Social Group Utility Maximization Game with Applications in Mobile Social Networks

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Abstract—In this paper, we develop a social group utility maximization game model that takes into account both social relationships and physical coupling among users. Specifically, instead of maximizing one’s individual utility, each user aims to maximize its social group utility that hinges heavily on its social ties with other users. A salient feature of this model is that it spans the continuum space between non-cooperative game and network utility maximization - two extreme paradigms based on drastically different assumptions that users are selfish and altruistic, respectively. Based on this model, we study two important applications in mobile social networks: random access control and power control, and quantify the impact of social ties on users’ strategies and network efficiency. In particular, our results show that, as the strength of social ties increases from the minimum to the maximum, the social-aware Nash equilibrium strategy of a player in this model migrates from the Nash equilibrium strategy in a standard non-cooperative game to the social-optimal strategy in network utility maximization. Therefore, the proposed social group utility maximization game model offers a general framework that encompasses non-cooperative game and network utility maximization as special cases, and we believe that it will open a new door to exploring the impact of social behavior on networking.

1. INTRODUCTION

Game theory has found a variety of important applications in many fields [1]. A basic assumption underpinning the existing game-theoretic models is that all players are selfish, i.e., each player acts without regard to the effects of its behavior on other players, aiming at maximizing its own benefit (e.g., utility). However, this assumption of selfishness has been repeatedly challenged by economists and sociologists. Extensive experiments have shown that the participants exhibit social behavior rather than being selfish [2], [3]. For example, in a mobile network where users are connected by an overlaying social network (as illustrated in Fig. 1), an individual user’s behavior depends on not only its physical coupling but also social relationships with other users, and hence it would often take into account the effects of its strategy on its social neighbors. Along a different line, network utility maximization has been extensively studied for resource allocation problems in various networks, assuming that all users have the same social objective of maximizing the total network utility. Clearly, this paradigm cannot capture the scenario where users have diverse social relationships from others.

In this paper, we advocate a social group utility maximization game model that takes into account both the players’ social relationships and physical coupling. In this model, instead of maximizing one’s individual utility, each player optimizes its strategy to maximize the weighted sum of its social neighbors’ utilities based on its social ties with them. One primary objective of this study is to establish a general framework that bridges the gap between non-cooperative game and network utility maximization - two traditionally disjoint paradigms for network optimization. These two paradigms are captured under the proposed framework as two special cases where no social tie exists and all players are connected by the strongest social ties, respectively (as illustrated in Fig. 2).

Based on the social group utility maximization game model, we study two important examples of its applications in mobile social networks: random access control and power control. Specifically, for the cases of $N$-user random access control and two-user power control, we show that there exists a unique social-aware Nash equilibrium (SNE). Furthermore, as the strength of social ties increases from the minimum to the maximum, a player’s SNE strategy (i.e., access probability or transmit power) migrates from the Nash equilibrium strategy in a standard non-cooperative game to the social-optimal strategy in network utility maximization. These results provide useful insight into the impact of social ties on users’ strategies and network efficiency.

Although there exists a significant body of work on non-

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cooperative game and network utility maximization, very little attention has been paid to the continuum space between these two extreme paradigms, especially in the context of mobile social networks. Recent works [4], [5] have studied the impact of altruistic behavior in a routing game. [6] has recently investigated a random access game between two symmetrically altruistic players. We emphasize that the social group utility maximization game is quite different from a coalitional game [7], [8], since each player in the latter aims to maximize its individual benefit (although it is achieved by cooperating with other players). Furthermore, while a player in a coalitional game can only participate in one social group (coalition), a player in our proposed game can be in multiple social groups associated with different players.

