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Application of Mathematical Model of Evacuation for Large Stadium Building

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Abstract: The statistics of sports arena accidents show that the main reasons which leading to crowd stampede are the exports blockage and the poor surrounding transportations. In the process of evacuation, the most common problem is that there are a large number of people are stranded and also they are the main carrier which leading to crowded stampede. With large amounts of data and reasonable evaluations on staffs and transportation instruments. We propose inflow model in the crowding state, principle of maximum flow on channel design, optimal model of vehicle parking, evacuation model of subways and buses, according to sections of evacuation in stadiums. We analyze their usage area, marginal conditions and real data. Finally, we get some valuable results, which are curves of density and flow, evacuation time, formula for channel design, optimal parking design and formulas for evacuation time of subways and buses. Such data suits the real data from varied references. With the help of models and results, we get the total time of evacuation, simulation of progress and give parts of real situations of evacuation. According to such results, 100000 people's evacuation can be finished in about 45 min. On such basis, we propose some optimal plans for stadium and its surroundings building.

Keywords: Evacuation, large flow, mathematical model, stadium

INTRODUCTION

The public places where have intensive persons are very dangerous, it could cause significant accidents with mass casualties easily. For example, the Atlanta Olympic games in 1996, when a lot of persons gathered in the Olympic Park, there was a sudden explosion which causing hundreds of people injured and dead. Fire, people over excessive. For example, on October 20, 1982, there were 340 people's dead by crowded. Riots, for example, when the game took place between Syria Super League of a Kameshli and Deiral-zour, there were riots in the fans and this leaded to over 100 persons injured and dead. On December 25, 2000, there was a fire accident in East building of Luoyang City, which leads to 309 persons dead. In 2001, the incident "911", which occurred in the high-rise building where people crowed, leading to 25,000 persons were evacuated emergency, 2000 persons dead and 6347 persons missing. In 2003, the subway arson which occurred in the crowd accumulation subway, leading 134 persons dead and 136 persons injured at least. In 2004, during the festival performances in Beijing, there occurred crowded accident leading to 17 persons dead. Therefore, study the safety evacuation of large public places has a great practical significance and social security value and in this process, the speed of evacuation is the most critical factor. At present, the typical models of evacuation velocity are as follows: the former Soviet Union Predtechenski and Milinskii and Togawa Ando in Japan. The space grid evacuation model SGM which developed by Wuhan University and City University of Hong Kong in China and the mathematical model of evacuation and escape speed. Compared the above models of crowd evacuation speed; it could take countermeasures to improve the design of Fire, which has a guiding significance for the study of crowd evacuation.

With the rapid development of sports competition in the world, the scale and the audience are all increasing; the safety management system of sport stadium has become the major constructing objects. The sport stadium is a place where always has crowed people, a large sport stadium can accommodate tens of thousands or even over 10% million persons. The key to hold a tournament is the transportation safety, especially the opening ceremony, closing ceremony and some wonderful competitions. The large numbers of people and centering time make it difficult for transportation and vehicles. The evacuation in stadiums should be considered seriously.

This study takes Chinese National Stadium as an example. It locates in suburbs of Beijing, contains 100 thousand people and has enough high-speed roads. This paper analyzes and designs the problems of exits, channels, places of parking, reasonable car styles, constitution of staff, arrangement of vehicles and acceptable waiting time, etc. This paper builds proper mathematical model of evacuation, simulates real situations and calculates the time for whole evacuation. The innovation of this study is that, according to the characteristics of the development of Beijing, analyze the possible factors that influence the crowd evacuation, which would play an important instruction role on the construction and planning of the sport places surroundings.

Fig. 1: Place of Chinese national stadium

RELATIVE ASSUMPTION

Places and scales of national stadium: According to plans of Olympic stadium building (Beijing 2008, http://www.beijing-2008.org/), the National Stadium (which is shown in Fig. 1), which holds the large Olympic opening ceremony and closing ceremony, is located in the center region of north Olympic park in Beijing. Its traffic is convenient and its place can promise no effects of outside transportations.

