Multi-region distribution of petroleum products in large quantities by a heterogeneous vehicles fleet

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Abstract—In this paper, we continue addressing the multi-region distribution problem of petroleum products in large quantities involving the use of a fleet of tankers parked in depots. This heterogeneous fleet consists of vehicles owned by the company or of renting, non-compartmentalized and without volumeters, to serve customers from removal sites.

The objective is to minimize transportation cost while streamlining the number of vehicles. This work is an extension of the article on the same issue [3]. It extends the mathematical model by taking into account the displacements in departure and in the return of depots.

This paper presents a mixed linear programming model and samples are afterwards solved using a mathematical programming solver.

Keywords- distribution; petroleum products in large quantities; multi-region; mathematical programming

I. INTRODUCTION

In this work, we address the problem related to the distribution, in large quantities of petroleum products requiring the use of tankers fleet, parked in depots. This heterogeneous fleet consisting of vehicles owned by the company or third non-compartmentalized and without volumeters can serve customers from several removal sites (R.S). The objective is double; minimize the total transportation cost while rationalizing the vehicles utilization.

The distribution problem in general, is not a recent problem. It started with two classical problems in combinatorial optimization; the Traveling Salesman Problem (TSP) and the Vehicle Routing Problem (VRP).

The basic problem, the TSP, consists from a departure point, to visit a set of customers with one single truck and to come back. It consists therefore to plan its tour by finding the sequence of customers with the lowest possible total cost. Historically, [1] is the first work that introduces TSP problem by proposing resolution methods.

The vehicle routing problem addresses the case where each customer has a given request. It consists in determining several tours that all start and end at the depot and where each customer is visited only once by a single truck. The first work that addresses the VRP is [2]. The VRP generalizes the traveling salesman problem (TSP). The VRP is much more difficult to solve than the TSP [4].


There are several variants of the VRP; the main ones are presented in [7]. Among the first variant, we find the periodic VRP (PVRP) in which customers are served in a period of time rather than a single day [8], the multiple depot VRP (MDVRP) in which a company may have multiple depots to serve all its customers [9][10][11][12][13][14], VRP with pickup and delivery (VRPPD) in which vehicles can both load and unload products at the customers [15][16][17], the VRP with time windows (VRPTW) which defines a time interval within which the customer must be served [18][19], the stochastic VRP (SVRP) where one or more components of the problem are random [20][21][22] and the VRP with the choice of the heterogeneous fleet of vehicles (FSMVRP) [23][24][25].

After this literature review summarized in Fig.1, we notice that no research has presented a comprehensive study of the fuel distribution problem as defined above. More specifically, in connection with the problematic issue, two types of problems have been addressed in the literature, the multiple depots VRP (MDVRP) and the fleet size and mix VRP. Indeed, most publications are studying routing problems variations with inventory management of a single product or multiple products that can be transported in the same compartment of a truck. Furthermore, no publication addresses the problem of distribution with both multiple depots and multiple removal sites.

We note that regardless of the constraints of the VRP problem, especially the variant of the problem we are dealing with, it is still NP-hard, meaning that no known algorithm can guarantee to find in polynomial time the exact solution of these problems.

Figure 1. VRP variants
In our preceding work, we formulated the simplifying assumption that tankers’ depots are near the removal sites and as a consequence of this one can assimilate depots to removal sites. Thus the depots did not appear in the model and only displacements between removal sites and customers were considered.

In this article, this hypothesis is raised to lead to a more complete model including on the one hand, displacements between depots and removal sites, and on the other hand, displacements between clients and depots. This new model allows giving a more accurate account of the situation of the petroleum products distribution in the real world.

This article is organized as follows. In Section 2, we describe the problem addressed. Section 3 is devoted to the mathematical formulation of the problem. Finally, some numerical results are presented in the last section.

II. PROBLEM DESCRIPTION

We consider the case of a fuel distribution company that has its own fleet of trucks. The fleet being generally insufficient to meet the demand of all its customers, the company may then use the services of a truck rental company. Since products are ordered in large quantities, we can assume that the amount ordered by each customer exceeds the volume of the smaller truck. Therefore, each round consists of a single customer.

The objective of this type of company is to:

• Satisfy, at lower costs, the daily orders of all its customers.
• Rationalize the trucks utilization.

