Is the Price Level Tied to the M2 Monetary Aggregate in the Long Run?

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A long-run link between money and prices is evident for the United States since the Korean War if the M2 *measure of money is used and the velocity of* M2 (V2) *is modeled as a mean-reverting series. This link between* M2 *and prices is the basis for a dynamic model of inflation that compares favorably in forecasting exercises with Phillips-curve and more typical monetarist approaches. The behavior of* V2 *is examined from 1870 to the present, providing a basis for reconsidering previous findings that* V2 *follows a random walk.* (*JEL* E30, E50)

During the last decade, the velocities of the monetary aggregates have varied considerably as accounts at depository institutions have become largely deregulated and as market interest rates have gone through substantial swings. These fluctuations have lead some economists (e.g., Benjamin M. Friedman, 1988a, b) to conclude that the monetary authority cannot rely on any of the aggregates to anchor the price level. However, the velocity of M2 (currency plus liquid balances and retail time deposits) relative to the gross national product (V2), while somewhat variable in the short run, has shown virtually no trend in the postwar period.[1] The absence of a trend, not evident for the velocities of M1, M1A, or the monetary base, provides a relatively reliable long-run link between M2 and the price level, particularly in the period since the Korean War. This link between M2 and the price level is used as the empirical basis for a dynamic model of inflation that is motivated by long-run quantity-theory considerations. The advantages of this approach are examined relative to simple Phillips-curve models and relative to more typical monetarist models. While this paper focuses on the period from 1955 to 1988, we also apply our methodology to the period from 1870 to the Korean War. Extending the period back to 1870 also enables us to reexamine the result of John P. Gould and Charles R. Nelson (1974) and of Charles R. Nelson

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This work is the result of a request by Alan Greenspan, who suggested the usefulness of M2 per unit of potential real output as an indicator of longer-term price trends. For an earlier version of this paper —which contains many of the results noted below and stated in the text as available from the authors—see Hallman et al. (1989). Single copies of Staff Study No. 157 are available from Publication Services, Board of Governors of the Federal Reserve System, Washington, DC 20551.

[1]The monetary aggregates M1A, M1, and M2 as used here follow the Federal Reserve Board definitions: M1A is currency, traveler's checks, and demand deposits; M1 is M1A plus other checkable deposits; and M2 is M1 plus passbook savings, money-market demand accounts, money-market mutual-fund balances, small time deposits, and overnight Eurodollar and repurchase agreements.

and Charles I. Plosser (1982) that velocity follows a random walk.

I. Basic Concepts

Our analysis of M2 and the price level starts with the long-term behavior of real output and the GNP velocity of M2 and asks what price level the current stock of M2 would support if output and velocity were to settle down to their long-run values. This long-run equilibrium price level at time t, P_t^*, is defined as the price level consistent with the current value of M2, the long-run equilibrium value of velocity (V_t^*), and the current value of potential real GNP (Q_t^*):[2]

$$P_t^* \equiv \frac{\mathrm{M2}_t V_t^*}{Q_t^*}.$$

In general, long-run equilibrium velocity may be a function of endogenous variables such as real GNP, the price level, market interest rates, and deposit rates and may also depend on the overall structure of the financial sector. A model relating long-run velocity to such variables may be appropriate not only to explain past developments, but also to analyze possible future movements in velocity. In this paper, we model M2 velocity over the period 1870–1954 as a function of institutional factors stressed by Michael D. Bordo and Lars Jonung (1987). For the period since 1954, we examine the validity and implications of the simplest possible assumption, namely that long-run velocity has been constant since the mid-1950's.[3] For this period, our estimate of V_t^* is the mean of V2 over our sample period of 1955:1–1988:4; that is, we let

$$(1) \qquad P_t^* = \frac{\mathrm{M2}_t V^*}{Q_t^*}$$

where V^* equals 1.65 (the sample mean of V_t).[4] If permanent shifts to velocity are empirically significant, actual prices would diverge from P^* in the long run.[5] However, the top panel of Figure 1 shows that, over the past three decades, P and P^* have tended to move together.[6]

[2]For the methodology underlying empirical estimates of Q^*, see Steven N. Braun (1990).