The stadium can hold 100 thousand people. A references shows (Cai, 1997) the criteria and fore-examples of previous Olympics. We estimate its area (except the surrounding facilities) is 120 thousand m^2.

People attended: The people are constituted of performance people, audiences, guests and staff. The audiences for ceremonies or competitions in the main stadium are more than 95% of the total ones. The main part is audience. So the model, except the special ones, is used for audience.

Arrangement for transportation tools: The stadium sits in the suburb. Most audiences will not be on foot. The transportation tools include:

- Subway and bus
- Small private car
- Taxi
- Client cars for private organization
- Is few, which can be ignored. (1) And (2) will be stressed in the following

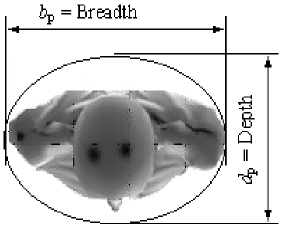

Fig. 2: Body's ellipse model

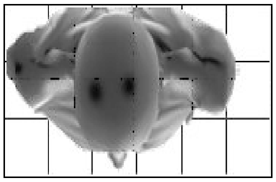

Fig. 3: Body's square model

FLOW MODEL OF CROWDING STATE

Physiological sizes: The area depends on maximum physiological sizes in all directions; usually the breadth of shoulder bp (Fig. 2) and the Depth of body dp (Fig. 3). To calculate conveniently, we subtract the individual as ellipse or square area.
The body's area S_p can indicated as:

$$S_{\mathrm{pE}} = \frac{1}{4}\pi b_{\mathrm{p}} d_{\mathrm{p}} \tag{1}$$

$$S_{\mathrm{pS}} = b_{\mathrm{p}} d_{\mathrm{p}} \tag{2}$$

Table 1: Physiological sizes in different regions

Nation (region)		Breadth b_p (m)	Depth d_p (m)	Area of ellipse S_{pE} (m^2)	Area of square S_{pS} (m^2)
British	M	0.5100	0.2850	0.1142	0.1454
	F	0.4350	0.2950	0.1008	0.1283
Japanese	M	0.4750	0.2300	0.0858	0.1093
	F	0.4250	0.2350	0.0785	0.0999
Hong Kong	M	0.4700	0.1250	0.0461	0.0588
	F	0.4350	0.2700	0.0923	0.1175
USA	M	0.5150	0.2800	0.1133	0.1442
	F	0.4700	0.2950	0.1089	0.1387
Indian	M	0.4550	0.2350	0.0840	0.1069
	F	0.3900	0.2550	0.0781	0.0995
Average		0.4850	0.2310	0.0887	0.1129

Table 2: Spatial occupation in typical situation

One person walks (normal)	650 mm
Two people walks from the opposite directions	1 350 mm
Normal spatial occupation of wheelchairs	900 mm
A person walks with an umbrella	1 150 mm
A person walks with a bag	800 mm

The data of physiological sizes in different regions are shown in the following, just as the following Table 1:

According to the results in Table 1, considering future development of population's quality, adding the convenient calculations, this paper set $b_p = 0.5$ m, $d_p = 0.25$ m, $S = 0.125$ m^2.

Group density: Group density is related to hysiological sizes and distances. The spatial occupation data in typical situation is shown as Table 2 (Stephen, 2001).

In view of large density while evacuating, the traverse distance between two people is 100 mm, the vertical distances change with the density.

In general, in view of safety, the maximum density must be less than 40 person/m^2 (Department of National Heritage, 1997). Combining the calculation above, the density can be accepted is (0.4) person/m^2 (the speed is ignored and the ideal value can be shown as follows).

Flow model in the crowding state:

Several assumptions:

- The flow comes in a unlimited channel with definite width in a single direction, comparative saturate, which is its speed is less than an extreme speed $V_{max} = 3$ m/sec.
- Any person has to conform to common principle: not trying to beyond the front body and without too large distance.
- Density ρ (person/m^2) equals everywhere and increases with the v decreases. Its values are between (ρ_{min}, ρ_{max}).
- Definite density flow (person/m·sec) is the number of people come across the unit area in a unit time, $q = \rho_v$.

