MMSEC-RAKE Receivers with Resolution Reduction of the Diversity Branches: Analysis, Simulation, and Applications

Mario R. Hueda, Graciela Corral-Briones, and Carmen E. Rodríguez

Abstract—In channels where the delay spread is smaller than the chip interval (e.g., an IS-95 system operating in indoor environments), spread-spectrum signals do not give rise to path diversity. In this situation, maximal-ratio combiner (MRC) RAKE receivers with resolution reduction (RR) of the diversity receiver branches may be used by the mobile stations to provide diversity gain, significantly improving system performance.

A new resolution reduction technique based on the use of a minimum mean-square-error diversity combiner (MMSEC) is proposed in this work. We show that, under very general assumptions, this new method of RR is optimal. A detailed study of the performance of a dual-branch MMSEC-RAKE receiver with RR in a typical indoor office environment is presented. In order to allow a simple practical implementation, a suboptimal structure of the MMSEC is also proposed. Numerical results show that this new receiver scheme provides a 1.2-dB improvement over the previously proposed RR technique based on MRC, and a 4.9-dB improvement over conventional MRC-RAKE receiver without RR, at a frame-error rate of 0.01 for the downlink of the IS-95 system in a typical indoor office environment.

Index Terms—CDMA, correlated link fading, diversity combining, PCS.

I. INTRODUCTION

T he direct-sequence code-division multiple-access (DS-CDMA) IS-95 standard with 1.23-MHz bandwidth [2] was originally designed for an outdoor cellular system where the delay spread is usually in the range of 10 μs. The delay spread for an indoor environment is typically around 100 ns, which cannot be resolved by a CDMA receiver with an 813-ns chip interval. In this situation, the path resolvability condition is not present, thus a conventional maximal-ratio combiner (MRC) RAKE receiver sets to unity the number of branches and diversity gain is not available.

The importance of using receivers especially designed to handle the unresolvability condition has been clearly demonstrated [3], [4]. Danilo and Leib [4] derived noncoherent receivers structures for an L-path Rician/Rayleigh channel assuming unresolved multipaths with known delays. In [4], it was shown that for Rayleigh fading channels the optimum receiver consists of a decorrelation stage followed by an optimum decision rule for resolved fading channels. Performance evaluation of this scheme demonstrated that significant performance improvements can be achieved when the knowledge of the unresolved multipath delays is available at the receiver. Special techniques such as super-resolution technique or sounding the channel with a wide-band delays are required to estimate the delays.

Yang showed in [1] that diversity-like gain can be also obtained from unresolved multipaths even if their delays are not known. The technique introduced by Yang (called resolution reduction) does not require multipath delays estimates and thus it can be easily incorporated in current CDMA schemes (e.g., IS-95), significantly improving the system capacity. The resolution reduction (RR) technique consists of using the RAKE receiver with branches spaced less than one chip period apart. It has been shown in [1] that this technique produces diversity gain in spite of the fact that the signals at the output of the RAKE are correlated. Several authors have analyzed the effects of the correlation of path signals on the system performance [5]–[8]. For example, Turin [5] derived the error probability of the optimal diversity receiver with correlated link fadings. Ling [6] found the matched filter bound for M correlated path channels. In [7], Al-Hussaini and Al-Bassiouni studied the effects of correlation on the performance of a dual-branch MRC, while Aalo [8] analyzed the performance of an M-branch MRC in a correlated Nakagami-fading channel. All these works show that even when the path signals are correlated, it is possible to obtain an important improvement with respect to the case of only one path. Based on this result, Yang [1], [9] proposed a reduction of the interval between diversity branches in order to improve the performance of an MRC-RAKE receiver on indoor radio channels. In this case, a reduction to one half of the chip interval significantly improves the frame-error rate (FER) in the forward link of the IS-95 standard. However, a detailed theoretical study of resolution reduction techniques has not been reported so far.

In this paper, we first analyze the signal at the mobile station, which encompasses the transmitted signal from its base station and the noise (thermal noise + interference from other cells). We show that owing to RR and the effect of the receive filter, the noise components at the output of the RAKE are correlated. In this situation, an MRC is not effective, therefore we introduce a minimum mean-square-error combiner (MMSEC) [10].

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We focus our study on a dual-branch MMSEC-RAKE receiver with RR in a typical indoor office channel. Similar to the work of DaSilva et al. [11], we take into account the chip pulse shape used in practical implementations. This is important because in an interference dominated system, the pulse shape determines not only the correlation of the signal, but also the correlation of the noise. Our study also includes the effects of uncorrelated noise components at the input of the RAKE, which are not affected by the RR (for example, quantization noise from A/D converters). By means of theoretical analysis and computer simulation, we find that MMSEC significantly outperforms MRC. To allow a simple practical implementation, a suboptimal structure of the MMSEC is also proposed. This only needs to estimate the signal components at finger outputs (which can be accomplished by using the pilot signal) rather than the gains, delays, and/or phases of the unresolved multipaths. Compared with an MRC, only a pair of additional operations per combiner coefficient is necessary. FER improvements are demonstrated by using a simulation system based on the forward link of the IS-95 standard.

