

Explanation of the Fine Structure of the Spectral Lines of Hydrogen in Terms of a Velocity-Dependent Correction to Coulomb's Law

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Central forces have been productive in describing nature. Newton's law of gravitational attraction describes to a high degree of accuracy the orbits of the planets; Coulomb's law of electrical attraction describes to a high degree of accuracy the orbits of the electron in a hydrogen atom. By applying the Theory of Relativity, where space and time are no longer independent and absolute quantities, Einstein extended Newton's law to describe and explain the anomalous precession of the perihelion of Mercury, and Sommerfeld extended Coulomb's law to characterize the precession of the electron orbit of hydrogen that gives rise to the 2.7 cm fine structure spectral line. Here I show that the introduction of a tangential velocity-dependent force that I have previously used to describe and explain the anomalous precession of the perihelion of Mercury can also be used to explain the fine structure in the spectral lines of the hydrogen atom without robbing the atom of Euclidean space and Newtonian time.

1. Introduction

Nearly a century ago, Sommerfeld [1] extended Bohr's [2,3] planetary model of the atom by successfully incorporating the Theory of Relativity into the model. By assuming that the relativity of time would cause the mass of an electron moving in an elliptical orbit to increase as it approaches perihelion (or perinucleon) where the tangential velocity is maximum and decrease as it recedes from perihelion, Sommerfeld introduced a perturbation into the equation of motion that produced a precession of the perihelion that accounted for the fine structure in the spectral lines. The fine structure in the spectral lines was first identified in 1887 by Michelson and Morley [4] yet it went unexplained by the Bohr model. Sommerfeld's correction to the hydrogen atom was based on an analogy with Einstein's [5] relativistic correction to Newton's model of the solar system. The anomalous precession of the perihelion of Mercury amounted to a fine structure of the planetary orbits that could not be explained by Newton's law of gravitation [6]. Einstein's relativistic correction accounted for the fine structure of the solar system; Sommerfeld's relativistic correction accounted for the fine structure of the spectral lines of hydrogen.

I recently re-interpreted Einstein's relativistic spacetime treatment of the anomalous precession of the perihelion of Mercury in terms of a friction-induced tangential velocity-dependent change $\left(1 + \left(\frac{v_\theta}{c}\right)^2\right)$ in the gravitational potential energy occurring in Euclidean space and Newtonian time [7]. Consistent with the orbital paradox [8], friction caused by radiation or cosmic dust would cause a decrease in Mercury's tangential velocity. This would result in Mercury moving closer to the sun and increasing the gravitational potential. As a consequence of the conservation of total energy and angular momentum, the increased gravitational potential would result in an increased kinetic energy. As long as the friction was greater at perihelion than at aphelion, a precession of the perihelion would result [9].

The force law developed here and elsewhere [7] that accounts for the anomalous precession of the perihelion is given by:

$$F_g = G \frac{Mm}{r^2} \left[1 + \left(\frac{v_\theta}{c}\right)^2\right] \quad (1)$$

Where, F_g is the force of gravity, G is the gravitational constant ($6.67384 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$), M and m are the masses of two bodies that are interacting gravitationally, r is the distance between the two bodies, which is equal to the radius of a circular orbit or the semimajor axis of an elliptical orbit, v_θ is the relative tangential

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velocity of the orbiting body, and c is the vacuum speed of light. When $v_\theta \rightarrow 0$, the above equation reduces to Newton’s law of gravitational attraction. The correction term becomes significant when the relative motion of the two bodies (v_θ) becomes large. The tangential velocity of the orbiting body is limited to the speed of light [10].

Here I show that Sommerfeld’s relativistic treatment of the hydrogen atom, which he used to explain the fine structure of the spectral lines of hydrogen, can also be interpreted in terms of a quantized friction-induced tangential velocity-dependent change ($\frac{1}{n_\theta} \left(\frac{v_\theta}{nc}\right)^2$) in the Coulomb force in the hydrogen atom in Euclidean space and Newtonian time.

$$F_{Coulomb} = \frac{Ze^2}{4\pi\epsilon_0 r^2} \left[1 + \frac{1}{n_\theta} \left(\frac{v_\theta}{nc}\right)^2 \right] \quad (2)$$

Where, $F_{Coulomb}$ is the Coulomb force of electrical attraction, Z is the atomic number of hydrogen, e is the elementary charge, ϵ_0 is the electrical permittivity of the vacuum, r is the radius of the orbit, v_θ is the relative tangential velocity of the electron and proton around the center of mass, n is the principal quantum number that is related to the semimajor axis of the elliptical orbit and n_θ is the azimuthal quantum number (n_θ) that is related to the semiminor axis of the orbit. For circular orbits, $n_\theta = n$. When $v_\theta \rightarrow 0$, the above equation reduces to Coulomb’s law. The correction term becomes significant when the relative motion of the two bodies (v_θ) becomes large. The tangential velocity of the orbiting body is limited to the speed of light by the electromagnetic force described by the time derivative of the product of the magnetic vector potential and the charge of the moving particles. The electromagnetic counterforce is equivalent to the optomechanical counterforce [11].