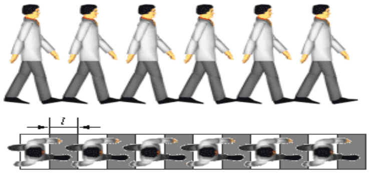

Fig. 4: Flow model

Then the model can be built:

The feet magnitude l(m) equals the distance between two neighboring people. With reference to Fig. 4 and the calculation of physiological sizes, we can get:

$$l = \frac{1}{(b_p + 0.1)\rho} - d_p \tag{3}$$

With the help of velocity and foot magnitude from reference (Jiang, 1993 and http://www.crowddynamics.com/), the relation between density ρ and walking frequency *f* can be definite:

$$f = K\rho^n \tag{4}$$

and a deeper proves: K = 1.36, n ≈ 0.5.

Indicate the group velocity in density:

$$v = l \cdot f = (\frac{1}{(b_p + 0.1)\rho} - d_p) \cdot K\rho^n \tag{5}$$

Definite the flow flux:

$$q = \rho \cdot v = (\frac{1}{b_p + 0.1} - d_p) \cdot K\rho^n \tag{6}$$

With the mathematical models above, relative parameters and marginal conditions, draw *v*-ρcurve and *q*-ρcurve, which are shown in Fig. 5 and 6:

It can be definite, when $\rho_0 = 2.22$ person/m^2 and $v_0 = 1.01$ m/sec, q gets the extreme, $q^* = 2.25$ person /m·sec.

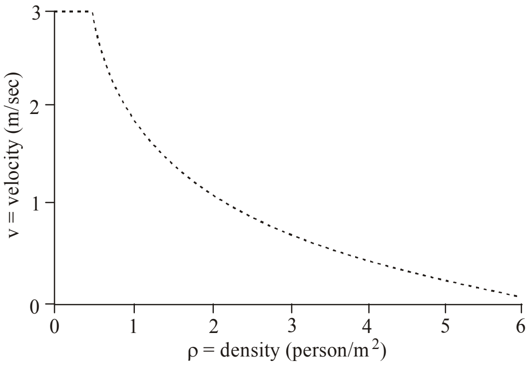

Fig. 5: Density-velocity curve

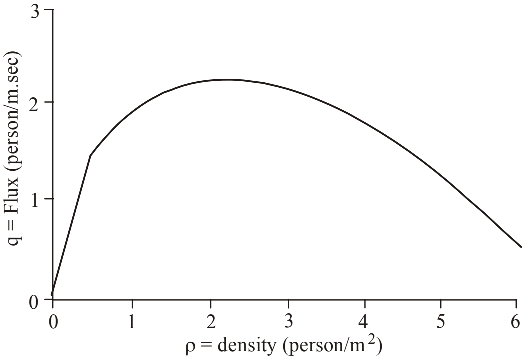

Fig. 6: Density-flux curve

Conclusion and analysis is as follows:
The theoretic prediction conforms to daily experiences and its values are similar to current data. By calculating the trends of flux, the maximum velocity and density can be kept.

The channels in the stadium are all narrow ones, with the enough density. So this model can be used for analyzing. To get the minimal evacuation time, the flow fluxes in all the channels should next to q*.

OPTIMAL DESIGN AND EVACUATION TIME

Maximum flux principle for channel: The analysis above says: to get the minimal evacuation time, the fluxes in all channels should be next to q* and be wide as possible as it could. The reference (Cai, 1997) summaries such principles:

According to Chinese shapes, the width of seat is 0.6 m. There are 50 groups of seats, which can hold 1600 people, in a circle. The distance between two groups is 1.0 m. To get the maximum flux, because the density of seats is next to the initial value of flow density, the seat density should be 2 person/m^2, which is a person occupies 0.5 m^2, the distance between the neighboring rows is 0.5/0.6 = 0.83 m. The perimeter is 50× (30×0.6+1) = 950 m. There are 31~32 rows in up and down layers. There are about 100000 seats, which are shown in Fig. 7.