[3]A theory of M2 demand—applicable to the 1980's, when there was extensive deregulation of deposit rates —that implies the long-run invariance of V2 to changes in general macroeconomic variables such as GNP and interest rates is one in which (i) M2 demand has a unitary long-run elasticity with respect to nominal GNP, (ii) interest rates enter the demand for M2 solely by way of opportunity costs (market rates less own rates), and (iii) key deposit rates adjust one-for-one in the long run with market rates. Complete adjustment of all deposit rates is not essential if, for example, incomplete adjustment of other checkable deposits (OCD) rates and complete adjustment of small-time deposit rates cause a reshuffling of balances between OCD's and small-time deposits but do not affect M2 as a whole. From the 1950's until the 1980's, the stability of V2 likely stemmed from the flexible administration of the Federal Reserve Board's Regulation Q, as well as from the introduction of new deposit instruments.

For models of deposit rates and money demand that possess the first two of the above three properties and approximate the third property, see George R. Moore et al. (1990). These models would imply a change in long-run velocity if, for example, there were either technological or regulatory changes that altered the cost to depository institutions of maintaining deposits.

[4]Since there is no official Federal Reserve series on M2 prior to 1959, we use the M2 series developed by Robert J. Rasche (1990) for the 1952–1958 period.

[5]In Hallman et al. (1989), we examined whether there may have been a permanent downward shift in V2 associated with the introduction of money-market deposit accounts and Super-NOW accounts beginning in late 1982. We were not able to reject the hypothesis of no permanent shift in the relationship.

[6]The same chart can be constructed using data back to 1959:1, rather than 1955:1, for M1, M1A, and the monetary base. Defining V2*, V1*, V1A*, and VB* as the means of the respective velocities from 1959:1 to 1988:4, P^*'s can be calculated for each aggregate. Over that period, the P^* plot using M2 intersects the plot of P seven times, while the comparable plots of P^* measures for M1, M1A, and the monetary base each cross the plot of P only once. At least one intersection of P and P^* (whichever monetary aggregate is used) is virtually assured by the use of the mean of velocity as the measure of V_t^*. Applying P^* to these other aggregates would require modeling their long-run velocities.

After Hallman et al. (1989) was written, we learned from Thomas Humphrey (1989) that Holbrook Working (1923) had presented a similar chart based on U.S. data from 1890 to 1921. While the details of Working's implementation are somewhat different, the overall approach is similar.