Sommerfeld realized that the ratio of the tangential velocity of the electron in the first circular orbit of the hydrogen atom to the speed of light ($\frac{v_\theta}{c}$) is equal to the fine structure constant ($\alpha = 7.2973525698(24) \times 10^{-3}$). The fine structure constant (α) is given by $\frac{e^2}{4\pi\epsilon_0 \hbar c}$. Thus it is reasonable to speculate that a tangential velocity-dependent change in the Coulomb potential might be given in terms of the fine structure constant.

2. Results and Discussion

The stability of the Bohr atom results from the equality of the Coulomb force and the centrifugal force:

$$\text{Coulomb force} = \frac{Ze^2}{4\pi\epsilon_0 r^2} = \frac{\mu v^2}{r} = \text{Centrifugal force} \quad (3)$$

Where, Z is the atomic number of hydrogen ($= 1$), e is the elementary charge, ϵ_0 is the electrical permittivity of the vacuum, r is the radius of a circular orbit or the semimajor axis of an elliptical orbit, μ is the reduced mass ($\frac{Mm_e}{M+m_e}$), m_e is the mass of the electron, M is the mass of the proton, and v is the relative velocity of the electron and proton around the center of mass. By multiplying both sides by $\frac{r}{2}$, we get the internal kinetic energy of the atom:

$$KE = \frac{\mu v^2}{2} = \frac{Ze^2}{8\pi\epsilon_0 r} \quad (4)$$

The Coulomb force ($F_{Coulomb}$) is equal to the negative gradient of the potential energy ($-\nabla PE$). Thus the potential energy of the atom is given by:

$$PE = -\frac{Ze^2}{4\pi\epsilon_0 r} \quad (5)$$

The total internal energy of the atom is equal to the sum of the kinetic and potential energy:

$$E = KE + PE = \frac{Ze^2}{8\pi\epsilon_0 r} - \frac{Ze^2}{4\pi\epsilon_0 r} = -\frac{Ze^2}{8\pi\epsilon_0 r} \quad (6)$$

The total internal energy or Hamiltonian can be given in alternative ways:

$$H = E = -KE = \frac{PE}{2} \quad (7)$$

Dimensionally, Planck’s constant is given in units of angular momentum which points to the significance of angular momentum in atomic theory. The foundational assumption is that the angular momentum is quantized:

$$L = \mu v r = n\hbar \quad (8)$$

Where, n is the total or principal quantum number, which is an integer and positive. We can look at the principal quantum number for an atom free from externally applied fields as being a generalized angular momentum composed of multiple components, including one due to the linear momentum parallel to the radius of the orbit (n_r) and one due to the eccentricity of the orbit (n_θ):

$$n = n_r + n_\theta \quad (9)$$

Where, n_θ must be nonvanishing by necessity since the electron cannot traverse through the nucleus. In order to ensure that the electron does not collide with the nucleus, $n \geq n_\theta$. Because the angular momentum is constant in time, the orbit is restricted to a plane.

The relation between the radius or semimajor axis of the atom and the quantized angular momentum is obtained by solving Eqn. (8) for r :

$$r = \frac{n\hbar}{\mu v} \tag{10}$$

The relation between the relative velocity of the electron and nucleus and the quantized angular momentum is obtained by solving Eqn. (8) for v :

$$v = \frac{n\hbar}{\mu r} \tag{11}$$

By substituting $\frac{n\hbar}{\mu v}$ for r in Eqn. (4) and solving for v , we can get the relative velocity in terms of the quantum number n and fundamental constants:

$$v = \frac{Ze^2}{4\pi\epsilon_0 n\hbar} \tag{12}$$

We see that the tangential velocity is quantized and inversely proportional to n . It follows that the square of the velocity, which is related to the kinetic energy, is inversely proportional to n^2 . By substituting $\frac{n\hbar}{\mu r}$ for v in Eqn. (12) we get the radius or semimajor axis of the orbit in terms of the quantum number n and fundamental constants:

$$r = \frac{4\pi\epsilon_0 n^2 \hbar^2}{\mu Z e^2} \tag{13}$$

We get the quantized total internal energy by substituting $\frac{4\pi\epsilon_0 n^2 \hbar^2}{\mu Z e^2}$ for r in Eqn. (6):

$$E = KE + PE = \frac{\mu Z e^2 Z e^2}{4\pi\epsilon_0 n^2 \hbar^2 8\pi\epsilon_0} - \frac{\mu Z e^2 Z e^2}{4\pi\epsilon_0 n^2 \hbar^2 4\pi\epsilon_0} \tag{14}$$