II. SOCIAL GROUP UTILITY MAXIMIZATION GAME

We consider a game consisting of players $\mathcal{N} \triangleq \{1, \ldots, N\}$ in a social network where two players are connected by a directed edge if one has a social tie towards the other. The strength of the social tie from player $i$ to player $j$ is quantified by $a_{ij}$ where $0 \leq a_{ij} \leq 1$. We assume that each player has a social tie with strength 1 to itself (i.e., $a_{ii} = 1, \forall i \in \mathcal{N}$). Let $u_i$ denote the individual utility obtained by player $i$. Based on the strength of social ties, each player $i$ aims to maximize its social group utility $f_i$ defined as

$$f_i(x_i, x_{-i}) \triangleq \sum_{j=1}^{N} a_{ij} u_i(x_i, x_{-i})$$

where $x_i$ denotes player $i$’s strategy and $x_{-i}$ denotes the vector consisting of the strategies of all the players except $i$. Note that both the individual utility and social group utility of a player potentially depend on the strategies of all other players. We say that a strategy profile $(x_1^{\text{NE}}, \ldots, x_N^{\text{NE}})$ is a social-aware Nash equilibrium$^1$ (SNE) if no player can improve its social group utility by unilaterally changing its strategy. It is worth noting that if no social tie exists (i.e., $a_{ij} = 0, \forall i \neq j$), our social group utility maximization game degenerates to a standard non-cooperative game where each player is selfish and only cares its individual utility, i.e.,

$$f_i(x_i, x_{-i}) = u_i(x_i, x_{-i})$$

$^1$For brevity, we may use “social Nash equilibrium”, and “Nash equilibrium” interchangeably.

In this case, a strategy profile $(x_1^{\text{NE}}, \ldots, x_N^{\text{NE}})$ is a Nash equilibrium (NE) if no player can improve its individual utility by unilaterally changing its strategy. If all the players are connected by the strongest social ties (i.e., $a_{ij} = 1, \forall i \neq j$), our game degenerates to network utility maximization where each player is fully altruistic and cares the network-wide utility, i.e.,

$$f_i(x_i, x_{-i}) = \sum_{j=1}^{N} u_j(x_j, x_{-j}).$$

We define the network utility $v$ as the total utility of all the players:

$$v(x) \triangleq \sum_{i=1}^{N} u_i(x)$$

where $x \triangleq (x_1, \ldots, x_N)$. We say that a strategy profile $(x_1^{\text{NO}}, \ldots, x_N^{\text{NO}})$ is network optimal (NO) if it maximizes the network utility.

To get a more concrete sense of our proposed social group utility maximization game model, in Section IV and III, we will study its applications in two scenarios: random access control and power control.

III. SOCIAL GROUP UTILITY MAXIMIZATION GAME FOR RANDOM ACCESS CONTROL

We consider a set of wireless communication links under the protocol interference model, where each player $i$ represents a link $i$ between a transmitter $T_i$ and a receiver $R_i$. Transmitter $T_i$ causes interference to receiver $R_j$ if $R_j$ is in the interference range of $T_i$ (as illustrated in Fig. 3). We use $\mathcal{I}_i^+$ to denote the index set of the receivers that transmitter $T_i$ causes interference to, and use $\mathcal{I}_i^-$ to denote the index set of the transmitters that causes interference to receiver $R_i$. Each player $i$ contends for the opportunity of data transmission with probability $q_i$ in a time slot where $0 \leq q_i \leq 1$. If multiple interfering links contend in the same time slot, a collision occurs and no link can grab the transmission opportunity. Then the probability $b_i$ that player...
i can grab the transmission opportunity is given by
\[ b_i(q_i, q_{-i}) = q_i \prod_{j \in I_i^+} (1 - q_j). \]

We assume that the individual utility of player i is given by
\[ u_i(q_i, q_{-i}) = \log(\theta_i b_i) \]
where \( \theta_i \) denotes player i’s efficiency of utilizing the transmission opportunity (e.g., transmission rate), which depends on the player’s local environment factors such as fading. Note that the logarithmic function is widely used in literature for modeling utility of wireless users [9], [10]. Next, given the social ties (i.e., \( a_{ij} \)) among the players, we study the social group utility maximization game for random access control \( G^R \triangleq (\mathcal{N}, \{q_i\}, \{f_i\}) \). We first have the following result.