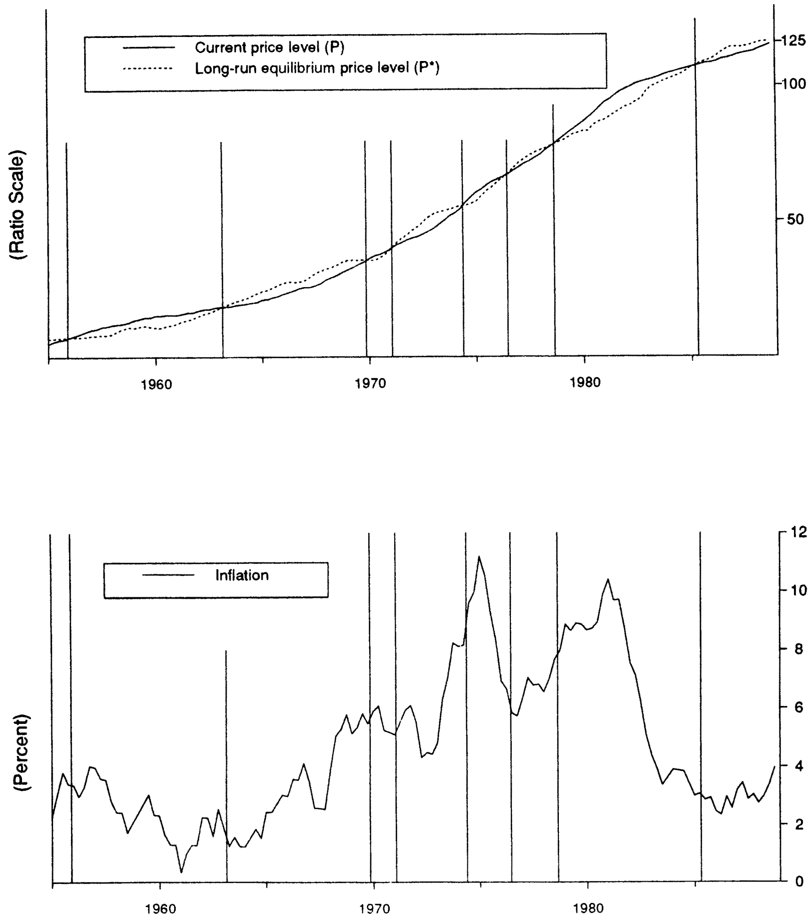

FIGURE 1. INFLATION INDICATOR BASED ON P^*

Notes: The current price level (solid line in top panel) is the implicit GNP deflator, which is set to 100 in 1982. Inflation (bottom panel) is the percentage change in the implicit GNP deflator from four quarters earlier.

In modeling inflation dynamics, we have leaned heavily on the constancy of V_t^* and the long-run neutrality of money implicit in P^* but have tried to cut through the problems of modeling the particular leads, lags, and expectations that influence short-run price dynamics. From the identity that

$$(2) \quad p - p^* \equiv (v - v^*) + (q^* - q)$$

(throughout the paper lowercase variables are the natural logarithms of their uppercase counterparts), we see that if the quantity of M2 is supporting P^* at a level above P then it is depressing V2 below V^* or is boosting real output above Q^* or both. As factors such as lags in money demand are worked out, velocity will rise to V^*, thereby shifting the nominal aggregate demand schedule to its long-run level consistent with current M2. As lags in the formation of inflation expectations and in the movement of nominal wages are overcome, the short-run aggregate supply curve will shift upward. After these adjustments take place, the long-run aggregate demand curve intersects the vertical long-run supply curve at the equilibrium price level. Our modeling strategy has been to identify this equilibrium price level through the construction of P^* and then to estimate reduced-form short-run dynamics that drive the actual price level to P^* and thereby are consistent with the long-run constraints imposed by P^*.

Modeling inflation as driving the actual price level to a measure of its equilibrium level has been placed on a more explicit theoretical basis by Michael Mussa (1981) and Bennett T. McCallum (1980).[7] Mussa proposed the following adjustment equation for inflation:

$$(3) \qquad \pi_t = \alpha[p_t - \bar{p}_t] + \bar{\pi}_t \qquad \alpha < 0$$

where $\bar{\pi}_t$ is a forward-looking estimate of the growth in the equilibrium price level $\bar{P}$.[8] In Mussa's model of sticky prices and market disequilibrium, $\bar{P}$ is the price level that equates money demand to money supply, but the variables affecting money demand need not be at their long-run equilibrium values. Therefore, $\bar{P}$ does not contain the forward-looking information of these other variables moving over time to their long-run equilibrium values.

McCallum's price-adjustment equation is

$$(4) \quad \pi_t = \alpha[p_{t-1} - \tilde{p}_{t-1}] + E_{t-1}[\tilde{p}_t - \tilde{p}_{t-1}] \qquad \alpha < 0.$$

As in Mussa's equation, inflation responds to the gap between the actual and equilibrium price levels. McCallum's equilibrium price level $\tilde{P}$ is forward-looking in that it represents the price level consistent with the economy being at capacity output, given the current level of aggregate demand. But unlike P^*, $\tilde{P}$ does not assume that aggregate demand is at its long-run equilibrium level given M2; that is, it does not incorporate future adjustments in velocity from its current level.

However, in both Mussa's and McCallum's models the second term on the right, the expected rate of change in the equilibrium price level, carries forward-looking information, whereas in the P^* model we put all such information into P^* itself. We leave only backward-looking information such as lagged inflation in place of the second term used by Mussa and McCallum. For example, an ad hoc assumption that seems to conform reasonably well with annual U.S. data over the 1955–1988 period is to use the previous year's inflation rate as the lagged information. Our analogue to equations (3) and (4) then is

$$(5') \quad \pi_t = \alpha[p_{t-1} - p^*_{t-1}] + \pi_{t-1} \qquad \alpha < 0.$$