The development thus far is based on the Bohr atom. I now extend the model by positing that a tangential velocity-dependent change in the Coulomb potential exists and is quantized with the following form: $(\frac{\alpha^2}{n^2 n_\theta})$. The n^2 represents the quantization of energy, which is inversely proportional to n^2 , and n_θ represents the quantization of orbital angular momentum. When

$n_\theta = n$, the orbit is circular and when $n > n_\theta$, the orbit is elliptical where the shape of the orbit [13] is described by the ratio of the semimajor axis (a) to the semiminor axis (b) given by $\frac{a}{b} = \frac{n}{n_\theta}$. The eccentricity of the orbit [14] is given by $\epsilon = \sqrt{1 - \frac{n_\theta^2}{n^2}}$ and $n_\theta = n\sqrt{1 - \epsilon^2}$. Given this *ansatz*, the tangential-velocity-dependent equation of motion is given by:

$$E = KE_{Bohr} + PE_{Bohr} \left[1 + \frac{\alpha^2}{n^2 n_\theta} \right] \tag{15a}$$

$$E = \frac{\mu Z e^2 Z e^2}{4\pi\epsilon_0 n^2 \hbar^2 8\pi\epsilon_0} - \frac{\mu Z e^2 Z e^2}{4\pi\epsilon_0 n^2 \hbar^2 4\pi\epsilon_0} \left[1 + \frac{\alpha^2}{n^2 n_\theta} \right] \tag{15b}$$

Where, n_θ is the azimuthal quantum number that quantizes the tangential velocity-dependent change in the Coulomb potential. After expanding Eqn. (15b), we get:

$$E = KE_{Bohr} + PE_{Bohr} + PE_{Bohr} \left[\frac{\alpha^2}{n^2 n_\theta} \right] \tag{16a}$$

$$E = \frac{\mu Z e^2 Z e^2}{4\pi\epsilon_0 n^2 \hbar^2 8\pi\epsilon_0} - \frac{\mu Z e^2 Z e^2}{4\pi\epsilon_0 n^2 \hbar^2 4\pi\epsilon_0} - \frac{\mu Z e^2 Z e^2}{4\pi\epsilon_0 n^2 \hbar^2 4\pi\epsilon_0} \frac{\alpha^2}{n^2 n_\theta} \tag{16b}$$

Using the definition of the fine structure constant ($\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c}$) to simplify Eqn. (16b), we get:

$$E = \frac{\mu c^2 Z^2 \alpha^2}{2n^2} - \frac{\mu c^2 Z^2 \alpha^2}{n^2} - \frac{\mu c^2 Z^2 \alpha^4}{n^4 n_\theta} \tag{17}$$

Where, the energy is presented in terms of the mass energy (μc^2). $\frac{\mu c^2 Z^2 \alpha^2}{2n^2}$ is the quantized kinetic energy, $-\frac{\mu c^2 Z^2 \alpha^2}{n^2}$ is the quantized potential energy when there is no tangential velocity-dependent change in the Coulomb potential and $-\frac{\mu c^2 Z^2 \alpha^4}{n^4 n_\theta}$ is the change in the Coulomb potential due to the coupling of the relatively moving charges to the frictional component. Since $\frac{\mu c^2 Z^2 \alpha^2}{2n^2} - \frac{\mu c^2 Z^2 \alpha^2}{n^2} = -\frac{\mu c^2 Z^2 \alpha^2}{2n^2}$, Eqn. (17) can be simplified:

$$\begin{aligned}
 E &= -\frac{\mu c^2 Z^2 \alpha^2}{2n^2} - \frac{\mu c^2 Z^2 \alpha^4}{n^4 n_\theta} \\
 &= -\frac{\mu c^2 Z^2 \alpha^2}{2n^2} \left[1 + \frac{2\alpha^2}{n^2 n_\theta} \right] \\
 &= -\frac{\mu c^2 Z^2 \alpha^2}{n^2} \left[\frac{1}{2} + \frac{\alpha^2}{n^2 n_\theta} \right]
 \end{aligned}
 \tag{18}$$

The spectral lines ($\lambda = \frac{hc}{E_f - E_i}$) of the Lyman series (Table 1), the Balmer series (Table 2), the Paschen series (Table 3), the Brackett series (Table 4) and the Pfund series (Table 5) can be obtained by calculating the difference in the total energy of the initial orbit and the final orbit using Bohr's equation, Sommerfeld's equation, and the new equation presented here:

Bohr:

$$E_{n_f} - E_{n_i} = -\frac{\mu c^2 Z^2 \alpha^2}{2n_f^2} + \frac{\mu c^2 Z^2 \alpha^2}{2n_i^2} \tag{19a}$$