**Theorem 1:** There exists a unique SNE \( (q_1^{SNE}, \ldots, q_N^{SNE}) \) in the social group utility maximization game for random access control, and
\[ q_i^{SNE} = \frac{1}{1+\sum_{j \in I_i^+} a_{ij}}, \forall i \in \mathcal{N}. \]

**Proof:** Since
\[ f_i(q_i, q_{-i}) = \log\left( \theta_i q_i \prod_{j \in I_i^+} (1 - q_j) \right) + \sum_{j \neq i} a_{ij} \log\left( \theta_j q_j \prod_{k \in I_i^+} (1 - p_k) \right), \]
we have
\[ \frac{\partial f_i(q_i, q_{-i})}{\partial q_i} = \frac{1}{q_i} - \sum_{j \in I_i^+} \frac{a_{ij}}{1 - q_i}. \]
Since
\[ \lim_{q_i \to 0} \frac{1}{q_i} - \sum_{j \in I_i^+} \frac{a_{ij}}{1 - q_i} = \infty, \lim_{q_i \to 1} \frac{1}{q_i} - \sum_{j \in I_i^+} \frac{a_{ij}}{1 - q_i} = -\infty \]
and
\[ \partial \left( \frac{1}{q_i} - \sum_{j \in I_i^+} \frac{a_{ij}}{1 - q_i} \right) = -\frac{1}{q_i} - \sum_{j \in I_i^+} \frac{a_{ij}}{(1 - q_i)^2} < 0, \]
there exists a unique value of \( q_i \) such that
\[ \frac{1}{q_i} - \sum_{j \in I_i^+} \frac{a_{ij}}{1 - q_i} = 0 \] (1)
which is also the value of \( q_i^{SNE} \). Solving (1), we obtain the desired result.

The next result directly follows from Theorem 1.

**Corollary 1:** The SNE strategy \( q_i^{SNE} \) is decreasing in \( a_{ij}, \forall j \in I_i^+, \forall i \in \mathcal{N} \).

Then we also have the following result.

**Proposition 1:** The network utility at the SNE \( v(q_1^{SNE}, \ldots, q_N^{SNE}) \) is increasing in \( a_{ij}, \forall j \in I_i^+, \forall i \in \mathcal{N} \), and achieves optimal when \( a_{ij} = 1, \forall j \in I_i^+, \forall i \in \mathcal{N} \).

![Fig. 4. In a two-player random access control game \( G^R \), as the social tie strength \( a_{12} \) increases from 0 to 1, player 1's social-aware NE strategy \( q_1^{SNE} \) decreases from the standard NE strategy \( q_1^{NO} \) to the network optimal strategy \( q_1^{NO} \), while the network utility \( v \) increases.](image-url)

**Proof:** Since
\[ v(q_1, \ldots, q_N) = \sum_{i=1}^{N} \log(\theta_i q_i \prod_{j \in I_i^+} (1 - q_j)), \]
we have
\[ \frac{\partial v(q_1, \ldots, q_N)}{\partial q_i} = \frac{1}{q_i} - \sum_{j \in I_i^+} \frac{1}{1 - q_i} = \frac{1}{q_i} - \frac{|I_i^+|}{1 - q_i}. \]
Using the same argument as in the proof of Theorem 1, the optimal value \( q_i^{NO} \) of \( q_i \) for \( v(q_1, \ldots, q_N) \) is the unique solution of
\[ \frac{1}{q_i} - \frac{|I_i^+|}{1 - q_i} = 0, \]
which is
\[ q_i^{NO} = \frac{1}{1 + |I_i^+|}. \]
In particular, we have \( q_1^{SNE} \geq q_1^{NO} \). Since \( \frac{\partial v(q_1, \ldots, q_N)}{\partial q_i} < 0 \) when \( q_i \geq q_i^{NO} \), \( v(q_1, \ldots, q_N) \) is decreasing in \( q_i \) when \( q_i \geq q_i^{NO} \). Using Lemma 1, \( q_i^{SNE} \) is decreasing in \( a_{ij}, \forall j \in I_i^+, \forall i \in \mathcal{N} \), and hence \( v(q_1^{SNE}, \ldots, q_N^{SNE}) \) is increasing in \( a_{ij}, \forall j \in I_i^+, \forall i \in \mathcal{N} \). \( \square \)