Moving π_{t-1} to the left-hand side of equation (5'), we see that in this simple case the acceleration in inflation is related to the price gap, the difference between the logs of the actual price level and P^*:

$$(5) \qquad \Delta\pi_t = \alpha[p_{t-1} - p^*_{t-1}] \qquad \alpha < 0.$$

The two panels of Figure 1 show that inflation generally accelerated, though with a lag, when P^* became greater than P and decelerated, after a lag, when P^* became less than P. The lags between money and prices are evidenced by several episodes: the imposition of wage and price controls in September 1971, which slowed inflation in 1971 and 1972 despite the fact that P^* exceeded P; the quadrupling of oil prices starting in October 1973, which increased inflation in 1974 despite the fact that P exceeded P^*; and the period from mid-1978 to early 1985 when P exceeded P^* but inflation continued to accelerate from mid-1978 through 1980, perhaps due in part to the oil-price shock of 1979.

[7]From the perspective of the quantity theory, however, the idea is not new. Humphrey (1989) has traced the antecedents of this approach in the writings of various quantity theorists going back to David Hume.

[8]Of course, in estimation, Mussa's model needs to address the occurrence of p_t on both sides of his model. One way to handle this would be to lag the price gap on the right-hand side, making his model more directly comparable to McCallum's specification in equation (4) and our model in equation (5'). Also, Mussa notes that in a stochastic version of his model the expected rate of change of the equilibrium price level, rather than the actual rate of change, should enter.

The conditions under which equation (5) is equivalent to a standard expectations-augmented supply curve can be seen by starting with a generalized form of equation (5′):

$$(6) \qquad \pi_t = \beta[q^*_{t-1} - q_{t-1}] + \psi[v_{t-1} - v^*] + \pi_{t-1}$$

where we have substituted the right-hand side of (2) for $p - p^*$ but have allowed for different coefficients on the two components, the velocity and output gaps.

The standard expectations-augmented aggregate supply curve can be written

$$(7) \qquad \pi_t = \beta'[q_t - q^*_t] + \pi^e_t \qquad \beta' > 0$$

where π^e_t is the expectation of π_t based on period-$(t-1)$ information. If the output gap is dated $t-1$ to make equation (7) a forecasting equation and if $\beta = -\beta'$, then a restriction on inflation expectations that makes equation (7) equivalent to equation (6) is:

ASSUMPTION 1:[9]

$$\pi^e_t = \pi_{t-1} + \psi(v_{t-1} - v^*).$$

To make (7) consistent with the equations (5) and (5′), one must further assume:[10]

ASSUMPTION 2: $\psi = -\beta'$.

This restriction allows the velocity and output gaps to collapse into just the price gap and therefore renders the price level neutral with respect to changes in fiscal policy. A fiscal stimulus that raises q above q^* must raise v above v^*—presumably through higher interest rates—so as to leave $p - p^*$ in equation (2) unchanged. Without this restriction, the effect of fiscal policy through the output gap could dominate the effect through the velocity gap.

Of course, the primary question is what sense can be made of Assumption 1. If y equals the log of nominal output, from the identities $y \equiv m + v$ and $y^* \equiv m + v^*$, we see that $v - v^* = y - y^*$, where y^* is the long-run equilibrium level of nominal income consistent with the current stock of M2. Upon making this substitution, Assumption 1 can be rewritten as

$$(8) \qquad \pi^e_t = \pi_{t-1} + \psi(y_{t-1} - y^*_{t-1}) \qquad \psi < 0.$$

Following an increase in the money supply, y^* will jump immediately, but y may change by less than y^* due to lags in aggregate demand. Agents in the economy may then expect that aggregate demand will rise to y^* over time. Therefore, their expectations of inflation are adjusted upward.

II. Empirical Models of M2 and Inflation: 1955–1988

We start our sample period in 1955 since, as shown in Figure 2, the time-series properties of inflation seem to be different before and after 1955.[11] The autocorrelations and unit-root test statistics for inflation in Table 1 also show marked differences across the two periods. The unit-root hypothesis cannot be rejected for inflation after 1955 but can be rejected before. Therefore, the ad hoc use of the lagged inflation rate as a

[9]A test of the usefulness of such a method of forming expectations is to regress actual inflation against the velocity gap and lagged inflation rates. This regression then takes the form of the velocity-gap model in Table 2, where the coefficient on the velocity gap is significantly negative at the quarterly, annual, and biennial frequencies.