Sommerfeld:

$$E_{n_f} - E_{n_i} = -\frac{\mu c^2 Z^2 \alpha^2}{2n_f^2} \left[1 + \frac{\alpha^2}{n_f} \left(\frac{1}{n_{\theta f}} - \frac{3}{4n_f} \right) \right] + \frac{\mu c^2 Z^2 \alpha^2}{2n_i^2} \left[1 + \frac{\alpha^2}{n_i} \left(\frac{1}{n_{\theta i}} - \frac{3}{4n_i} \right) \right] \tag{19b}$$

Wayne:

$$E_{n_f} - E_{n_i} = -\frac{\mu c^2 Z^2 \alpha^2}{n_f^2} \left[\frac{1}{2} + \frac{\alpha^2}{n_f^2 n_{\theta f}} \right] + \frac{\mu c^2 Z^2 \alpha^2}{n_i^2} \left[\frac{1}{2} + \frac{\alpha^2}{n_i^2 n_{\theta i}} \right] \tag{19c}$$

Table 1: Spectral lines in the Lyman series observed and predicted by Eqns. (19a), (19b) and (19c). The value of each equation in predicting the observed data was tested by Analysis of Variance. The significance (p) of the regression is given in the last row. While the predictions of all three models are highly significant, the Sommerfeld model is the best predictor.

n_f	n_i	λ_{obs} (nm) (NIST [12])	λ_{Bohr} (nm)	$\lambda_{Sommerfeld}$ (nm) $n_\theta = n$	λ_{Wayne} (nm) $n_\theta = n$
1	2	121.56701	121.604952	121.602929	121.588765
1	3	102.5768	102.604	102.603	102.592
1	4	97.2517	97.284	97.2826	97.273
1	5	94.9742	95.0039	95.0026	94.9933
			$p = 2.84 \times 10^{-8}$	$p = 2.37 \times 10^{-8}$	$p = 2.71 \times 10^{-8}$

Table 2: Spectral lines in the Balmer observed and predicted by Eqns. (19a), (19b) and (19c). The value of each equation in predicting the observed data was tested by Analysis of Variance. The significance (p) of the regression is given in the last row. While the predictions of all three models are highly significant, the Wayne model is the best predictor.

n_f	n_i	λ_{obs} (nm) (NIST [12])	λ_{Bohr} (nm)	$\lambda_{Sommerfeld}$ (nm) $n_\theta = n$	λ_{Wayne} (nm) $n_\theta = n$
2	3	656.279	656.667	656.664	656.641
2	4	486.135	486.420	486.418	486.404
2	5	434.0472	434.303	434.302	434.29
2	6	410.1734	410.417	410.415	410.405
			$p = 8.83 \times 10^{-11}$	$p = 7.22 \times 10^{-11}$	$p = 7.03 \times 10^{-11}$

Table 3: Spectral lines in the Paschen series observed and predicted by Eqns. (19a), (19b) and (19c). The value of each equation in predicting the observed data was tested by Analysis of Variance. The significance (p) of the regression is given in the last row. While the predictions of all three models are highly significant, the Wayne model is the best predictor.

n_f	n_i	λ_{obs} (nm) (NIST [12])	λ_{Bohr} (nm)	$\lambda_{Sommerfeld}$ (nm) $n_\theta = n$	λ_{Wayne} (nm) $n_\theta = n$
3	4	1875.13	1876.19	1876.19	1876.16
3	5	1281.8072	1282.55	1282.55	1282.53
3	6	1093.817	1094.44	1094.44	1094.43
3	7	1004.98	1005.52	1005.52	1005.51
			$p = 1.12 \times 10^{-9}$	$p = 1.12 \times 10^{-9}$	$p = 1.02 \times 10^{-9}$

Table 4: Spectral lines in the Brackett series observed and predicted by Eqns. (19a), (19b) and (19c). The value of each equation in predicting the observed data was tested by Analysis of Variance. The significance (p) of the regression is given in the last row. While the predictions of all three models are highly significant, the Bohr model is the best predictor.

n_f	n_i	λ_{obs} (nm) (NIST [12])	λ_{Bohr} (nm)	$\lambda_{Sommerfeld}$ (nm) $n_\theta = n$	λ_{Wayne} (nm) $n_\theta = n$
4	5	4052.279	4053.5	4053.49	4053.45
4	6	2625.871	2626.67	2626.66	2626.64
4	7	2166.1178	2166.78	2166.78	2166.76
4	8	-	1945.68	1945.68	1945.66
			$p = 2.12 \times 10^{-7}$	$p = 2.5 \times 10^{-6}$	$p = 7.51 \times 10^{-7}$

Table 5: Spectral lines in the Pfund series observed and predicted by Eqns. (19a), (19b) and (19c). The value of each equation in predicting the observed data was tested by Analysis of Variance. The significance (p) of the regression is given in the last row. While the predictions of all three models are highly significant, the Wayne model is the best predictor.

