A Novel GNSS Weak Signal Acquisition Using Wavelet Denoising Method

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BIOGRAPHY
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ABSTRACT
With the increasing demands of precise positioning in weak signal environment, high sensitive GNSS receiver research and development has been pushed badly in need. Conventional GNSS signal acquisition techniques are considered inadequate when the incoming signal is too weak. In this paper we have mainly consider wavelet de-noising algorithm applying in weak GNSS signal acquisition. Conventional wavelet de-noising algorithms include regional scale transformation method and threshold method. The first method requires less limitation about the noise type, but the latter one is applied only in Gauss noise conditions. Besides wavelet de-noising process is done when the signal is independent in time sequence, therefore our work has done based on the traditional correlation acquisition. The first method requires less limitation about the noise type, but the latter one is applied only in Gauss noise conditions. Besides wavelet de-noising process is done when the signal is independent in time sequence, therefore our work has done based on the traditional correlation acquisition. When the non-correlation or differential correlation has done, the noise distribution and property has been also changed. If the noise pre-processed is Gauss distributed, the post-processed noise is no longer Gauss white noise. Under this circumstance we conduct statistics analysis to estimate the derivation of noise, and assume a new Gauss noise. Then the wavelet de-noising process is done. Our algorithm contains three key steps. Firstly, correlation and differential correlation method are used to acquire the very weak signal; secondly, noise derivation is estimated and noise model is established; then the wavelet de-noising process is applied. The result turns out fine for the signal lower than other acquisition method.

INTRODUCTION
There are several weak GPS signal acquisition methods introduced in recent years. Most of them focus on how to increase the PIT (Predict Integrated Time) of the GPS signal and how to predict the data bit transit of GPS signal. Based on these researches and wavelet de-noising theory, we will introduce a new weak GPS signal acquisition method to increase at least 1dB the sensitivity of GPS acquisition.

The typical GPS signal arrive at earth is about -160dBW. Typical GPS algorithm use 1ms PIT correlation time to detect this GPS signal. When in in-door usage, the GPS signal could reduce to -180dBW or even lower. By using coherent/non-coherent combined acquisition method and differential acquisition method, -182dBW GPS signal could be detect under 100ms PIT of GPS signal. But the weaker GPS signal detection need much longer PIT due to the signal square loss and doppler frequency change. So, in this paper we introduce the wavelet de-noising method to increase the sensitivity without increasing PIT time. We use wavelet de-noising method to decrease the background noise, thus to increase the signal peak of weak signal acquisition.

WEAK GPS SIGNAL ACQUISITION
1. Coherent/Non-coherent (NC)
Coherent integration is referred as the regular correlation between the received signal and local generated replica. Usually there are three methods to do a coherent integration: sequential acquisition, parallel phase domain acquisition (IFFT) and parallel frequency domain acquisition (FFT). For regular GPS received signal a 1ms coherent integration can contribute 30dB energy accumulation, which is enough to reach the threshold. But that is not the condition of a weak GPS signal environment. Long time coherent integration is limited by the unknown message of navigation data transferring. For example if the bit edge occurs in the middle of a data sequence, the total coherent integration can be zero due to two magnitude equally integration blocks that have the opposite signs. To eliminate the effect of navigation data on integration non-coherent process is used by squaring the in-phase and quad-phase coherent integration results. This is called coherent/non-coherent combined method.
With the help of this method weak signal acquisition is luckily improved and to some extent undoubted. But non-coherent also has its inherent drawback because of two reasons. The first reason is the Doppler frequency effect on the code length, which will be discussed in detail later on and the second reason is the square loss. Square loss comes when the non-coherent process is done, by squaring the magnitude of signal is enlarged and at the same time the magnitude of noise is also enlarged. A squaring integration of Gauss Noise is no longer normally distributed, which brings difficulty to calculate SNR.