The paper is organized as follows. In Section II, we derive expressions for the signal statistics at the output of the RAKE. The optimal RR receiver is derived in Section III. In Section IV, we introduce a near-optimal MMSEC, and concluding remarks are given in Section V.

II. RECEIVED SIGNAL AT THE MOBILE

Throughout this paper, we concentrate on the forward link of the IS-95 standard [2]. This section considers the signal received at the mobile terminal.

A. Channel Model

We assume a Rayleigh fading model for the transmission channel from the base station to the mobile terminal. Then, the multipath radio channel may be described as a wide-band tapped delay line model

\[ h(t) = \sum_{l=0}^{L-1} \beta_l(t) \delta(t - \tau_l) \]  

(1)

where \( L \) is the total number of multipaths, \( \beta_l(t) \) and \( \tau_l \) are the complex amplitude and the delay of the \( l \)th multipath, respectively (specifically, \( \tau_l \) is the relative time delay respect to the first arriving component). Note that, for a Rayleigh fading channel, \( \beta_l(t) \) are zero-mean complex Gaussian random variables. We assume that the multipath components are independent, thus

\[ E\{\beta_l(t)\beta_{l'}^*(t)\} = \begin{cases} 0, & m \neq l, \ m = 0, 1, \ldots, L - 1 \\ \frac{1}{\tau_l}, & m = l \end{cases} \]  

(2)

where \( E\{\} \) denotes the expectation operator and the superscript * indicates complex conjugation. The delays \( \tau_l \)'s are assumed to be real constants for a particular channel model. The tapped delay line parameters for indoor office channel shown in Table I [1] are used in this work. Note that the mean delay spread of the channel (denoted by \( d_{mean} \)) is around 100 ns.

### Table I

<table>
<thead>
<tr>
<th>Tap</th>
<th>Rel. Delay</th>
<th>Av. Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 ns</td>
<td>0.0 dB</td>
</tr>
<tr>
<td>1</td>
<td>100 ns</td>
<td>-3.6 dB</td>
</tr>
<tr>
<td>2</td>
<td>200 ns</td>
<td>-7.2 dB</td>
</tr>
<tr>
<td>3</td>
<td>300 ns</td>
<td>-10.8 dB</td>
</tr>
<tr>
<td>4</td>
<td>500 ns</td>
<td>-18.0 dB</td>
</tr>
<tr>
<td>5</td>
<td>700 ns</td>
<td>-25.2 dB</td>
</tr>
</tbody>
</table>

B. RAKE Receiver

For a slow fading channel, the equivalent low-pass received signal (assuming perfect carrier recovery) can be expressed as

\[ r(t) = \sum_{l=0}^{K-1} \sum_{k=0}^{N_l} \alpha_k a_k \cdot \sum_{i} b_k[N_i] a_k[i] \delta(t - zT_c - \tau_l) + z(t) \]  

(3)

where \( K \) is the total number of users (\( k = 0 \) denotes the pilot signal), \( T_c \) is the chip period (e.g., \( T_c = 813 \) ns for IS-95), \( \alpha_k \) and \( A_k \) are the voice activity factor and the signal amplitude for the \( l \)th user, respectively.

\[ \phi(t) = \frac{1}{\sqrt{T_c}} \frac{\sin(\pi t / T_c)}{(\pi t / T_c)} \]

is the impulse response of the transmit filter, which corresponds closely to the chip pulse shape used in the IS-95 standard [2]. \( a_k[i] \) is the quadrature phase-shift keying chip sequence for the \( l \)th user (i.e., \( a_k[i] = (\pm 1 \pm j)/\sqrt{2}, \ j = 1 \); we assume that the different chip sequences are orthogonal because Walsh functions are used in the code sequence. \( b_k[N] = \sum_i b_k[m] p_{iN} \) is the information sequence, where \( b_k[m] = \pm 1 \) is the data symbol (e.g., the output of the channel coder and the interleaver in the IS-95 standard). \( N \) is the number of chips per data symbol, and \( p_{iN} = 1 \) for \( i = 0, \ldots, N - 1, p_{N} = 0 \) elsewhere. \( z(t) \) is zero-mean additive white Gaussian noise (AWGN) due to combined effects of thermal noise and interference from adjacent cell-site base stations; we will assume that the power spectral density function (psdf) of \( z(t) \) is \( I_{\infty} \), that is \( E\{z(t + \tau)z^*(\tau)\} = I_{\infty} \delta(\tau) \).