n_f	n_i	λ_{obs} (μm) (NIST [12])	λ_{Bohr} (μm)	$\lambda_{Sommerfeld}$ (μm) $n_\theta = n$	λ_{Wayne} (μm) $n_\theta = n$
5	6	7.45990	7.46212	7.46196	7.46204
5	7	4.65378	4.65519	4.65512	4.65515
5	8	3.740576	3.74169	3.74164	3.74167
5	9	3.29698	3.29799	3.29795	3.29797
			$p = 2.38 \times 10^{-11}$	$p = 3.11 \times 10^{-11}$	$p = 1.4 \times 10^{-11}$

All three equations describe the spectral lines in each series and the predictions made by all three models are close to the observed values. Eqns. (19b) and (19c) reduce to the Bohr Eqn. (19a) when the quadratic terms with respect to α in the square brackets are neglected.

In the above examples, the orbits are all circular and $n_\theta = n$. When precessing elliptical orbits are taken into consideration, the degeneracy of energy levels is broken and the fine structure of the spectral lines emerges. The fine structure is

obtained when either $n_i \neq n_{\theta i}$ or $n_f \neq n_{\theta f}$, the orbit is elliptical, and there is a precession of the perihelion. When $n_f = n_i = 2$, $n_{\theta f} = 1$ and $n_{\theta i} = 2$, the initial orbit is circular ($\epsilon = 0$), the final orbit is elliptical ($\epsilon = 0.866$), and the energy difference between the two orbits given by Eqns. (19b) and (19c) produces a spectral line of 2.7403236 cm, consistent with the observed value of 2.7330560 cm (Table 6). It is worth noting that Sommerfeld’s solution to the relativistic Kepler problem, according to Kragh [15] “was mathematically

demanding but not an impossible problem for a mathematical physicist of Sommerfeld's caliber." On the other hand, in this paper only elementary

mathematics is involved in the solution to the relativistic Kepler problem necessary to describe the fine structure of the spectral lines of hydrogen.

Table 6: The Fine Structure Spectral Line.

n_f	n_i	$n_{\theta f} = k_f$	$n_{\theta i} = k_i$	λ_{obs} (cm) (NIST [12])	$\lambda_{Sommerfeld}$ (cm)	λ_{Wayne} (cm)
2	2	1	2	2.7330560	2.7403236	2.7403236

While for the given conditions, Eqns. (19b) and (19c) give the exact same answer for the fine structure spectral line, the two equations are not algebraically identical. That is, they give different energy levels for the orbits involved in the fine structure but the same difference between the energy levels. Sommerfeld's equation is based on a series expansion of the Lorentz factor:

$$m = \frac{m_o}{\sqrt{1-\beta^2}} = m_o \left(1 + \frac{1}{2}\beta^2 + \dots \right) \quad (20)$$

Where, m is the relativistic mass, m_o is the rest mass and β is the ratio of the velocity of the electron to the speed of light and equals the fine structure constant for the first orbit of hydrogen. Sommerfeld [16] wrote that the success of his theory was "the experimentum crucis for the truth of the theory of relativity" and that "the absolute theory comes to grief owing to spectroscopic facts and has to give up to the theory of relativity the position of sovereignty which it formerly occupied." Pauli [17] wrote that Sommerfeld's theory of the fine structure of spectra "leads to a complete confirmation of the relativistic formula, which can thus be considered as experimentally verified." Eqn. (19c), which is based on the postulate of a tangential velocity-dependent Coulomb potential energy in Euclidian space and Newtonian time, gives an identical prediction of the fine structure of spectral lines in the hydrogen atom and provides an alternative to the Theory of Relativity for describing and explaining the atomic structure of hydrogen. The model of the atom presented here, like the model in Sommerfeld's old quantum theory, describes the movement in Newtonian time of an electron in a precessing elliptical orbit that is restricted to a single plane that is arbitrarily-oriented in absolute space (Fig. 1).

What provides the friction that will result in the precession of the perihelion? As long as the electron has extension [14,18], which it must if it is to have a measurable magnetic moment and

mechanical moment, the electron can act upon itself. As a consequence of its movement, the orbiting and spinning electron induces a magnetic field (Ampere's law) that in turn produces an electromotive force (Faraday's law) that has a polarity [Lenz's law] that resists the movement of the charged particle. Depending on the model used, this electromagnetic counterforce is equivalent to the optomechanical counterforce, friction, an increase in the inertia of the electron, an increase in the self-energy of the electron, a dressing or renormalization of the electron, or a relativistic dilatation of time [11].

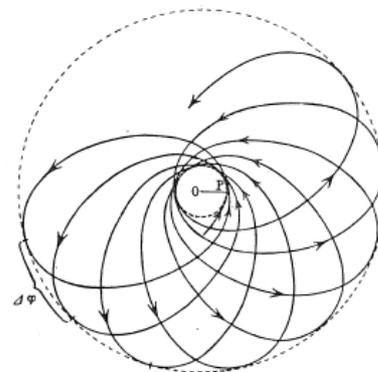


FIG. 110.

