increases, and a satisfactory BER performance can be achieved with a reasonable number of bits (6 bits).
Experiment 3: Furthermore, the BER performance of ZF and MMSE linear receivers was compared for three substreams 6 x 3 system using 4 bits of feedback and 4-QAM modulation scheme. The (SC-MSV) is used for the ZF receiver, whereas the (SC-MSE) with trace cost function is used for the MMSE receiver. Fig. 4 and Fig. 5 depict the BER performance of 'CSI', 'Conv', and 'Pred' situations. It is observed that the proposed scheme (Prediction) outperform the conventional case for the ZF and the MMSE receivers. Also we observe that the MMSE receiver performs better than the ZF receiver by a small margin, at the cost of SNR knowledge at the receiver.

IV. CONCLUSIONS

In this paper we assessed the performance of a precoded spatial multiplexing multiple-input multiple-output (MIMO) systems due to feedback channel delay. A prediction method based on a Kalman filter has been proposed to overcome the feedback delay effect. The effectiveness of this method was evaluated using computer simulation, and it is shown through improved BER performance, that the proposed method mitigates the adverse effect of the feedback delay.

REFERENCES