At the receiver, \( r(t) \) is filtered by \( \phi^*(t) \) and sampled each \( T_c/M \) s (e.g., \( M = 2, 4, 8 \)). Then, the samples are processed by synchronization stages (code acquisition, alignment, and tracking) and by the RAKE fingers (despreaders) [12]–[14]. After the searching process, the output of the \( j \)th finger at the \( m \)th instant for the \( k \)th user is (see Fig. 1)

\[ y_{ij}[m] = \frac{1}{N} \sum_{n=mN+n_j}^{mN+n_j+N-1} a_{k1}[n - n_j] \cdot (r_f(nT_c + \Delta \tau_j) + \mu_j[n]) \]

(4)

where \( \mu_j[n] = \mu^[\phi^*(t)]_j(t) \) is the signal at the receive filter output (the operator \( \oplus \) denotes convolution), \( J \) is the
total number of RAKE fingers, $a_{kj}[n]$ is the chip sequence of the $k_{th}$ user, $nT_c + \Delta \tau_j$ is the sampling instant, and $\Delta \tau_j = q_j T_c / M$ with $q_j = 0, 1, 2, \ldots, M-1$ is the sampling phase. Note that
\[
\tau_j = n_j T_c + \Delta \tau_j, \quad j = 0, 1, \ldots, J-1 \tag{5}
\]
with $n_j = 0, 1, 2, \ldots, N-1$, is the time delay corresponding to the $j$th finger. The proper values of $n_j$ and $q_j$ are determined by synchronization stages. For the finger with the strongest path or main branch (which is denoted with $j = 0$), the time delay is $\tau_0 = n_0 T_c, (q_0 = 0)$. We have included $\mu_j[n]$ in our model to consider the effects of uncorrelated noise components present after the receive filter (e.g., quantization noise from A/D converters). We will refer to this component as “internal noise.” In this work, we will assume that $\mu_j[n]$ is zero-mean AWGN with $E[\mu_j[m] \mu_k[n]] = \delta_{mn} \delta_{jk} k_{\text{in}}$ where $\delta_{mn}$ is the Kronecker’s delta function (i.e., $\delta_{mn} = 1 \text{ m = n, } \delta_{mn} = 0 \text{ m \neq n}$).

Equation (4) can be rewritten as
\[
y_j[m] = \frac{1}{N} \sum_{n=mN+n_0}^{mN+N-1} a_{kj}[n-n_0] \cdot (r_j(nT_c + t_j) + \mu_j[n]), \quad j = 0, 1, \ldots, J-1 \tag{6}
\]
where
\[
t_j = \tau_j - \tau_0 = n_j T_c + q_j T_c / M - n_0 T_c \tag{7}
\]
is the finger delay from the main branch.

We show in the Appendix that the despreader output (6) can be expressed in the following compact form:
\[
y_j[m] = b_{kj}[m] s_j[m] + \chi_j[m] + n_j[m] + c_j[m], \quad j = 0, 1, \ldots, J-1 \tag{8}
\]

The first term on the right-hand side of (8) is the desired signal component; the second is the interference from the same cell-site users due to the presence of the multipaths and imperfect chip timing (i.e., the interpath interference or IPI). The third term represents the interference from adjacent cell-site base stations plus thermal noise, while the last term corresponds to the internal noise. In the following subsections, we analyze each one of the components of (8).

1) Desired Signal Component: The desired signal component $s_j[m]$ is given by
\[
s_j[m] = \alpha_{kj} A_{kj} \sum_{l=0}^{L-1} \beta_l[m] \Theta(\tau_{jl}) \tag{9}
\]
where $\tau_{jl} = \tau_j - \gamma_l = \tau_0 + t_j - \gamma_l, \beta_l[m] = \beta_l((mN+n_j)T_c + \Delta \tau_j) \Phi(t)$ is assumed constant in a time interval of $T = NT_c$ [15] and
\[
\Phi(t) = \phi(t) + \phi^*(-t) = \phi(t) + \phi(t) = \frac{\sin(\pi t / T_c)}{(\pi t / T_c)}, \tag{10}
\]

Note that the different multipaths interfere among themselves because $|\gamma_l - \gamma_j| < T_c, \forall l \neq j$ (i.e., different multipaths cannot be resolved by a RAKE receiver having fingers spaced at the chip period when the mean delay spread is smaller than the chip interval). Also, from (9) it can be seen that $s_j[m]$ results in a zero-mean complex Gaussian random variable.