[10]This restriction is tested in Subsection II-C in the estimation of equation (13), where we find that the coefficient on the velocity gap is not statistically different from the coefficient on the output gap.

[11]This chart is based on annual data prior to 1954 from Nathan S. Balke and Robert J. Gordon (1986 appendix B) and uses data on M2 from Rasche (1990) for the period 1954–1958. A referee has noted that the M2 numbers from Balke and Gordon prior to 1948 are not based on the same concept as the current official M2 series.

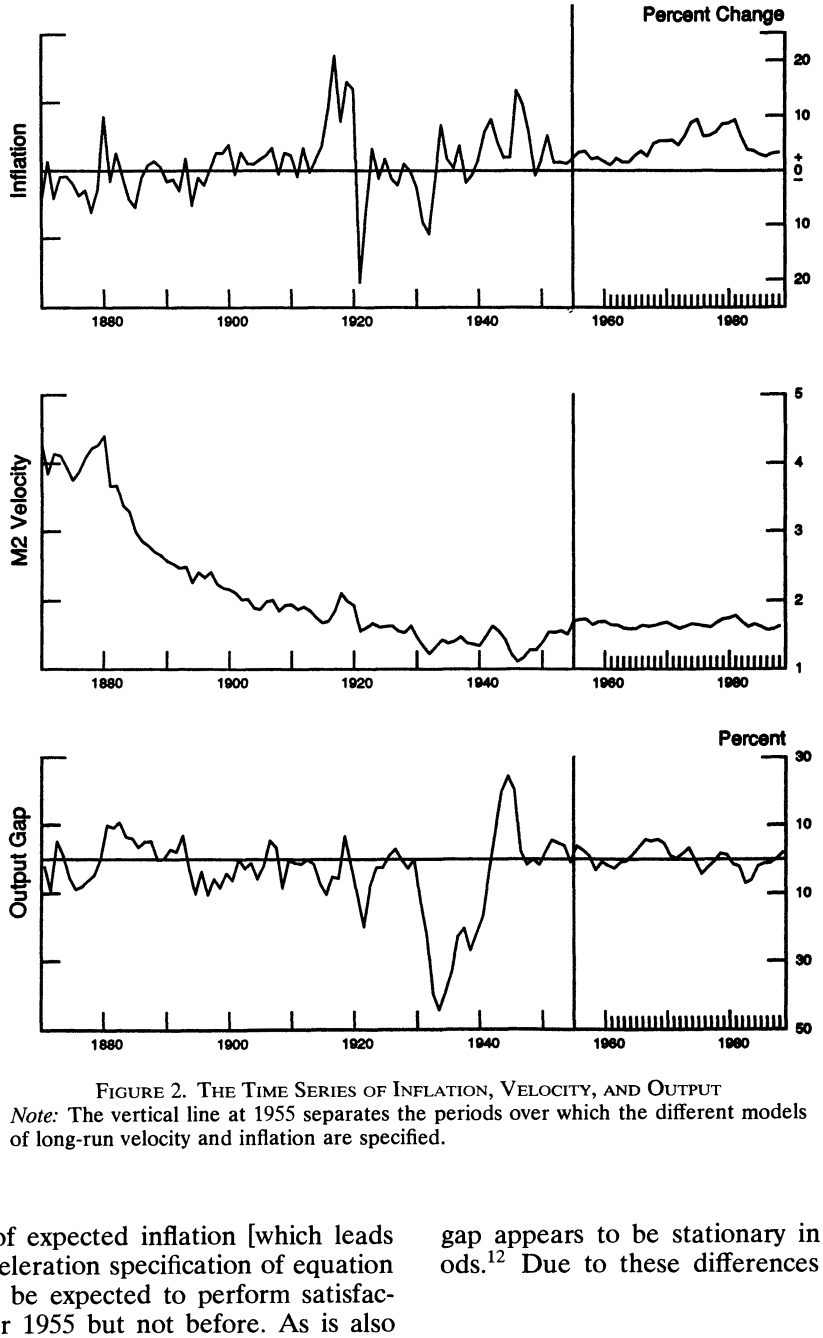

FIGURE 2. THE TIME SERIES OF INFLATION, VELOCITY, AND OUTPUT
Note: The vertical line at 1955 separates the periods over which the different models of long-run velocity and inflation are specified.