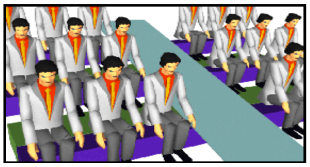

Fig. 7: Seats allocation and channel setting

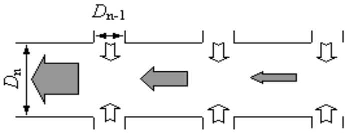

Fig. 8: Channel joining part

The channel between two rows (0 channels) can hold only a flow, whose width can make a person to walk through. The flow in 0 channels cannot reach ideal flux q*. So the length should be as short as possible (suggesting 15 times as the length of seat). The total length of 0 channels is related to the numbers of seats.

Others design depends on varied internal channels. The width should be controlled properly. Promise the inflowing of flow in the last level, to keep the mean flux as high as possible in the stable state, which is shown in Fig. 8.
Due to such principle, we can get:

$$D_n = \frac{k}{2} D_{n-1} \tag{7}$$

K : The total number of joints between n channel and n-1 channel
D_i: width of i channel

Design of exit, the relation formula:

$$D = BC \tag{8}$$

B : The number of flow in evacuation exit (channel)
C : A flow density. Generally, C = b_p+0.1 = 0.6 m

Other details include: The down-walking, horizon and slope are designed to increase the velocity. When the widths of stairs and slopes are large (>3 m), the middle railing to help evacuate.

Calculate evacuation time: Due to large number of audiences and centralized time, the unblocked channels and fine exits should be designed to evacuate all audiences in a time. The formula is shown in the following Cai (1997):

$$T_s = \frac{S}{V} + \frac{N}{BA} \tag{9}$$

T_s = Evacuation time
V = Evacuation velocity (m/min)
A = Single flux person/min)
B = The number of groups in a channel
N = Number of evacuation people
S = Evacuation distance (m)

Analyze the factors of evacuation time:
Single flux A (person/min):

$$A = \frac{V}{\frac{1}{C\rho}} = VC\rho \tag{10}$$

C = Single width of flow. Generally, C = Bp + 0.1 = 0.6 m
P = Group density

Number of exits n_b: The more exits, the smaller total distance between exit and out and it is better for shorting the evacuation time Ts. But the number should not too large, or the flows are too many and scattering, which is bad for controlling. It also increases the loads and makes it easy to forming bottleneck, so as to be dangerous.

Considering foreign large stadiums, n_b is 4, with symmetric distribution. The total number of exits reaches 8 or more, to evacuate in any accident.

The number of groups in a channel B: It is the key that can be controlled. With reference to design criteria and its design scale, estimate the evacuation time T_o = 15 min. The number of audiences is 95% of all. N = 100000:

$$B = \frac{N}{T_o A n_b} \tag{11}$$

Evacuation velocity V (m/min): Flow model in the crowding state quantitatively shows the relation between density and velocity. To get minimal evacuation time, the fluxes in all channels should be next to q*.

The velocities should also be next to v_0. Evacuation velocity V = v_0 = 60 m/min

Evacuation distance S (m): According to the real distance between the entrance and exit, calculate the total distance, which is the weighted distance. The formula is shown as follows:

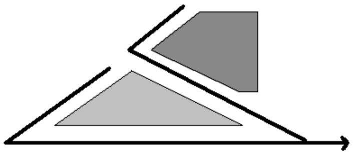

Fig. 9: Two-layer evacuation channel

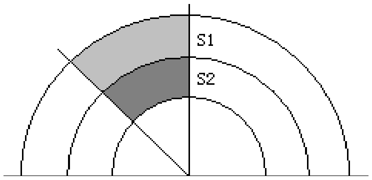

Fig. 10: Sector model

$$S = \frac{\sum_{i=1}^{n} S_i \cdot b_i}{\sum_{i=1}^{n} b_i} \tag{12}$$

$b_1, b_2, \ldots$: The numbers of flow in the first, second… evacuation channel
$S_1, S_2, \ldots$: The distances of the first, second… evacuation channel

The evacuation distance S should be as small as possible. In view of current references, the audience seats can be classified into two layers. The evacuation form is shown as Fig. 9.