2) Interference Component from Same Cell-Site Users: As shown in the Appendix, this interference component is given by
\[
\chi_j[m] = \sum_{l=0}^{L-1} \beta_l[m] \sum_{i \neq j} \psi[m, i] \Theta(\tau_{ij} - iT_c) \tag{11}
\]
where
\[
\psi[m, i] = \frac{1}{N} \sum_{k=0}^{K-1} \alpha_{ki} A_{ki} \sum_{n=(mN+n_0)}^{(mN+N-1)} \alpha_{kj}[n-n_0 + i] a_{kj}[n-n_0], \tag{12}
\]
with $\alpha_{ki}[n] = b_{ki}[n] N \mu_k[n]$. Next, we analyze the statistics of $\psi[m, i]$. The data symbols $b_{ki}[m]$ are zero-mean, independent and identically distributed binary $\{\pm 1\}$ random variables with $E[b_{ki}[m] b_{ki}[m']'] = \delta_{km} \delta_{mm'}$ (the standard IS-95 uses a long pseudorandom masking sequence to provide voice privacy). The user chip sequences $\alpha_{ki}[n]$ are orthogonal because Walsh functions are used in the forward link of IS-95. Thus, the spread sequences $\alpha_{ki}[n]$ can be treated as zero-mean random processes with autocorrelation function given by $E[\alpha_{ki}[n] \alpha_{ki}[n']'] = \delta_{nm} \delta_{kk'}$ [15]. Then we use the central limit theorem to approximate (12) by a Gaussian process. This is a frequently used assumption in the analysis of DS-CDMA.
systems and the results obtained using this assumption are reasonably accurate when $N$ is sufficiently large [15]. Thus, $\psi[n]$, results in a zero-mean Gaussian process with autocorrelation function given by

$$E \{ \psi[n], \psi[r] \} = \delta_{nr} \delta_{ij} \frac{I_{or}}{N}, \quad i \neq 0, j \neq 0 \tag{13}$$

where $I_{or} = A^2_o + \sum_{k=1}^{K} E \{ a^2_k \} E \{ A^2_k \}$ and $A_o$ is the pilot signal amplitude. From [19], $I_{or}$ represents the psdf of the wide-band signal transmitted from the base station (approximately white within the frequency band of interest). In this work, we assume that $E \{ a^2_k \} = 0.4$ and $A_k = A_T$, $k = 1, 2, \ldots, K$ (i.e., the transmitted signal amplitudes for the same cell-site users are all equal) [2], [15], therefore

$$I_{or} = A^2_o + 0.4K A^2_T. \tag{14}$$

Using (11), (13), and the fact that the different paths are independent, $X_j[m]$ results in a zero-mean random process with

$$E \{ X_j[m]X^*_k[m] \} = \frac{I_{or}}{N} \sum_{l=0}^{j-1} \frac{\tau l}{2} \cdot \sum_{i=0}^{\infty} \Theta(\tau_{jl} - i T_c) \Theta(\tau_{ik} - i T_c), \quad j, k = 0, 1, \ldots, J - 1 \tag{15}$$

where $\frac{\tau l}{2} = E \{ |\beta_k[m]|^2 \}$. From (11) and (15), it can be seen that $X_j[m]$ is caused by the nonorthogonality among user sequences of the same cell, resulting from the presence of multipaths and imperfect chip timing (e.g., if $L = 1$ and $\tau_{jl} = 0$ this interference component is zero). Note also that for a given set of fades values, the IPI component $X_j[m]$ is a Gaussian random variable.

3) Thermal Noise and Interference from Adjacent Cell-Site Base Stations: The interference component $\eta_j[m]$ can be expressed as

$$\eta_j[m] = \sum_{n=m-N+n_0}^{mN+n_0+N-1} \frac{1}{N} a^*_k[n-n_0] \delta(n T_c + t_j) \tag{16}$$

where

$$\delta(t) = \begin{cases} 1 & \text{if } t = 0 \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

It is easy to verify that $\delta(t)$ is a zero-mean Gaussian process with autocorrelation function $E \{ \delta(t + \tau) \delta^*(t) \} = I_{or} \Theta(\tau)$. Since $N \gg 1$, the spread sequence $a_k[n]$ can be treated as a coin-flipping sequence [12], thus we can assume that $a_k[n]$ is a zero-mean random process with $E \{ a_k[n] a^*_k[m] \} = \delta_{nm}$. Then, using the central limit theorem, it can be shown that $\eta_j[m]$ can be modeled as a zero-mean Gaussian process with

$$E \{ \eta_j[m], \eta^*_k[m] \} = \delta_{nm} \frac{I_{or}}{N} \Theta(t_j - t_k). \tag{18}$$

From (10) and (18), it is possible to verify that if we decrease the time interval between diversity branches ($|t_j - t_k| < T_c, j \neq k$), the noise components at the output of the RAKE will be correlated.

4) Interference from “Internal Noise”: This component is given by

$$\zeta_j[m] = \sum_{n=m-N+n_0}^{mN+n_0+N-1} \frac{1}{N} a^*_k[n-n_0] \mu_j[n] \tag{19}$$

where $\mu_j[n]$ is the internal noise at the input of the $j$th finger. Similar to previous analysis, assuming $N \gg 1$, $\zeta_j[m]$ can be modeled as a zero-mean Gaussian process with

$$E \{ \zeta_j[m], \zeta^*_k[m] \} = \delta_{nm} \frac{I_{in}}{N}. \tag{20}$$

Note that $\eta_j[m]$, $X_j[m]$, and $\zeta_j[m]$ are mutually uncorrelated.