2. Differential (DF)
Differential correlation is somewhat like the coherent/non-coherent method. But in the differential case the in-phase and quad-phase are not squared. Supposed that our coherent integration is done sequentially by a fixed time interval in a received signal, the differential correlation is then processed by multiplying one time of in-phase result with the next time of in-phase result, and the same calculation will be done on the quad-phase result. Then the in-phase and quad-phase are added together to form the final determination. The advantage of differential correlation is said that it has better performance dealing with the noise. Simple speaking it has less energy loss than the non-coherent method.

3. CCMDB
The full name of CCMDB method is Circular Correlation with Multiple Data Bits [7]. This acquisition type is based on the basic circular correlation of PRN code, usually done in parallel code acquisition determination. Nevertheless traditional GPS acquisition involves two arguments to be estimated or to further extent be refined, but acquisition in this method extend the arguments from two to four: including estimation of C/A code initial phase, Doppler frequency, data combination of navigation and the edge when the data symbol transfers from one to zero or from zero to one. To accomplish such a more complex acquisition procedure, longer data will be used for energy accumulation. Besides the data combination and data transferring edge are guessed by a maximum accumulation result method. It is supposed that each edge of 1ms data may be the right data transferring place according to equal possibility occurrence. The calculation will be time consuming since long data process and complex data combination possibility enumerations. Consequently it is encouraged to develop algorithm to reduce less efficient searching and total calculation time can be decreased.

4. MDBZP
MDBZP stands for Modified Double Block Zero Padding [7]. The DBZP calculates the coherent integration at all of the Doppler bins and all of the code delays at the same time. Thus it requires less processing compared to the circular correlation. In this method there is no need to multiple the received signal with a local generated carrier which has a estimated frequency of the received carrier. Local produced PRN code is divided into several blocks as it needs and each of those blocks is padded with an all zero valued block of the same side. The new built block, which has twice sides of its previous one, correlates with received signal of the same side. A maximum correlation result will also be used as a determination and select data results for FFT calculation, which can be helpful to find the estimated Doppler frequency. The problem of DBZP is that only one replica code is used in the correlation calculation at all the Doppler bins. The replica code is not compensated by the Doppler effect on the code length. Thus there will be a difference in the length between the received and replica codes. Consequently subsequent incoherent integrations will be added at different code delays relative to each other. This problem increases as the integration length increases. Therefore a limitation to the maximum integration length exists. This means there is limitation to the minimum SNR that can be acquired. The MDBZP circumvents the limitation of DBZP by inviting Doppler frequency compensating models.

**WAVELET DE-NOISING**

1. Theory
Compared with the traditional de-noise method, the technique of wavelet de-noising based on the wavelet transform has many distinctive virtues. It can reduce the noise of signal keeping the singularity of signal. Now wavelet de-noising is commonly used in video image denoising. Some reports[3] said it also can be used in real-time signal processing. Because of some key advantages over Fourier analysis, wavelet analysis has become a widely used tool in signal estimation, classification, and compression. Wavelet transform tends to concentrate the signal energy into a relatively small number of large coefficients. On this basis, a method called wavelet shrinkage to use threshold in wavelet domain was proposed, and it was shown to be asymptotically near optimal for a wide range of signals corrupted by additive Gaussian noise [1]. Commonly steps to reduce high frequency noise by wavelet de-noising are [2]:

- A direct wavelet transform is computed from the original image.
- Noise level at each wavelet scale is estimated separately.
- This defines a threshold for zeroing wavelet coefficients.
- Other wavelet coefficients are shrieked according to local derivation estimation (Soft threshold).
- After inverse wavelet transform, the image is renormalized.
The signal with Gaussian noise is divided into discrete detailed signal and discrete approached signal after wavelet transform. It is proved[6] that the amplitude and derivation decreased when the scale level decreased. For all wavelet scale levels, the derivation of white noise detailed signal decreased when the scale increased. But the signal does not fit the criteria. According to this method, we could choice a threshold to filter out the Gaussian white noise to achieve de-noising effect for original signal.