**Remarks:** In the random access control game \( G^R \), each player’s strategy at the SNE is a **dominant strategy** (i.e., the optimal strategy is independent of other players’ strategies), but depends on its social ties with other players. Since each player makes **negative effects** (i.e., increases probability of collision) on the individual utilities of those within its interference range by raising its access probability, it would reduce the access probability when its social ties with them become stronger (as illustrated in Fig. 4 left). As the players’ individual utilities are **equally weighted** in the network utility, the SNE strategy of each player becomes closer to the network optimal one (i.e., the network utility increases) when other players’ individual utilities weigh more (i.e., the social ties become stronger) in that player’s social group utility (as illustrated in Fig. 4 right). Therefore, as the social tie strengths increase from 0s to 1s, the SNE strategy of each player migrates from the NE strategy in a standard non-cooperative game to the social-optimal strategy for network utility maximization. This demonstrates that the proposed
social group utility maximization game spans the continuum space between these two extreme paradigms.

IV. SOCIAL GROUP UTILITY MAXIMIZATION GAME FOR POWER CONTROL

We consider a set of wireless communication links under the physical interference model, where each player $i$ represents a link $i$ between a transmitter $T_i$ and a receiver $R_i$. The channel gain of communication link $i$ is $h_i$, and the channel gain of the interference link between transmitter $T_i$ and receiver $R_j$ is $g_{ij}$ (as illustrated in Fig. 5). The noise at receiver $R_i$ is $n_i$. Then the signal to interference and noise ratio (SINR) $\gamma_i$ of link $i$ is given by

$$\gamma_i(p_i, \mathbf{p}^{-i}) = \frac{h_i p_i}{n_i + \sum_{j=1}^{N} g_{ij} p_j}$$

where $p_i$ denotes the transmit power of $T_i$. We assume that the individual utility $u_i$ of player $i$ is given by

$$u_i(p_i, \mathbf{p}^{-i}) = \log(\gamma_i) - c_i p_i$$

where $c_i$ denotes the cost of per unit power consumption. Similar to Section III, we also use the logarithmic function to model the utility of a player. For example, $\log(\gamma_i)$ can be a good approximation for the channel capacity $\log(1 + \gamma_i)$ under the high SINR regime. Also, $\log(\gamma_i)$ can be used to quantify the satisfaction of wireless users’ requirements in the high SINR regime. Next, given the social ties (i.e., $a_{ij}$) among the players, we study the social group utility maximization game for power control $G^P \triangleq \langle \mathcal{N}, \{p_i\}, \{f_i\} \rangle$. We first have the following result.

**Theorem 2:** The social group utility maximization game for power control is a supermodular game, and it follows that there exists at least one SNE.