III. MMSEC RAKE RECEIVER WITH RR

Path diversity is not available when $d_{\text{spread}}$ is smaller than $T_c$. Similar to [1], we decrease the resolution between diversity branches with the objective of obtaining diversity gain (i.e., $|\tau_{jl} - \tau_{jk}| < T_c, j \neq k$) [16]. We consider a RAKE receiver with two branches ($J = 2$). In this case, the second finger is used to demodulate the received signal with a delay $t_k$, respect to the main finger, less than $T_c$ (that is, $|t_k| = |\tau_{jl} - \tau_{jk}| < T_c$).

The RAKE output for the $k$th user can be expressed as (for simplicity we omit the time index $m$)

$$Y = b_k S + Z \tag{21}$$

where $Y = [y_0, y_1]^T$, $S = [s_0, s_1]^T$, and $Z = [\chi_0 + \zeta_0, \eta_1 + \chi_1 + \zeta_1]^T$ (the superscript $T$ denotes transposition). Components of $S$ and $Z$ are obtained from (9), (11), (16), and (19) with $j = 0$ and 1. According to the analysis presented in previous subsections, for a given set of fade values, the components of $Z$ (and $Y$) are Gaussian random variables. Since $|t_k| < T_c$, from (15) and (18) we can verify that the noise correlation factor is different from zero, thus the MRC is not so effective. In this case, it is well known that the optimal combiner is the minimum mean square error one (MMSEC) [10].

The coefficients of the optimal MMSEC are given by

$$W_{MMSEC}^T = S R^{-1} \tag{22}$$

where $R$ is the correlation matrix of the noise components at the finger outputs, defined by $R = E \{ ZZ^T \}$ (superscript $^T$ indicates conjugate transpose).

A. Performance of the MMSEC

Using (21) and (22), the output of the MMSEC can be expressed as a simple decision variable in the form [17]

$$u_r = \Re \{ W_{MMSEC}^T Y \} = \Re \{ b_k S^T R^{-1} S + S^T R^{-1} Z \}. \tag{23}$$

For a given set of fade values, $u_r$ is a Gaussian random variable with mean $b_k S^T R^{-1} S$ and variance $\sigma^2 = 0.5 S^T R^{-1} S$. Then, since binary phase-shift keying modulation is used (i.e., $b_k = \pm 1$), the probability of error is

$$P_e(\gamma) = Q \left( \sqrt{2\gamma} \right) \tag{24}$$

where $Q(x) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{x}{\sqrt{2}} \right) \right]$, and $\text{erf}(x)$ is the error function.
where $Q(x) = \int_x^{\infty} (1/\sqrt{2\pi}) \exp(-t^2/2) \, dt$ and $\gamma$ is the signal-to-noise ratio (SNR) defined by

$$\gamma = S^t R^{-1} S.$$  \hspace{1cm} (25)

Denote by $f_\gamma(\gamma)$ the probability density function (pdf) of $\gamma$. The average bit-error rate (BER) is obtained by averaging (24) over the distribution of $\gamma$

$$P_e = \int_0^\infty P_e(\gamma) f_\gamma(\gamma) \, d\gamma.$$  \hspace{1cm} (26)

Since the elements of matrix $R$ depend on the fade values (due to interference from the same cell-site users or IPI), it is difficult to find a close analytical expression for (26). However, we can obtain a simple lower bound ignoring the IPI components. As we will show later, since $\rho_{\text{spread}}$ is smaller than $T_c$, the lower bound obtained with this assumption is reasonably close to (26).

### B. Lower Bound on the Bit-Error Probability

Let $C_S$ be the correlation matrix of the signal component vector $S$, defined by $C_S = E[SS^t]$. The lower bound is obtained ignoring the IPI components, $\chi_0$ and $\chi_1$, thus the new noise vector is $\mathbf{Z}_0 = [\mathbf{Z}_0 + \boldsymbol{\eta}_t + \boldsymbol{c}_1]^T$. Using (18) and (20), it is possible to verify that the elements of $R_0 = E[\mathbf{Z}_0 \mathbf{Z}_0^t]$ are independent of the fade values. The SNR defined in (25) becomes $\gamma_0 = S^t R_0^{-1} S$. Then, the average BER may be easily derived \cite{6,17,18}

$$P_{e\text{lb}} = \int_0^\infty Q(\sqrt{2\gamma_0} f_{\mathbf{Z}_0}(\gamma)) \, d\gamma_0 
= \text{0.5} \cdot \left\{ \sum_{j=0}^{1} \frac{p_j}{\frac{1}{1+\frac{\delta_j}{1+\delta_j}}} \right\}$$  \hspace{1cm} (27)