Taking into account the following constraints:

• Respecting the maximum number of work hours allowed for drivers.
• Full load transportation: the truck must leave entirely loaded towards the customers and return empty.

This problem is characterized by:

• The use of a heterogeneous tankers fleet without volumeters;
• The presence of several tankers depots;
• The truck loading is made from several removal sites.

A. Assumptions

In the remainder of this article, we are considering the following assumptions:

• A single product.
• The tours' planning horizon is daily
• A region is an area with a set of customers, a set of vehicles and one removal site. The customers of a region can be served only by the removal site associated with this region.

• A vehicle can be used in any region. It starts from the depot to which it is associated and must return there after visiting the last customer.
• The customer demands are linear combinations of the vehicles' capacities.
• Priority is given to company’s owned vehicles.

B. Constraints

• Constraint 1: each vehicle must leave empty its depot towards a removal site for a first loading
• Constraint 2: each vehicle must leave loaded from a removal site and return empty to any removal site or to its depot.
• Constraint 3: each vehicle visits only one customer at a time.
• Constraint 4: compliance with labor's legislation in terms of working hours.

C. The objective

The objective is twofold:

• Minimize the total transportation cost.
• Rationalize the vehicles utilization.

D. Vehicles displacements

The diagram in Fig. 2 shows an example of a general vehicle route between three regions, each formed by a single removal site and comprising a total of 7 customers. A vehicle displacement can be of four types:

• Type 1: Initial displacement from a depot to a R.S. It represents the first empty displacement of the vehicle from the depot to which it is associated to a removal site to serve customers associated with it. In Fig. 2, the empty displacement between the depot D1 and the removal site RS1 is an example of type 1. This is shown by a dashed arrow from D1 to RS1.
• Type 2: Round trip between a R.S and a customer. It represents a finite round trips' number between a removal site and a customer who is allocated to it. An example of this displacement type is a 3 times round trip between the removal site RS2 and the customer C3 (Fig. 2). It's shown by two arrows, the first solid one from RS2 towards customer C3 representing a loaded trip and the second dotted one from the customer C3 towards RS2 representing an empty return.

![Figure 2. Vehicle route diagram](image-url)
• Type 3: Displacement from an R.S towards another while passing through a customer. It represents a displacement of a removal site towards another while passing through a customer. The vehicle thus leaves a region to no longer come back there. In Fig.2, an example of this displacement type is presented between RS2 and RS3 while passing through the customer C4. It is represented by two arrows. The first solid one from the source removal site RS2 towards the customer C4 (representing a loaded displacement) and the second from the customer C4 towards the destination removal site RS3 (representing an empty displacement).

• Type 4: Final displacement from a customer towards the depot. It represents the final empty displacement from the last served customer to the depot to which the vehicle is associated. In Fig.2, the empty displacement between the customer C7 and the depot D1 is an example of type 4. It is represented by a dotted arrow from C7 towards D1.

III. MATHEMATICAL MODELING

The problem comes in the form of binary or integer variables linear program (MILP).

A. Data

• n: The number of customers to serve
• dj: The demand of customer j
• m: The number of vehicles
• V: The set of vehicles; \( V = \{1, ..., e\} \)
• r: The number of depots
• qk: The capacity of vehicle k
• Tk: Maximum travel duration time for vehicle k
• e: The number of removal sites
• Ck: The use cost of vehicle k
• \( c_{pj}^k \): The loaded transportation cost from the removal site p towards the customer j by the vehicle k
• \( c_{jp}^k \): The empty transportation cost from customer j towards the removal site p by the vehicle k
• \( c_{op}^k \): The empty transportation cost from depot o towards the removal site p by the vehicle k
• \( c_{jo}^k \): The empty transportation cost from customer j towards the depot o by the vehicle k
• \( t_{jp}^k \): The displacement time of vehicle k from the removal site p towards le client j
• \( t_{op}^k \): The displacement time of vehicle k from the depot o towards the removal site p
• \( t_{jo}^k \): The displacement time of vehicle k from the client j towards the depot o
• \( s_j \): The index of the removal site to which the client j is assigned.
• \( u_k \): The index of the depot where the vehicle k is initially parked.
• M: A large number whose greatness is superior than the number of routes between depots, removal sites and clients for any solution of the problem
• \( \alpha \): Weighting coefficient on the objective of minimization of the total kilometric cost
• \( \beta \): Weighting coefficient on the objective of minimization of the vehicles use cost