**Proof:** Since

$$f_i(p_i, \mathbf{p}^{-i}) = \log \left( \frac{h_i p_i}{n_i + \sum_{j \neq i} g_{ij} p_j} \right) - c_i p_i + \sum_{k \neq i} a_{ik} \log \left( \frac{h_k p_k}{n_k + \sum_{j \neq k} g_{jk} p_j} \right) - c_k p_k,$$

we have

$$\frac{\partial f_i(p_i, \mathbf{p}^{-i})}{\partial p_i} = \frac{1}{p_i} - \frac{\sum_{k \neq i} a_{ik} g_{ik}}{n_k + \sum_{j \neq k} g_{jk} p_j} - c_i.$$

Since each term in the above summation term is decreasing in $p_j$, $\forall j \in \mathcal{N} \setminus i$, it follows that

$$\frac{\partial^2 f_i(p_i, \mathbf{p}^{-i})}{\partial p_i \partial p_j} > 0, \forall j \in \mathcal{N} \setminus i$$

which implies that the social group utility function $f_i(p_i, \mathbf{p}^{-i})$ is supermodular. It follows from [11] that there exists at least one NE.

Since $G^P$ is a supermodular game, it follows from [12] that each player $i$ can use the best response starting from $p_i = 0$ to update its strategy, such that all the strategies will monotonically converge to a SNE.

For ease of exposition, in the rest of this section we will focus on $G^P$ with two players, because the two-player game can provide useful insight into the impact of social ties on the players’ strategies and network utility. Furthermore, in general, the game with more than two players does not yield closed-form strategies at the SNE, and hence it is much more difficult to quantify the impact.

**Theorem 3:** In the two-player case, there exists a unique SNE $\{p_1^{SNE}, p_2^{SNE}\}$ in the social group utility maximization game for power control, and

$$p_1^{SNE} = \sqrt{\alpha_1^2 + \beta_1 - \alpha_1,} \quad p_2^{SNE} = \sqrt{\alpha_2^2 + \beta_2 - \alpha_2}$$

where

$$\alpha_1 \triangleq \frac{a_{12} g_{12} + c_1 n_2 - g_{12}}{2 c_1 g_{12}}, \quad \beta_1 \triangleq \frac{n_2}{c_1 g_{12}},$$

and

$$\alpha_2 \triangleq \frac{a_{21} g_{21} + c_2 n_1 - g_{21}}{2 c_2 g_{21}}, \quad \beta_2 \triangleq \frac{n_1}{c_2 g_{21}}.$$

**Proof:** Since

$$u_1(p_1, p_2) = \log \left( \frac{h_1 p_1}{n_1 + g_{21} p_2} \right) - c_1 p_1 + a_{12} \log \left( \frac{h_2 p_2}{n_2 + g_{12} p_1} \right) - a_{12} c_2 p_2,$$

we have

$$\frac{\partial u_1(p_1, p_2)}{\partial p_1} = \frac{1}{p_1} - \frac{a_{12} g_{12}}{n_2 + g_{12} p_1} - c_1.$$

Since

$$\lim_{p_1 \to 0} \left( \frac{1}{p_1} - \frac{a_{12} g_{12}}{n_2 + g_{12} p_1} \right) \geq \lim_{p_1 \to 0} \left( \frac{1}{p_1} - \frac{a_{12}}{p_1} \right) = \infty$$

and

$$\lim_{p_1 \to \infty} \left( \frac{1}{p_1} - \frac{a_{12} g_{12}}{n_2 + g_{12} p_1} \right) = 0$$

and

$$\frac{\partial}{\partial p_1} \left( \frac{1}{p_1} - \frac{a_{12} g_{12}}{n_2 + g_{12} p_1} \right) = -\frac{1}{p_1^2} + \frac{a_{12} g_{12}^2}{(n_2 + g_{12} p_1)^2} = \frac{(a_{12} - 1) g_{12}^2 p_1^2 - 2 a_{12} g_{12} p_1 - n_2^2}{p_1^2 (n_2 + g_{12} p_1)^2} < 0,$$

there exists a unique value of $p_1$ such that

$$\frac{1}{p_1} - \frac{a_{12} g_{12}}{n_2 + g_{12} p_1} - c_1 = 0,$$

which is also the value of $p_2^{SNE}$. Solving (2), we obtain the desired result. Similarly, we can obtain $p_2^{SNE}$. \qed

Next we have the following results.
Corollary 2: In the two-player case, the SNE strategy \( p_1^{SNE} \) is decreasing in \( a_{12} \) and \( p_2^{SNE} \) is decreasing in \( a_{21} \).