where $f_{\mathbf{Z}_0}(\gamma_0)$ is the pdf of $\gamma_0$ and $p_j = \delta_j/(\delta_j - \delta_k)$, $j = 0, 1$ with $j \neq k$. Components $\delta_j$ are the eigenvalues of matrix $D$ given by

$$D = \lambda^{1/2} P^t R_0^{-1} \lambda^{1/2}$$  \hspace{1cm} (28)

where $P$ is the matrix of eigenvectors of $C_S$, and $\lambda$ is the diagonal matrix of real, positive eigenvalues of $C_S$. For example, if $E[|s_0|^2] = E[|s_1|^2]$, the error probability of the MMSEC is given by (27) with

$$D = \frac{N \epsilon E[|s_0|^2]}{\epsilon_{\text{oc}} + \epsilon_{\text{in}}} \begin{bmatrix} 1 + \rho_{\text{sig}} & 0 \\ 0 & 1 + \rho_{\text{noise}} \\ 0 & 0 \end{bmatrix}$$  \hspace{1cm} (29)

where $\rho_{\text{sig}}$ and $\rho_{\text{noise}}$ are the correlation factors of the signal and noise components, respectively. From (29), note that if $\rho_{\text{noise}} = \rho_{\text{sig}}$ (both smaller than one), the MMSEC performance is similar to an MRC with two independent paths.

Using (9), (18), and (20), $\rho_{\text{sig}}$ and $\rho_{\text{noise}}$ are as follows:

$$\rho_{\text{sig}} = \frac{\sum_{t=0}^{L-1} \Theta_0(t_0 - t_1) \Theta(t_0 + t_1 - t_0)}{\sqrt{\sum_{t=0}^{L-1} \Theta_0^2(t_0 - t_1) \left(\sum_{t=0}^{L-1} \Theta_0^2(t_0 + t_1 - t_0)\right)}}$$  \hspace{1cm} (30)

$$\rho_{\text{noise}} = \frac{\Theta(t_0)}{1 + \epsilon_{\text{in}} / \epsilon_{\text{oc}}}.$$  \hspace{1cm} (31)

From (30) and (31), it can be seen that the chip pulse shape affects both correlation factors. Note also that $\lim_{t_1 \to 0} \rho_{\text{sig}} = 1$ and $\lim_{t_1 \to 0} \rho_{\text{noise}} = (1 + \epsilon_{\text{in}} / \epsilon_{\text{oc}})^{-1}$, therefore $\lim_{t_1 \to 0}(1 - \rho_{\text{sig}}/1 - \rho_{\text{noise}}) = 0$. Then, from (29), we conclude that diversity gain is not available for $t_1 \to 0$ due mainly to the effect of the uncorrelated noise at the RAKE input.

#### C. Numerical Results and Discussion

Performances of the MRC and the MMSEC with two branches for different values of $t_1$ are investigated. The objective of this study is to identify the behavior and gains that can be achieved with the proposed receiver. For this purpose, we assume perfect knowledge of the signal vector $S$ and the correlation matrix $R$ at the mobile station (practical implementation issues will be addressed in Section IV). The channel of Table I is used ($L = 6$) and the chip duration is $T_c = 813$ ns.

Note that since $\delta_{\text{spread}}$ is smaller than $T_c$, the lower bound obtained with this assumption is reasonably close to (26).

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with the exact value $P_e$. Note also that $P_{e,0}$ is closer to $P_e$ for lower values of $t_0$ because $\mathbb{E}[[X_1^2]]$ tends to $\mathbb{E}[[X_0^2]]$, which is negligible since $d_{\text{spread}} < T_c$ (in a limit case, $\mathbb{E}[[X_0^2]] = 0$ for only one path and perfect chip timing).

As can be seen, for $t_1 \to 0$ the MRC performance is similar to the case of only one path (diversity gain is not available). Note the following when $t_1$ grows:

a) the correlation factor between signal components decreases \(\Rightarrow\) it gives rise to diversity gain;

b) the correlation factor between noise components decreases \(\Rightarrow\) MRC tends toward the optimal operation (uncorrelated noise);

c) the average power on the second finger is reduced.

Then, starting with zero, the behavior of the MRC improves when $t_1$ grows, until a minimum SNR is reached. This improvement occurs because the effects a) and b) are dominant. Continuing to increase the value of $t_1$, the gain of the MRC decreases due to the average power reduction on the second finger [effect c)].

In the case of the MMSEC, it can be seen that when $t_1 \to 0$, performance is similar to the MRC with one finger as was expressed in the previous section (when $t_1 \ll T_c$, $\mathbb{E}[[s_0^2]] \approx \mathbb{E}[[s_1^2]]$ while $\mathbb{E}[[X_0^2]]$ and $\mathbb{E}[[X_1^2]]$ are negligible, then (27)–(31) can be used to analyze the MMSEC performance). Thus, from Fig. 2 we conclude that a reduction of the resolution to less than about $0.25T_c$ is not advantageous due to the effects of the “internal noise.” As $t_1$ is increased, performance improves compared to MRC. This is because the MMSEC performs a better cancellation of the noise which tends to enhance the diversity gain from the correlated signal components $s_0$ and $s_1$. When $t_1$ is larger than about $0.6T_c$, the gain of the MMSEC decreases due to the reduction of the average power of the second finger. Finally, when $t_1 \to T_c$ the performances of the MMSEC and MRC are similar [see effect b)].