The stadium has symmetric structure. To calculate conveniently, only consider the sector. Just as shown in Fig. 10.
Due to formula (13), here:

$$S = \frac{(S_1 \cdot s_1 + S_2 s_2)}{s_1 + s_2} \tag{13}$$

S_1 = Mean evacuation distance of upper layer audiences
S_2 = Mean evacuation distance of down layer audiences
s_1 = Area of sector in the upper layer stands
s_2 = Area of sector in the down layer stands

This sector area replaces the number of flows. In this sector, the middle row has 1600/8 = 200 seats. Imagine the neighboring rows have 2 seats' difference. Both the upper layers and down layers have about 30 rows. Therefore, the nearest row has 140 seats and the furthest one has 260 seats. Calculate the area of sector:

$$S = (Z_J + Z_Y) * P_S \tag{14}$$

Z_J : Number of seats in the nearest row
Z_Y : Number of seats in the furthest row
P_S : Number of rows

Then:

$$\frac{s_1}{s_2}=\frac{23}{17} \tag{15}$$

The distance of each circle lc ≈ 120 m. The slope angle of stair and stands is α = 30º. The height of audience seats h = number of rows × height of each row (about 0.47 m) = 14.1 m. The mean distance between upper layer and down layer is 14.1/sin30º = 28.2 m. Then:

$$S_1 = \frac{1}{2} l_c + h = 74 \text{ m} \tag{16}$$

$$S_2 = S_1 + \frac{h}{2\sin\alpha} = 88 \text{ m} \tag{17}$$

Bring into formula (18):

$$S = 82.05\text{m} \tag{18}$$

Calculate evacuation time:

$$T_s = \frac{S}{V} + T_o = 16.4 \text{ min} \tag{19}$$

PARKING PLAN AND EVACUATION TIME

Parking scale: The researches above say, the subway-bus and private cars is the main. The parking is for private vehicles. The following calculate the proportion of audiences with private vehicles.

In 2001, the total number of private cars is 500 thousand and keeps the increasing ratio of 15% (http://vinnie.myrice.com/news_3/rushiyiwei.htm).

Meanwhile, the number of population is 13.8 million and the year increasing ratio is 2.4% (http://www.cpirc.org.cn/new0406-6.htm). It can be deduced that, in 2008, the occupation ratio of vehicles in Beijing and other surrounding regions is 8.16 per hundred people. Adding the consideration of future increase, the scale is 10000, as 3 people per car. The number of evacuation is 27000 people.

To reduce the walk time, build more than two parking lots. Each can hold 5 thousand cars. To save costs, decrease sizes of parking lots as possible as it could and increase the spatial occupation ratio.

Table 3: common vehicle size

Type	People	Length mm	Width mm	Area m^2
Santana 2000AT	5	4546	1710	7.77
Jetta King	5	4385	1674	7.34
Jetta	5	4428	1660	7.35
Toyota Sienna CE	7	4932	1862	9.19
Mercedes-Benz S600	5	5154	1857	9.57
Average	5.4	4689	1753	8.22

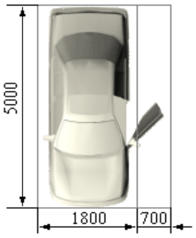

Fig. 11: Average size of vehicle

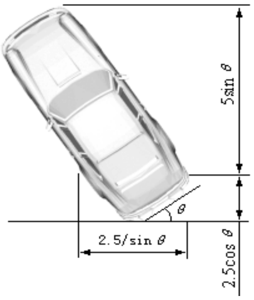

Fig. 12: Area for a car

Vehicle sizes: Common sizes from http://www.chinacars.com can help to estimate the mean size of private ones as follows Table 3.