B. The variables

• \( x_j^k \): Integer variable indicating the number of round trips made by the vehicle k between the removal site Sj and the customer j
• \( \delta_p^k \): 1 if the vehicle k performs the route from the depot uk towards the removal site p. 0 otherwise.
• \( \lambda_j^k \): 1 if the vehicle k performs the route from the removal site Sj towards the depot uk while passing through the customer j. 0 otherwise.
• \( y_{jp}^k \): defined only for p \( \neq \) Sj, 1 if the vehicle k performs the route between the removal sites Sj and p while passing through the customer j. 0 otherwise.

Thus, the variables’ domains are as follows:

\[
\begin{align*}
x_j^k & \in N & j \in \{1, ..., n\} \quad k \in \{1, ..., m\} \\
y_{jp}^k & \in \{0, 1\} & p \in \{1, ..., e\} \quad j \in \{1, ..., n\} \quad k \in \{1, ..., m\} \\
\delta_p^k & \in \{0, 1\} & p \in \{1, ..., e\} \quad k \in \{1, ..., m\} \\
\lambda_j^k & \in \{0, 1\} & j \in \{1, ..., n\} \quad k \in \{1, ..., m\}
\end{align*}
\]

C. The constraints

\[
\sum_{p=1}^{e} \delta_p^k \leq 1 \quad k \in \{1, ..., m\}
\]

(1)
\[ \sum_{p} \delta_{p}^{k} = \sum_{j} \lambda_{j}^{k} \quad k \in \{1, \ldots, m\} \]  

(2)

\[ \delta_{p}^{k} + \sum_{q \neq p} v_{pq}^{k} \leq 1 \quad p \in \{1, \ldots, e\} \quad k \in \{1, \ldots, m\} \]  

(3)

Where:

\[ v_{pq}^{k} = \sum_{j \neq p} y_{jq}^{k} \quad p, q \in \{1, \ldots, e\} \quad p \neq q \quad k \in \{1, \ldots, m\} \]

Note that:

\[ v_{pq}^{k} : 1 \text{ if the vehicle } k \text{ performs the displacement from region } p \text{ towards region } q. 0 \text{ otherwise.} \]

\[ \delta_{p}^{k} + \sum_{q \neq p} y_{jq}^{k} = 1 \quad p \in \{1, \ldots, e\} \quad k \in \{1, \ldots, m\} \]  

(4)

\[ \sum_{k} q_{k} x_{j}^{k} + \sum_{p \neq s_{j}} q_{k} y_{jp}^{k} + \sum_{k} q_{k} \lambda_{j}^{k} = d_{j} \quad j \in \{1, \ldots, n\} \]  

(5)

\[ \sum_{j} x_{j}^{k} (s_{j}^{k} + t_{j}^{k}) + \sum_{p} \delta_{p}^{k} t_{ukp}^{k} + \sum_{p \neq s_{j}} y_{jp}^{k} (s_{j}^{k} + t_{jp}^{k}) \]  

(6)

\[ \sum_{k} \lambda_{j}^{k} (s_{j}^{k} + t_{uk}^{k}) = T_{k} \quad k \in \{1, \ldots, m\} \]

\[ \frac{1}{M} \sum_{j} x_{j}^{k} \leq \delta_{p}^{k} + \sum_{q \neq p} v_{pq}^{k} \quad k \in \{1, \ldots, m\} \quad p \in \{1, \ldots, e\} \]  

(7)

\[ \frac{1}{M} (\sum_{j} x_{j}^{k} + \sum_{j \neq p} y_{jp}^{k}) \leq \sum_{p \neq s_{j}} \delta_{p}^{k} \quad k \in \{1, \ldots, m\} \]

(8)

\[ \sum_{p \in S} \sum_{q \in S} v_{pq}^{k} \leq |S| - 1 \quad k \in \{1, \ldots, m\} \quad S \subset V, 2 \leq |S| \leq e - 1 \]  

(9)