BER of $10^{-2}$. The corresponding gain at a BER of $3 \times 10^{-3}$ is about 4.3 dB. Also, note that the gain of the MMSEC over a classical MRC with two fingers ($t_1 = T_c$) is not so large. However, for $t_1 = T_c$, the average signal power on the second finger is very small with respect to the main finger ($\approx 17$ dB lower in our case), which may be problematic in practical implementations. This difficulty may be minimized with the MMSEC using, for example, $t_1 = T_c/4$ or $T_c/2$ (note also that in these cases MMSEC outperforms MRC).

Fig. 3 shows the cumulative distribution function of the SNR at the combiner output obtained from computer simulations for $E_b/N_0 \approx 12.6$ dB. The dual-branch MRC and the MMSEC with RR of $T_c$ and the one-finger MRC are considered. It is possible to verify that the MMSEC outperforms the MRC. Moreover, we can see that the behavior of the MRC with RR improves with respect to the one-finger MRC at low values of SNR, which is owing to the diversity benefits. Note that this improvement decreases as the SNR grows. This is because when the signal power at the main branch grows, the IPI power at the output of the second finger grows too [see (11)], which tends to deteriorate the SNR at the combiner output. Nevertheless, since the error probability is strongly influenced by the behavior of the pdf at low values of SNR, the overall system performance obtained with the dual-branch MRC with RR is significantly better than that derived with the classical one-finger MRC, as was shown in Fig. 2.

1) Effects of the Uncorrelated Noise Components: To analyze the effects of the uncorrelated noise components, we do not consider the interference components $X_0$ and $X_1$ [in this case, the MMSEC performance can be exactly derived from (27)].

Fig. 4 shows $E_b/N_0$ for different values of $t_1$ required to achieve a BER of $10^{-2}$ and $3 \times 10^{-3}$. To evidence the effects of $\zeta[m]$, we include the performance of the MMSEC for $I_{in} = 0$ (i.e., without uncorrelated noise components). In this figure, we show simulation results and theoretical values obtained from (27). Note the extremely close accordance between theory and simulation. Moreover, it is possible to see that the effects of the
uncorrelated noise components are not so important for \( t_1 > 0.25T_c \) (compare curves “a” and “c”). This important result will be used in the next section to derive a suboptimal approximation to the MMSEC.

IV. IMPLEMENTATION OF THE MMSEC

In practical situations, one cannot expect to know the correlation matrix \( \mathbf{R} \), and hence the MMSEC coefficients solution (22) is not applicable. As shown by Winters in [10], one possible approximation to implement the MMSEC is using the least mean square algorithm (the pilot channel can be used as reference signal in our case). However, since

- the effects of the uncorrelated noise components \( \zeta_j \) are not important for \( t_1 > 0.25T_c \) (see Fig. 4) and
- the effects of the IPI component \( \chi_j \) are not significant because \( d_{\text{STREAI}} \) is smaller than \( T_c \)

it is possible to derive a simple near-optimal structure for the MMSEC [20]. Based on previous observations, the suboptimal MMSEC coefficients result in

\[
\mathbf{W}^{T}_{\infty} = \mathbf{S}^{T} \mathbf{R}^{-1}_{\infty}
\]

where \( \mathbf{R}_{\infty} = \left[ \frac{1}{\delta(t_2)} \Theta(t_1) \right] \). These coefficients are derived from (22), neglecting \( \zeta_j \) and \( \chi_j \) [note that for \( t_1 = T_c \), \( \mathbf{W}_{\infty} \) results in the MRC coefficients given by (32)]. Since the channel varies slowly in indoor environments, the signal components \( \mathbf{S} \) can be accurately estimated by simply averaging the finger outputs corresponding to the pilot channel over an adequate number of chips [12]. Therefore, the suboptimal MMSEC structure given by (34) is simple to implement because it does not require any additional estimation compared with an MRC (for a given value of \( t_1 \) and receive filter, \( \Theta(t_1) \) is known).

Fig. 5(a) shows results for the optimal (22) and suboptimal (34) MMSEC. Note that the suboptimal combiner (34) introduces larger noise enhancement for \( t_1 \ll T_c \), thus the receiver performance is drastically deteriorated. However, for \( t_1 > 0.25T_c \), the performance of both combiners is practically the same, therefore the suboptimal solution (34) can be used with negligible performance degradation. Fig. 5(b) shows the BER at the combiner output for the suboptimal and optimal MMSEC with \( t_1 = T_c/2 \). The excellent results obtained with the new set of combiner coefficients defined by (34) are evident from this figure.