Proof: Since
\[
p_1^{SNE} = \sqrt{\alpha_1^2 + \beta_1 - \alpha_1}
\]
and
\[
a_{1} = \frac{a_{12}g_{12} + c_1n_2 - g_12}{2c_1g_{12}}, \quad \beta_{1} = \frac{n_2}{c_1g_{12}} > 0,
\]
we have
\[
\frac{\partial p_1^{SNE}}{\partial a_{12}} = \frac{\partial}{\partial a_1} \left( \sqrt{\alpha_1^2 + \beta_1 - \alpha_1} \right) \frac{\partial a_{11}}{\partial a_{12}}
= \left( \frac{\alpha_1}{\sqrt{\alpha_1^2 + \beta_1}} - 1 \right) \frac{1}{2c_1} < 0.
\]
So \( p_1^{SNE} \) is decreasing in \( a_{12} \). Similarly, we can show that \( p_2^{SNE} \) is decreasing in \( a_{21} \).

Proposition 2: In the two-player case, the network utility at the SNE \( v(p_1^{SNE}, p_2^{SNE}) \) is increasing in \( a_{12} \) and \( a_{21} \), and achieves optimal when \( a_{12} = a_{21} = 1 \).

Proof: Since
\[
v(p_1, p_2) = \log \left( \frac{b_1p_1}{n_1 + g_{12}p_2} \right) - c_1p_1 + \log \left( \frac{b_2p_2}{n_2 + g_{12}p_1} \right) - c_2p_2
\]
we have
\[
\frac{\partial v(p_1, p_2)}{\partial p_1} = \frac{1}{p_1} - \frac{g_{12}}{n_2 + g_{12}p_1} - c_1.
\]
Using the same argument as in the proof of Theorem 3, the optimal value \( p_1^{NO} \) of \( p_1 \) for \( v(p_1, p_2) \) is the unique solution of
\[
\frac{1}{p_1} - \frac{g_{12}}{n_2 + g_{12}p_1} = 0.
\]
In particular, we have \( p_1^{SNE} \geq p_1^{NO} \). Since \( \frac{\partial v(p_1, p_2)}{\partial p_1} < 0 \) when \( p_1 \geq p_1^{NO} \), \( v(p_1, p_2) \) is decreasing in \( p_1 \) when \( p_1 \geq p_1^{NO} \). Using Lemma 2, \( p_1^{SNE} \) is decreasing in \( a_{12} \), and hence \( v(p_1^{SNE}, p_2^{SNE}) \) is increasing in \( a_{12} \) since \( p_2^{SNE} \) is independent of \( a_{12} \). Similarly, we can show that \( v(p_1^{SNE}, p_2^{SNE}) \) is increasing in \( a_{21} \).

Remarks: Similar to the random access control game \( G^R \), the SNE strategy of each player in the two-player power control game \( G^P \) is also a dominant strategy. Each player would reduce its transmit power when its social tie with the other becomes stronger (as illustrated in Fig. 6 left). Also, the network utility improves when the other player’s individual utility weighs more in a player’s social group utility (as illustrated in Fig. 6 right). Therefore, as the social tie strengths increase from 0 to 1, the SNE strategy of each player also migrates from the NE strategy in a standard non-cooperative game to the social-optimal strategy in network utility maximization.

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V. CONCLUSION

In this paper, we have developed a general social group utility maximization game framework that bridges the gap between non-cooperative game and network utility maximization. In particular, we have studied two applications in mobile social networks under this framework: random access control and power control. Our findings provide useful insight into the impact of social ties on users’ strategies and network efficiency. Future work is needed to study the social group utility maximization game in more complex scenarios and applications. We believe that this work will open a new door to exploring the impact of social behavior on networking.

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