This study uses following model to calculate relative parameters, which is shown in Fig. 11:

Optimal vehicle parking: The relation between area for a car and parking angle θ is shown as Fig. 12. Imagine l_c and w_c is length and width of a parking place, respectively:

$$l_c = 5\sin\theta + 2.5\cos\theta \tag{20}$$

$$w_c = 2.5/\sin\theta \tag{21}$$

Then a place's area is:

$$S_c = \frac{2.5(5\sin\theta + 2.5\cos\theta)}{\sin\theta} \tag{22}$$

Its changes are shown as Fig. 13. Sc increases as θ decreases, but θ's decreasing is good for vehicle's

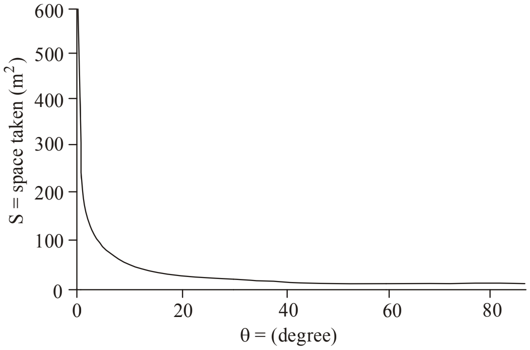

Fig. 13: Sc-θ curve

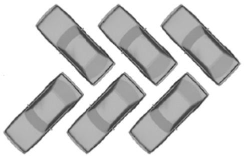

Fig. 14: 45 parking

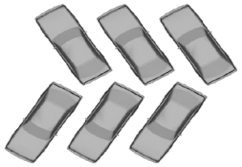

Fig. 15: The spatial waste of non-45 parking

leaving. When θ is 45º, the area for a car changes little. The car is easy to leave. Besides, the car can parking interlocked to save space, which are shown in Fig. 14 and 15.

When θ = 45º, the mean area of a car is:

$$l_c = 4.4\text{m} \tag{23}$$

$$w_c = 3.5\text{m} \tag{24}$$

Parking lot and arrangement of vehicles: To optimize the parking lots and reduce or avoid block, some principles are proposed:

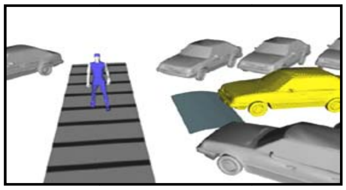

Fig. 16: The protection of obstacles for limiting velocity on passengers

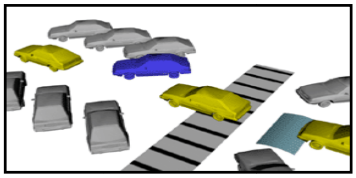

Fig. 17: The protection of obstacles for limiting velocity on cars

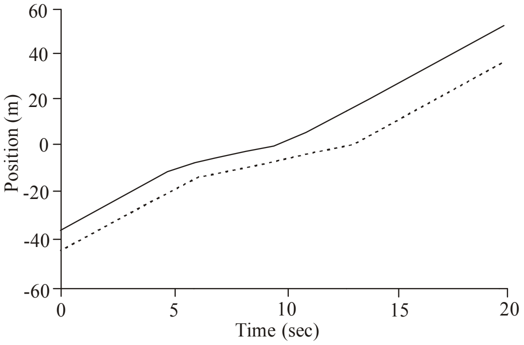

Fig. 18: The effects of obstacles for limiting velocity on neighbouring distance

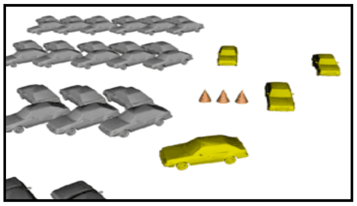

Fig. 19: Groups of parking

- Reduce the ratio of length and width of parking lots to shorten the distances for an audience to walk that is to short evacuation time as possible as it could.
- Set zebra lines at the cross of sidewalk and laneway, meanwhile set obstacles for limiting velocities, just as shown in Fig. 16 and 17. Such can keep passengers safe, increase the distance after crossing the deceleration zone, to help other cars to go into the flow. The influencing factors of setting the obstacles for limiting velocity on adjacent among vehicles are shown in Fig. 18.
- In lane 4. The two lanes can only be used for 7 to 12 groups, to avoid intercross or mutual blocking, which is shown in Fig. 19.
- The narrow shape can be good for connection between out lane and roads.
- When evacuation people flow into parking lots, they can be gone into through in lane. At the same time, the cars are banned.