Next, we analyze the proposed receiver using a rate-1/2 constraint length–9 convolutional code with interleaving as specified in the IS-95 standard, and soft-decision decoding. The channel of Table I is used. The sample rate is eight times the chip rate (\( M = 8 \)), the data rate is set to full voice rate 9600 b/s (\( \alpha_{\text{M}} = 1 \)), the symbol rate of \( b_{\text{M}} \) [\( m \)] is 19 200 symbols/s, the carrier frequency is 1800 MHz, and the Doppler frequency...
is \( f_d = 2 \text{ Hz} \). The coefficients of MRC and MMSEC are obtained from (32) and (34), respectively. Since we are using a rate-1/2 code, the SNR per bit is defined by \( E_b/N_0 = 2E_b/N_0 \) with \( E_b/N_0 \) given by (33). BER and FER at the output of the Viterbi decoder for different values of the signal level are shown in Fig. 6. An RR of \( T_1 = T_c/2 \) is used. We present both theoretical and simulation results. Theoretical estimates are obtained using the method described in [21]. Note the excellent agreement between simulation and theoretical values. Gains of about 4.9 and 1.2 dB over an MRC-RAKE receiver with one and two fingers with RR \( (T_1 = T_c/2) \), respectively, are obtained at an FER of 0.01. Equivalently, from (14) it is possible to verify that the total number of users per cell \( (K) \) that can be supported by the forward link is increased from about 5 or 11 (for an MRC-RAKE receiver with one or two branches with RR, respectively) to 15. Note that these important gains are achieved with small additional receiver complexity.

V. CONCLUDING REMARKS

In this paper, we showed that significant extra diversity gain can be achieved when a RAKE receiver with reduction of the interval between diversity branches is used in combination with an MMSEC. This technique is attractive when the bandwidth of the spread-spectrum signal is not sufficiently large to give rise to path diversity (e.g., IS-95 in indoor radio channels). By means of theoretical analysis and computer simulations, we showed that an important performance improvement can be achieved in a typical indoor office environment. We also showed that an RR to one quarter of the chip interval for the IS-95 parameters is enough to provide almost all the achievable improvement. In practice, an RR to less than one quarter of the chip interval is not useful as a result of uncorrelated noise components such as quantization noise from A/D converters. To allow a simple practical implementation, a suboptimal structure of the MMSEC was also proposed. Simulations of this receiver scheme in the forward link of the digital cellular standard IS-95, demonstrated that a gain of 4.9 dB over a classical MRC-RAKE receiver at an FER of 0.01 can be obtained. It is important to realize that this gain is achieved without the need of multiple antennas. Furthermore, the additional processing required by the RR technique takes place at the output of the RAKE at a low sampling rate (for example, 19.2 kHz for an IS-95 receiver). Finally, we have found that if the proposed receiver is used in combination with a distributed antenna system, a significant increase of the capacity of DS-CDMA systems can be obtained [22].

APPENDIX

In this appendix we analyze the signal at the output of a RAKE finger defined by (6). Since the channel varies slowly, \( \beta_k(t) \) can be assumed constant within the duration of the chip pulse. Then the despread output (6) can be expressed as

\[
\begin{align*}
  y_k[n] = & \sum_{k=0}^{K} \frac{1}{N} \alpha_k A_k \\
  & \times \sum_{m=N+n+1}^{m+N+n+N-1} \sum_{l=0}^{L} \beta_l[m] \sum_{i=0}^{K} \delta_i[n] \gamma_i[k] n_0 + i \delta_i[k] n_0.
\end{align*}
\]

\[\Theta(t) \text{ and } \phi(t) \text{ are given by (10) and (17), respectively; } \tau_j = \tau_0 + \tau_i, \quad \gamma_k[n] = \beta_k[n] N_q[k][n] \text{ and } \beta_l[m] = \beta_l((mN + n) + \Delta \tau_j). \text{ The third and the last terms in (A-1) } \]

\[\text{correspond to the interference components } \Theta_l[m] \text{ and } \gamma_k[m], \text{ respectively. The desired signal component } b_k[m] [m] \text{ is obtained setting } i = 0 \text{ in the first term on the right in (A-1). Note that in this case the users’ sequences are perfectly orthogonal. Finally, with } i \neq 0 \text{ in the first term on the right in (A-1), we obtain the interference component } \gamma_i[k][n], \text{ that is }
\]

\[
\chi_i[k] = \sum_{k=0}^{K} \frac{1}{N} \alpha_k A_k \\
  \times \sum_{m=N+n+1}^{m+N+n+N-1} \sum_{l=0}^{L} \beta_l[m] \sum_{i=0}^{K} \delta_i[n] \gamma_i[k] n_0 + i \delta_i[k] n_0.
\]

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