Fig.1: Sommerfeld's figure of the form of the relativistic Kepler orbit of the electron precessing around the nucleus (O) of a hydrogen atom [15]

In circular orbits, the electrical energy and the magnetic energy of the atom are constant. By contrast, in elliptical orbits, the magnitudes of the electric and magnetic energies are not uniform because the distance between the charged bodies, which determines the electrical energy, and the relative velocity of the charged bodies, which determines the magnetic energy, vary in a periodic fashion throughout the orbit. The correction term can be seen as nothing more than a second order correction to the coupling between the electric

energy and the magnetic energy that has the consequence of adding a precession of the perihelion of the orbit with consequent splitting of the spectral lines.

The correction term can also be interpreted as a perturbation energy that slows down the tangential velocity of the moving charges (decreasing the magnetic energy and electromotive force) and causes the charges to move closer together (increasing the electrical force). As a consequence of the conservation of total energy and angular momentum, the closer orbit will result in a greater relative velocity and a precession of the perihelion in Euclidean space and Newtonian time.

The magnetic field produced by an orbiting electron has the correct orientation both inside and outside the orbit to stabilize an extended electron in the orbit through the Lorentz force. Thus the transformation of electrical energy and magnetic energy by an extended body does not result in radiating energy and the collapse of the orbit as predicted by classical electromagnetic theory but in the stabilization of the orbit. Emission of energy results when an electron in an atom with more energy than its environment drops to a lower orbit with the loss of one unit of angular momentum and absorption results when an electron in an atom with less energy than its environment rises to a higher orbit with the gain of one unit of angular momentum. The spectral lines represent an exchange of a photon with energy $\frac{hc}{\lambda}$ and angular momentum $\pm\hbar$, consistent with the conservation laws for energy and angular momentum.

Sommerfeld's formula for the fine structure of the spectral lines of hydrogen gives the same discrete energy levels as Dirac's [19] relativistic equation as long as the quantum numbers are given different definitions [Appendix, 20-22]. Depending on the equation used, n_θ is equivalent to later definitions of the azimuthal quantum number such as k , $l + 1$, $l + \frac{1}{2}$, or $\sim\sqrt{l(l+1)}$ [23]. Unlike Sommerfeld's relativistic orbital mechanics, Dirac's relativistic quantum mechanics does not restrict an electron to a planar orbit but allows it to exist in a three-dimensional volume described by the quantum numbers.

The Dirac equation, which combined matrix mechanics with the Theory of Relativity, necessitates that an electron has spin. Eqn. 19c in the current paper contains the factor $\left[\frac{1}{2} + \frac{\alpha^2}{n^2 n_\theta}\right]$. Is it possible that the $\frac{1}{2}$ represents the contribution to the energy of the atom due to the intrinsic spin of

an electron while $\frac{\alpha^2}{n^2 n_\theta}$ represents the contribution to the energy of the atom due to the orbit, whose size and shape is characterized by n^2 and n_θ , respectively?

Griffiths [24] describes the fine structure constant as “undoubtedly the most fundamental pure (dimensionless) number in all of physics. It relates the basic constants of electromagnetism (the charge of the electron), relativity (the speed of light), and quantum mechanics (Planck's constant).” We can develop an idea of a dynamic meaning of the fine structure constant by rearranging Eqn. (18):

$$\frac{2E}{Z^2 \mu c^2} = -\frac{\alpha^2}{n^2} - \frac{2\alpha^4}{n^4 n_\theta} \tag{21}$$

The fine structure constant is, to a first approximation, equal to the negative square root of the ratio of twice the Hamiltonian energy to the mass-energy where the constant of proportionality is equal to the ratio of the principle quantum number to the atomic number. Because of the relationship between the Hamiltonian, the potential energy and the kinetic energy given in Eqn. (7), to a first approximation, the fine structure constant is also related to the negative square root of the ratio of the potential energy to the mass-energy and the square root of the ratio of twice the kinetic energy to the mass-energy.

$$\alpha \sqrt{1 + \frac{2\alpha^2}{n^2 n_\theta}} \cong \alpha \cong -\frac{n}{Z} \sqrt{\frac{2E}{\mu c^2}} = -\frac{n}{Z} \sqrt{\frac{PE}{\mu c^2}} = \frac{n}{Z} \sqrt{\frac{2KE}{\mu c^2}} \tag{22}$$

This gives special meaning to the mass energy outside of the Theory of Special Relativity. Thus for an electron in the first orbit ($n = 1$) of the hydrogen atom ($Z = 1$), the fine structure constant is given by:

$$\alpha \cong -\sqrt{\frac{2E}{\mu c^2}} \cong -\sqrt{\frac{PE}{\mu c^2}} \cong \sqrt{\frac{2KE}{\mu c^2}} \tag{23}$$

Thus α , which I propose couples tangential velocity-dependent electromagnetic frictional perturbations to changes in the electrical potential energy, is a measure of the ratio of the traditional electrodynamic energies to the mass energy [25].