- Constraint (1) imposes that a vehicle can leave its assignment depot at most once.
- Constraint (2) imposes the flow conservation for the assignment depot of vehicle k.
- Constraint (3) imposes that a vehicle can visit a region at most once.
- Constraint (4) imposes the flow conservation for each region visited by each vehicle, i.e. if a vehicle k visits a region then it must leave it.
- Constraint (5) ensures the satisfaction of the demand of each customer.
- Constraint (6) requires that each vehicle must respect the maximum duration authorized for one day’s work.
- Constraint (7) imposes that if a vehicle performs round trips towards a client of a region, then this vehicle must necessarily visit this region.
- Constraint (8) imposes that if a vehicle performs displacements towards clients, then this vehicle must have left its assignment depot.
- Constraint (9) imposes the sub tours elimination for the regions visits. This constraint is similar to the sub tours elimination constraint for the case of the mono-depot VRP where the graphs vertices correspond to regions.

**D. The objective function**

The objective function in (10) represents the total distribution cost, which consists of all displacements cost z1 in (11) and vehicles use cost z2 in (12):

\[ z = \alpha z_{1} + \beta z_{2} \]  

(10)

Where:

\[ z_{1} = \sum_{k} \sum_{j} (c_{s_{j}^{k}} + c_{p_{j}^{k}}) x_{j}^{k} + \sum_{k} \sum_{j \neq p} y_{jp}^{k} (c_{s_{j}^{k}} + c_{p_{j}^{k}}) + \sum_{k} \sum_{p \neq s_{j}} \sum_{p} c_{u_{k}p} \delta_{p}^{k} + \sum_{k} \sum_{j \neq p} (c_{s_{j}^{k}} + c_{j}^{k}) \lambda_{j}^{k} \]  

(11)

\[ z_{2} = \sum_{k} (\sum_{p \neq s_{j}} \delta_{p}^{k}) C_{k} \]  

(12)

The first term of z1 is the sum of transportation costs within each region, its second term represents the sum of transportation costs between two removal sites while passing through a customer, its third term represents the sum of transportation costs from depots towards removal sites, and the
fourth term represents the sum of transportation costs from removal sites to depots while passing through a customer.

Les Weightings α and β represent the quantification of the various terms to value. The industrial decision maker will be able to modulate the relative importance of criteria by exploiting parameters α and β values.

IV. NUMERICAL RESULTS

This section describes the first numerical results in-tended, on the one hand to validate the model and, on the other hand, to test the limits of the solver with big size instances.

A. Case study with 2 depots, 3 removal sites, 30 customers and 16 vehicles

This first test case aims to study a fuel distribution case in large quantities that consists of 2 depots, 3 removal site, 30 customers and 16 vehicles. The rental vehicles are of sufficient number. The vehicles capacities used are of 3 types: 7 tons, 9 tons and 18 tons.

Table I shows the vehicles distribution over depots according to their capacities and their belonging. Table II shows the vehicles use cost according to their capacities and their belonging. Table III shows the customers distribution as well as their demands by region (removal site). On the other hand, the time matrices and cost matrices are generated based on real cases.

The model contains 2449 variables and 303 constraints. The case study was solved using the CPLEX 9.0 solver on an Intel Core Duo 2 PC, 3 GHz with 3.25 GB of RAM under the Windows XP operating system. The computation time is expressed in seconds.

Table IV presents the results of the 11 vehicles tours related to the configuration α=β=0.5. For example in Fig.3, the tour of the company vehicle V15, with capacity 9T, starts at depot D2. Then the vehicle moves empty to RS3, serves the customers C21 then C25, afterwards it moves empty to RS1, serves customers C1, C10 (2 times), C2, C4, C8 and C6 belonging to the same region 1 before returning empty to the departure point D2 (depot to which vehicle V15 is assigned).

![Diagram](image-url)

Figure 3. Vehicles tours example- case of V15

To satisfy its customers demands, the company needed 9 vehicles of its own fleet besides 2 rental vehicles of 7T. That confirms the rational use of the owned vehicles before calling upon rental vehicles.