The evacuation time:
Several assumptions:

- The parking lot is single inflow multi outflow. It has 4 lanes. The width is 10 m. The two rows in the same time are a group. The width of out lane is 4 m. Set sidewalk between certain distances, whose width is 2.5 m.
- The numbers of sidewalks change little, to calculate conveniently and apply widely, there are 6 sidewalks.
- While evacuating, the flow comes in from in lane, avoiding the intercross with people and vehicles effectively. Besides, the obstacles for limiting velocity can keep the car running slowly in front of the sidewalks. So ignore the effect of people flow.
- The average of vehicle velocities is 5 m/sec and the velocity of walking is 1.3 m/sec

  Variable introduction:

  n = Number of groups
  w_p = Total width of sidewalk
  w_i = Width of in lane
  t_1 = The maximum driving time
  t_2 = The maximum walking time from the entrance to a vehicle

  Formula:

$$t_1 = \frac{5000 \quad w_c}{2n} \tag{25}$$

$$t_2 = \frac{1}{1.3}\left((2l_c + 4)n + \frac{\frac{5000\, w_c}{2n} + w_p + w_i}{2}\right) \tag{26}$$

  After calculating, we can get $t_1 \ll t_2$. Therefore, evacuation time depends on t_2. When $n = 19$, t_2 is the minimal one. The evacuation time:

$$T_p = \min(t_1 + t_2) = 7\,\text{min} \tag{27}$$

  We can list the optimal parameters in a deeper way: the total number of single row is 132. The total number of vehicles is 5016. Set a sidewalk per 22 vehicles. There are 4 sidewalks. The total length of vehicle is 500(487) m. The total width is 250 (244) m. The area is 125 (119) thousand m^2.
- **Evacuation time of subway and bus:** Calculation from above, 27%, that is 27000, people use private vehicles. Imagine all the resident ones choose rail transportation tools and bus. In view of their time principle, their abilities are constant and models are simple. Give the assumption and results directly:

Evacuation time of subway and bus:

$$T_b = \frac{N}{N \cdot N_c} t_g \tag{28}$$

t_g = Waiting time between neighboring bus (min).
N = Number of people using transportation tools 73000 persons
N_l = Number of lines available
N_c = Evacuated ability per car (person/time)

According to values given by Xu Yanli "Beijing light-rail railway will come true", N_c is 103. Set 2000 person/time. And set ideal waiting time tg = 2.5 min. According to news from (Beijing 2008 Olympic official website, 2008), Beijing will build 7 subway lines in 2008. Imagine $N_l = 3$ locates near the main stadium. Then according to formula 29, we get:

$$T_b = 30\,\text{min} \tag{29}$$

CONCLUSION

According to the flow model in the crowding state above: all the channels and exits should be designed with enough density ρ and essential flowing velocity v, so as to makes q to be next to the extreme value q* as possible as it could. On such basis, give some quantitative analysis of channel, exits and layout of parking lots and details of vehicle arrangement principle. Then get estimations of evacuation time. Give the total evacuation time T:

$$T = T_i + \max(T_p, T_b) \tag{30}$$

T_i = Evacuation time in stadium
T_p = Evacuation time in parking lot
T_b = Evacuation time in subway and bus

Summarized from analysis and results above, the total evacuation time is about 46 min to the scale of 100000 people. It is easy to get: evacuation time in subway and bus T_b is the key. To decrease T_b, add the lines available and density of vehicles. On the other hand, due to the low standard of private vehicles occupations, T_p is so small to be covered by T_b. But it can be predicted, that, the number of private vehicles will go up seriously, then more problems will have to be tackled.

REFERENCES

Beijing, 2008. Olympic Official Website. Retrieved from: http://www.beijing-2008.org/.

Cai, Z., 1997. 7th Set of Architecture Designs. China Architecture Industry Press, Beijing.

Department of National Heritage, 1997. Guide to Safety at Sports Grounds. 4th Edn., H.M.S.O. Publications.

Jiang, Q., 1993. Mathmatical Model. 2nd Edn., High-Level Education Press, Beijing.

Stephen, P., 2001. Bodyspace: Anthropometry, Ergonomics and the Design of the Work. 2nd Edn., USA Taylor & Francis Inc., USA.