3. Conclusion

The goal of classical mechanics and the old quantum theory was to describe and explain the unseen material world in terms of understandable

pictures. The methods used by astronomers were utilized by atomic physicists. In 1828, Fechner [26] used an astronomical picture to describe the inner workings of the atom. He wrote, “the atoms simulate in small dimensions the situations of the astronomical objects in large dimensions, being animated in any case by the same forces; and each body may be regarded as a system of innumerable small suns, floating at comparatively large distances from one another, such that each or several of them together are surrounded by orbiting atoms.” Weber [26] extended Fechner’s model of the atom by considering the Coulomb force to be more important than the gravitational force in determining the motions of subatomic charged particles. Lodge [27] wrote about the “dawn of atomic astronomy.” And according to Born [28], “If one succeeds in understanding the structure of the atom by means of perturbation calculation, then the similarity of cosmic and atomic processes has been demonstrated and it provides an indication of the unity of the processes and the laws of the world.”

The model of the atom presented here is similar to the Bohr-Sommerfeld model of the atom by using optical line spectra data to describe the internal arrangement and motion of subatomic particles through Euclidean space and Newtonian time. Quantum mechanics, on the other hand, abrogates the idea of particles orbiting through space and time, and describes the world in terms of numbers and eigenvalues, where \hbar no longer describes mechanical angular momentum but the uncertainty relation that results from noncommutativity of matrices. After the acceptance of the Copenhagen interpretation of quantum mechanics [24], probabilities were reified and mechanical orbits were eschewed. Sommerfeld himself lost faith in mechanical models [29]. Sommerfeld [30] wrote, “According to all results of experiment the formula doubtless remains correct but the model by means of which it was derived has to be abandoned.” After describing the Dirac theory of fine structure, Bethe and Salpeter [31] wrote, “Remarkably enough (17.1) has already been derived by SOMMERFELD from the ‘old’ BOHR theory, although the interpretation of the quantum numbers, statistical weights, etc. was different (and wrong) in the old theory!” Likewise, Richtmyer et al. [32] wrote, “By accident, Sommerfeld’s relativistic correction gave the same set of distinct energies as Eq. (159) [Dirac’s equation].” Eisberg [33] wrote “These results are in exact agreement with the predictions of Sommerfeld’s theory. Since the Sommerfeld theory

was based on the Bohr theory, it is only a rough approximation to physical reality. In contrast, the Dirac theory represents an extremely refined expression of our understanding of physical reality. From this point of view the agreement between equations (11-57) and (11-58) is one of the most amazing coincidences to be found in the study of physics.” Brehm and Mullin [34] wrote, “The two quantum numbers were employed by Sommerfeld and others to account for a broader array of quantized energy states in the analysis of the hydrogen atom. Of course, all these quasiclassical considerations passed rapidly into history with the coming of quantum mechanics.” Perhaps history still has something to teach us [35,36]. By taking orbits seriously again and by considering the possibility that the orbit can be perturbed by a tangential velocity-dependent force when the orbiting body is moving with a velocity greater than $1.5 \times 10^{-4}c$ [37], it may make it possible to model and describe the structure of an atom without robbing it of Euclidean space and Newtonian time.

Einstein wrote to Born [38] on December 4, 1926: “Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot, but does not really bring us any closer to the secret of the ‘old one’. I, at any rate, am convinced that He is not playing at dice.” Perhaps the consideration of the effect of a tangential velocity-dependent velocity dependent force that describes both the anomalous precession of the perihelion of Mercury and the fine structure of the spectral lines of the hydrogen atom will bring us closer to the secret.

Appendix

According to Bethe and Salpeter [30], the fine structure formula for hydrogen from the Dirac equation [19] is:

$$E = \mu c^2 \left[1 + \left(\frac{\alpha}{n-k+\sqrt{k^2-\alpha^2}} \right)^2 \right]^{-\frac{1}{2}} - 1 \quad (A1)$$

Letting $k = n_\theta$, the solution is:

$$E = - \frac{\mu c^2 Z^2 \alpha^2}{2n^2} \left[1 + \frac{\alpha^2}{n} \left(\frac{1}{n_\theta} - \frac{3}{4n} \right) \right] \quad (A2)$$

References

- [1] A. Sommerfeld, Eur. Phys. J. H. **39**, 179 (2014).
- [2] N. Bohr, Phil. Mag. **26**, 1 (1913).

