Dual representations for positively homogeneous functions: an application to constrained extremum problems

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Abstract

The aim of the present paper is to provide a unitary exposition on possible representations for positively homogeneous functions in the dual space. Starting from the classical Minkowski–Hörmander duality result of convex analysis, we develop a scheme in order to associate to a positively homogeneous function families of sublinear functionals representing it. Such dual description allows us to characterize several classes of positively homogeneous functions (for instance MSL or semicontinuous ones). Thus, by means of theorems of the alternative, an approach for deriving Lagrangean type optimality conditions is proposed for inequality constrained directionally differentiable extremum problems.

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