2. Wavelet method choice
The soft and hard threshold could be used for wavelet de-noising filter. Assuming Δx is the threshold, for soft threshold:

\[
y = \begin{cases} 
\text{sign}(x) \cdot (|x| - Δx) & |x| > Δx \\
0 & \text{else}
\end{cases}
\]

For hard threshold:

\[
y = \begin{cases} 
x & |x| > Δx \\
0 & \text{else}
\end{cases}
\]

The hard threshold means value is set to zero when its abstract value is lower than threshold, and the other data does not changed. The soft threshold shrinks the other data to zero. By comparison, soft threshold does not contain in-continuous value, whereas hard threshold contain in-continuous value of ±Δx. Commonly speak, soft threshold is much effective than hard threshold.

3. Acquisition method choice
The NC and DF acquisition method is first used to get a two-dimension correlation power grid.
In NC method, the test statistics Y is following:

When code/doppler matched,

\[
Y = A^2 \frac{\sin^2(\Delta w TN)}{2\sin^2(\Delta w T)} N L \cos(\Delta w TN) + \sum_{i=1}^{L} Z_i
\]

When code/doppler not matched,

\[
Y = \sum_{i=1}^{L} Z_i
\]

where, \( Z_i = noise_{t,i} \times noise_{t,i} \)

T is coherent time
L is differential time
N is differential number
Δw is difference of estimation frequency
A is power noise ratio of GPS signal

In DF method, the test statistics Y is following:

When code/doppler matched,

\[
Y = A^2 \frac{\sin^2(\Delta w TN)}{2\sin^2(\Delta w T)} N L \cos(\Delta w TN) + \sum_{i=1}^{2L} Z_i
\]

When code/doppler not matched,

\[
Y = \sum_{i=1}^{2L} Z_i
\]

where, \( Z_i = noise_{t,i} \times noise_{t,i-1} \)

T is coherent time
L is differential time
N is differential number
Δw is difference of estimation frequency
A is power noise ratio of GPS signal

The probability distributions of these two methods are:

Fig 3 Distribution of NC(right) and DF(left) method
The mean value of correlation power of NC method is not zero, that mean the noise can not be filtered by wavelet de-noising. On contrary, the mean value of correlation power of DF method is zero and the probability distribute is similar as additive white gauss noise. So, we can use wavelet de-noising method to decrease the noise level of DF method. Because the peak value has different distribution as noise level, the signal noise ratio of peak will be increased after the noise is filtered by wavelet.

The probability distribution of DF method is similar to additive gauss noise.

In order to get the probability density of test statistics Y, we should firstly get the probability density of noise Zi. Though Zi is not exactly fit the Gaussian distribution. But for convenient, we assume it is an Gaussian distribution, the average is zero and derivation is \( 4LN^2\sigma^4 \).

Furthermore, \( \sum_{i=1}^{2L} Z_i \) is also assumed to be a Gaussian distribution with zero average and \( 8LN^2\sigma^4 \) derivation.

Two hypothesis testing are H0(Signal not detected) and H1(Signal detected). So, \( f(Y|H0) \) and \( f(Y|H1) \) are both Gaussian distribution with different derivation. Thus we can use wavelet de-noising method to filter out Gaussian noise in Y.

4. De-noising method
Similar as image wavelet de-noising, the signal wavelet de-noising method contains following steps:

- First transform the 2-D acquisition grid to 1-D acquisition array.
- A direct wavelet transform is computed from the original acquisition array.
- Soft threshold is used for zeroing wavelet coefficients.
- After inverse wavelet transform, the acquisition array is renormalized and the SNR of peak increased.

- Reconstruct the acquisition grid from the array. Then these will make it easier to find out the peak of doppler and code phase.

![Wavelet de-noising flow](image)

Fig 4 Wavelet de-noising flow

For example, when the sampler is 12MHz and 100ms PIT data (25x4ms correlation) is used by DF method for acquisition. Because the correlation time is 4ms, the doppler step will be 125Hz. Assuming the doppler is limited in +/-5KHz, the total search span will be 81. The acquisition grid has 81*12000 (972000) values. We first transform the 2-D acquisition grid into 1-D acquisition array. Five-layer wavelet transform is done, and then use soft threshold wavelet method to filter out the noise. After that, we will use inverse wavelet transform to rebuild the acquisition array.