TABLE I. VEHICLES DISTRIBUTION OVER DEPOTS

<table>
<thead>
<tr>
<th>Vehicle Capacity</th>
<th>Company</th>
<th>Third</th>
</tr>
</thead>
<tbody>
<tr>
<td>7T</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>9T</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>18T</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

D.1

<table>
<thead>
<tr>
<th>Vehicle Capacity</th>
<th>Company</th>
<th>Third</th>
</tr>
</thead>
<tbody>
<tr>
<td>7T</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>9T</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>18T</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

D.2

TABLE II. VEHICLES USE COST

<table>
<thead>
<tr>
<th>Vehicles types</th>
<th>7T</th>
<th>9T</th>
<th>18T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company</td>
<td>350</td>
<td>450</td>
<td>800</td>
</tr>
<tr>
<td>Third</td>
<td>700</td>
<td>900</td>
<td>1600</td>
</tr>
</tbody>
</table>

TABLE III. CUSTOMERS’ DISTRIBUTION OVER REGIONS

<table>
<thead>
<tr>
<th>Region 1</th>
<th>Region 2</th>
<th>Region 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Customer</td>
<td>Demand</td>
<td>Customer</td>
</tr>
<tr>
<td>C1</td>
<td>27</td>
<td>C11</td>
</tr>
<tr>
<td>C2</td>
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<td>C12</td>
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<tr>
<td>C3</td>
<td>34</td>
<td>C13</td>
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<td>C4</td>
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<td>C14</td>
</tr>
<tr>
<td>C5</td>
<td>25</td>
<td>C15</td>
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<tr>
<td>C6</td>
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<td>C16</td>
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<td>C7</td>
<td>27</td>
<td>C17</td>
</tr>
<tr>
<td>C8</td>
<td>25</td>
<td>C18</td>
</tr>
<tr>
<td>C9</td>
<td>23</td>
<td>C19</td>
</tr>
<tr>
<td>C10</td>
<td>27</td>
<td>C20</td>
</tr>
</tbody>
</table>

TABLE IV. TOURS RESULTS FOR EACH VEHICLE

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Capacity</th>
<th>Company /Third</th>
<th>Daily tour</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>7T</td>
<td>Third</td>
<td>D1 S1 C2 S1 C3 S1 C6 S1 C6 S1 C8 S1 C9 S1 C9 S1 C5 D1</td>
</tr>
<tr>
<td>V4</td>
<td>7T</td>
<td>Third</td>
<td>D2 S3 C21 S3 C25 S3 C25 S3 C30 D2</td>
</tr>
<tr>
<td>V7</td>
<td>7T</td>
<td>Company</td>
<td>D1 S2 C11 S2 C15 S2 C15 S2 C15 S2 C17 S2 C18 S2 C20 S2 C20 D1</td>
</tr>
<tr>
<td>V8</td>
<td>7T</td>
<td>Company</td>
<td>D1 S2 C13 S2 C12 S2 C16 S2 C19 S2 C20 S2 C13 D1</td>
</tr>
<tr>
<td>V9</td>
<td>9T</td>
<td>Company</td>
<td>D1 S2 C11 S2 C15 S2 C17 S2 C18 S2 C20 D1</td>
</tr>
<tr>
<td>V11</td>
<td>18T</td>
<td>Company</td>
<td>D1 S1 C11 S2 C12 S2 C14 S2 C17 S2 C19 S2 C13 D1</td>
</tr>
<tr>
<td>V12</td>
<td>7T</td>
<td>Company</td>
<td>D2 S3 C22 S3 C22 S3 C23 S3 C24 S3 C26 S3 C27 S3 C28 S3 C29 C3 C30 D2</td>
</tr>
<tr>
<td>V13</td>
<td>7T</td>
<td>Company</td>
<td>D2 S3 C25 S3 C28 S3 C29 D2</td>
</tr>
<tr>
<td>V14</td>
<td>9T</td>
<td>Company</td>
<td>D2 S3 C10 S1 C3 S1 C4 S1 C7 S1 C8 S1 C9 D2</td>
</tr>
<tr>
<td>V15</td>
<td>9T</td>
<td>Company</td>
<td>D2 S3 C21 S3 C25 S1 C1 S1 C10 S1 C10 S1 C2 S1 C4 S1 C8 S1 C6 D2</td>
</tr>
<tr>
<td>V16</td>
<td>18T</td>
<td>Company</td>
<td>D2 S1 C1 S1 C3 S1 C7 S1 C5 S2 C16 S3 C30 D2</td>
</tr>
</tbody>
</table>
Results are presented in Table V. The 10 columns of this table correspond respectively:

- Column 1: the value of the weighting parameter $\alpha$. The parameter $\beta$ is computed by the formula $\alpha + \beta = 1$;
- Columns 2 to 4: the values of $z_1$ and $z_2$ as well as that of the objective function $z$;
- Column 5: the execution time (CPU);
- Columns 6 to 8 (respectively 9 to 11): the internal company’s indicators (resp. external indicators) in terms of their numbers nb1 (resp. nb2), their transportation costs $z'1$ (resp. $z''1$) and their use cost $z'2$ (resp. $z''2$). Note that $z_1 = z'1 + z''1$ and $z_2 = z'2 + z''2$.

Note that on one hand the execution time is not stable when varying the weightings $\alpha$ and $\beta$. On the other hand, the value of the objective function increases according to $\alpha$.

We note that the values of $z_1$ and $z_2$, in the case where $\alpha$ equals 0.4, 0.5 or 0.6, keep their values. This is explained for $z_2$ by the use of the same vehicles types (9 vehicles of the company and 2 rental vehicles). For $z_1$, we note that the values of $z'1$ and $z''1$ offset themselves. Indeed, vehicles of same company and 2 rental vehicles). For $z_1$, we note that the values of $z_1 = z'1 + z''1$ and $z_2 = z'2 + z''2$.

Table VI summarizes the numerical results by problem size.

The execution time of the model is related to the structure of each configurations case (number of depots, number of R.S., number of customers and number of vehicles). As the configuration complexity increase, the value of the objective function and the execution time also increase.

### Table V. Numerical result for the case study

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$z_1$</th>
<th>$z_2$</th>
<th>$z$</th>
<th>CPU</th>
<th>Internal indicators</th>
<th>External indicators</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$nb1$</td>
<td>$z'1$</td>
</tr>
<tr>
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<td></td>
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<td>$z'2$</td>
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<tr>
<td>0.0</td>
<td>14562</td>
<td>5750</td>
<td>5750.0</td>
<td>6225</td>
<td>9</td>
<td>12460</td>
</tr>
<tr>
<td>0.2</td>
<td>13805</td>
<td>5750</td>
<td>7361.0</td>
<td>6922</td>
<td>9</td>
<td>11867</td>
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<tr>
<td>0.4</td>
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<td>8954.4</td>
<td>7167</td>
<td>9</td>
<td>11910</td>
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<tr>
<td>0.5</td>
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<td>1451</td>
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<td>11865</td>
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</tbody>
</table>

Thus, as soon as one exceeds some size (for example, 3 depots, 4 R.S., 32 customers and 16 vehicles), we do not find any more optimal solution in a reasonable time of about 100 min. In this case, the result selected is the best solution found by the solver. We note that this solution is very close to the lower bound (Bound), also provided by the solver. The gap is presented in the last column of Table VI.

### V. Conclusion and Perspectives

Our study was focused on the multi-region distribution problem of petroleum products in large quantities involving the use of a fleet of heterogeneous tankers. This fleet is consisting of vehicles owned by the company or of renting, non-compartmentalized and without volumeters. We introduced a mixed linear programming model solved with the aid of a standard solver. First numerical results were introduced on one hand to validate the model and, on the other hand, to test the limits of the solver with big instances sizes.

The main perspective in future developments consists in the conception of the approached resolution methods especially via metaheuristics adapted to our case study like genetic algorithms family and ant colony algorithms family.

### Table VI. Numerical results by problem size

<table>
<thead>
<tr>
<th>Depots</th>
<th>R.S.</th>
<th>Customers</th>
<th>Available vehicles</th>
<th>Used vehicles</th>
<th>CPU</th>
<th>$z_1+z_2$</th>
<th>Bound</th>
<th>Gap</th>
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<td>4</td>
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</tr>
</tbody>
</table>

Thus, as soon as one exceeds some size (for example, 3 depots, 4 R.S., 32 customers and 16 vehicles), we do not find any more optimal solution in a reasonable time of about 100 min. In this case, the result selected is the best solution found by the solver. We note that this solution is very close to the lower bound (Bound), also provided by the solver. The gap is presented in the last column of Table VI.
REFERENCES