- [3] N. Bohr, *Phil. Mag.* **30**, 394 (1913).
- [4] A. A. Michelson and E. W. Morley, *Amer. J. Sci.* **34**, 427 (1887); *Phil Mag.* **24**, 463 (1887).
- [5] A. Einstein, *Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin* **11**, 831 (1915).
- [6] I. Newton, *The Mathematical Principles of Natural Philosophy* (Printed for B. Motte, London, 1729).
- [7] R. Wayne, "Explanation of the perihelion motion of Mercury in terms of a velocity-dependent correction to Newton's law of gravitation", *The African Review of Physics* submitted.
- [8] L. Blitzer, *Am. J. Phys.* **39**, 882 (1971).
- [9] O. Lodge, *Nature* **101**, 125 (1918).
- [10] R. Wayne, *Acta Physica Polonica B* **41**, 1001 (2010). <http://arxiv.org/abs/1103.3697>
- [11] R. Wayne, *African Review of Physics* **8**, 283 (2013).
- [12] A. Kramida, Y. Ralchenko, J. Reader and NIST ASD Team (2014). *NIST Atomic Spectra Database* (ver.5.2), <http://physics.nist.gov/asd> [2015, June 9].
- [13] G. Herzberg, *Atomic Spectra and Atomic Structure* (Dover, New York, 1944).
- [14] L. Pauling and S. Goudsmit, *The Structure of Line Spectra* (McGraw-Hill Book Co., New York, 1930).
- [15] H. Kragh, *Niels Bohr and the Quantum Atom: The Bohr Model of Atomic Structure 1913-1925* (Oxford Scholarship Online, 2012) p.160.
- [16] A. Sommerfeld, *Atomic Structure and Spectral Lines* (Methuen & Co., London, 1923) p.528.
- [17] W. Pauli, *Theory of Relativity* (Dover, New York, 1958) p.83.
- [18] H. G. Demhelt, "Experiments with an isolated subatomic particle at rest," Nobel Lecture, December 8 (1989).
- [19] P. A. M. Dirac, *The Principles of Quantum Mechanics*, third edition (Clarendon Press, Oxford, 1947).
- [20] W. Heisenberg and P. Jordan, *Zeit. Für Physik* **37**, 263 (1926).
- [21] H. E. White, *Introduction to Atomic Spectra* (McGraw-Hill Book Co., New York, 1934).
- [22] M. Born, *Atomic Physics*, fourth edition, (Blackie & Son, London, 1946).
- [23] L. B. Loeb, *Atomic Structure* (John Wiley & Sons, New York, 1938).
- [24] D. J. Griffiths, *Introduction to Quantum Mechanics*, second edition (Pearson/Prentice Hall, Upper Saddle River, NJ, 2005) p.267.
- [25] A. Einstein, Does the inertia of a body depend upon its energy content? In: *The Collected Papers of Albert Einstein*. Volume 2. Doc. 24. Translated by Anna Beck (Princeton University Press, Princeton, NJ, 1989) p.172.
- [26] J. Mehra and H. Rechenberg, *The Historical Development of Quantum Theory. Volume 1, Part 1. The Quantum Theory of Planck, Einstein, Bohr and Sommerfeld: Its Foundation and the Rise of its Difficulties 1900-1925* (Springer-Verlag, New York, 1982). p.169.
- [27] O. Lodge, *Atoms and Rays* (George H. Doran, New York, 1924) p.203.
- [28] M. Born, *Naturwissenschaften* **11**, 537 (1923).
- [29] S. Seth, *Stud. Hist. Phil Sci.* **39**, 335 (2008).
- [30] A. Sommerfeld, *Three Lectures on Atomic Physics* (E. P. Dutton, New York, 1926) p. 24.
- [31] H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One-and Two-Electron Atoms* (Plenum/Rosetta, New York, 1977) p.84.
- [32] F. K. Richtmyer, E. H. Kennard and T. Lauritsen, *Introduction to Modern Physics*, fifth edition (McGraw-Hill Book Co., New York, 1955) p.260.
- [33] R. M. Eisberg, *Fundamentals of Modern Physics* (John Wiley & Sons, New York, 1961) p.356.
- [34] J. J. Brehm and W. J. Mullin. *Introduction to the Structure of Matter. A Course in Modern Physics* (John Wiley & Sons, New York, 1989) p.178.
- [35] S. Seth, *Crafting the Quantum: Arnold Sommerfeld and the Practice of Theory, 1880-1926* (MIT Press, Cambridge, 2010).
- [36] M. Eckert, *Arnold Sommerfeld: Science, Life and Turbulent Times. 1868-1951* (Springer, New York, 2013).
- [37] O. Lodge, *Atoms and Rays* (Ernest Benn, London, 1931).
- [38] M. Born, *The Born-Einstein Letters. Friendship, Politics and Physics in Uncertain Times* (Macmillan, New York, 2005) p.88.

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