There are two wavelet de-noising methods to process the non-coherent result. The first is to filter out the noise of each 4ms correlation result, and then add 25 de-noising data blocks together. The second is to add the acquisition grid firstly, then use wavelet de-noising method to filter out the noise. Because the wavelet de-noising function is a linear system, so in fact, these two methods will get the same result. By comparison, the second method will use fewer computation time. All these procedures had been tested upon MATLAB Wavelet Toolbox.

EXPERIMENT
The actual GPS signal is sampled in BeiHang University campus (N:39.979, E: 116.344, Alt:98m) at Nov, 6th, 2007. The signal strength covers -160dBW to -180dBW. Satellite 17 is trying to be searched in our research.

1. Comparison of NC and DF
According to paper [4], the DF method has about 1.2dB sensitivity increased compared to NC method. In our experiment, with 100ms PIT time, the sensitivity of NC and DF method are:

<table>
<thead>
<tr>
<th>Sample</th>
<th>Bit</th>
<th>NC Sensitivity</th>
<th>DF Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>12MHz</td>
<td>1bit</td>
<td>-177dBW</td>
<td>-178dBW</td>
</tr>
<tr>
<td>12MHz</td>
<td>2bit</td>
<td>-179dBW</td>
<td>-180dBW</td>
</tr>
<tr>
<td>24MHz</td>
<td>2bit</td>
<td>-181dBW</td>
<td>-182dBW</td>
</tr>
</tbody>
</table>

Based on DF method, the wavelet de-noising could still increase the sensitivity by 1dB. That means the sensitivity will reach -183dBW when using 24MHz sample and 2bit quantization. We select two signal streams from sampled signal, in which strong GPS signal power is about -160dBW, and weak GPS signal power is about -175dBW.
The NC and DF acquisition grids without wavelet denoising over a strong signal are:

![Fig 5 NC method for Strong Signal](image1)

![Fig 6 DF method for Strong Signal](image2)

The antenna position of strong signal is:

![Fig 7 Strong Signal Sample](image3)

The NC and DF acquisition grids without wavelet denoising over a weak signal are:

![Fig 8 NC method for Weak Signal](image4)

![Fig 9 DF method for Weak Signal](image5)

The antenna position of weak signal is:

![Fig 10 Weak Signal Sample](image6)

2. De-noising effect of NC strong signal

![Fig 11 Strong NC Signal before de-noising](image7)
The SNR of original signal is 92.46 (45.27dB), and the SNR of de-noised signal is 91.85 (45.20dB). The mean level of noise level is same. That means the wavelet de-noising method does not decrease the average noise when the noise is not a gauss noise. So wavelet de-noising method could be used on NC method.

3. De-nosing effect of DF strong signal

The SNR of original signal is 436.30 (60.78dB), and the SNR of de-noised signal is 692.45 (65.40dB). In strong signal environment, the wavelet de-noise can archive about 5dB increase to signal power.

4. De-noising effect of DF weak signal
The SNR of original signal is 24.27 (31.89dB), and the SNR of de-noised signal is 33.47 (35.11dB). In weak signal environment, the wavelet de-noise can archive about 3dB increase to signal power. Furthermore, from previous figure, the high frequency part (d1 coefficient) does not contain peak power. So, wavelet de-noising method could use d1 coefficient to determinate the noise level as threshold, and then filter out the gauss noise.

CONCLUSION
The result shows that the wavelet de-noising method could be used to help acquisition of weak GPS and GNSS signal. Under strong signal environment, the peak will increase about 5dB after de-noised. And under weak signal environment, the peak will increase about 3dB after de-noised. This will increase the probability of detection of weak signal and reduce the false alarm ratio.

FUTURE WORK
Future work will focus on how to de-noising the signal which could not be detected under CF or NC method. We will consider on how to de-noise the noise without decrease the original signal. Then this method will help to increase the sensibilities of GNSS receiver rather than increase the probability of detection.

REFERENCE