measure of expected inflation [which leads to the acceleration specification of equation (5)] might be expected to perform satisfactorily after 1955 but not before. As is also shown in Table 1, a unit root for velocity can be rejected for the period after 1955, thereby justifying the use of the sample mean of velocity as a measure of long-run velocity. The unit root for velocity is not rejected for the earlier period. The output gap appears to be stationary in both periods.[12] Due to these differences across the

[12]As a referee has pointed out, the typical construction of a real-potential-output measure virtually guarantees that the output gap will appear to be stationary. These measures usually impose the condition that output reverts to its potential over the course of one or more business cycles, so that the gap displays the mean-reverting behavior of a stationary series.

TABLE 1—AUTOCORRELATION AND UNIT-ROOT TESTS

Sample autocorrelations	1870–1954			1955–1988		
	π	V2	$q-q^*$	π	V2	$q-q^*$
ρ_1	0.41	0.96	0.85	0.83	0.63	0.64
ρ_2	0.14	0.91	0.64	0.65	0.19	0.22
ρ_3	0.14	0.88	0.43	0.54	0.04	−0.01
ρ_4	−0.02	0.84	0.27	0.42	−0.03	−0.12
ρ_5	0.06	0.81	0.16	0.34	−0.18	−0.18
Unit-root test, |ADF|:	5.89*	2.22	3.28*	1.85	2.95*	3.03*

Notes: All autocorrelations are for annual observations. For the 1870–1954 period annual averages are used, but the 1955–1988 data are Q4 observations of the series. The unit-root tests used annual data for 1870–1954 and quarterly data for 1955:1–1988:4. The adjusted Dickey-Fuller (ADF) regression for inflation included an intercept term, while the output gap regressions did not. The velocity test included a constant in both periods and also included a time trend in the ADF regression for 1870–1954. The time trend did not enter significantly into any of the other test regressions, so it was omitted from them to preserve the power of the tests.

*Statistically significant at the 95-percent level.

two periods, we estimate our model over each period independently. In this section, we examine the period after 1955. The following section examines the period before 1955 and addresses in more detail the issue of whether or not the behavior of long-run velocity has changed across the two periods.

A. *A Price-Gap Model for Inflation*

Here we estimate inflation models of the general form of equations (5′) and (5) but allow for more than a single lag of inflation at the quarterly frequency. Our model, which we will call the price-gap model, for the period since the Korean War, is

$$(9)\quad \Delta\pi_t = \underset{(4.4)}{-0.148}(p_{t-1}-p^*_{t-1}) \underset{(8.0)}{-0.667}\Delta\pi_{t-1} \underset{(4.9)}{-0.463}\Delta\pi_{t-2} \underset{(3.6)}{-0.338}\Delta\pi_{t-3} \underset{(1.6)}{-0.124}\Delta\pi_{t-4}$$

($\bar{R}^2 = 0.336$, SE $= 1.56$, serial LM $= 3.12$ [P value $= 0.56$]; sample period $=$ 1955:1–1988:4). Absolute values of the t statistics are in parentheses.[13] Throughout this paper, the serial Lagrange-multiplier (LM)

[13]For ease of exposition, the inflation rate is measured as an annualized rate of growth by multiplying the $\log_e$ change in the implicit deflator by 400. Also, we have multiplied the "raw data" $(p-p^*)$ by 100 to express the price gap as a percentage deviation.

A referee pointed out that the smooth decline of the estimated coefficients on the lags of $\Delta\pi_t$ are close to those obtained by inverting an ARIMA(0,2,1) model for p_t with a moving-average parameter of about 0.65. Following up on this suggestion, we estimated a more parsimonious version of equation (9) that allows for both these univariate ARIMA effects and those of the price gap:

$$\Delta\pi_t = \underset{(5.5)}{-0.0469}(p_{t-1}-p^*_{t-1})+\varepsilon_t \underset{(13.0)}{-0.772}\varepsilon_{t-1}$$

($\bar{R}^2 = 0.356$, SE $= 1.54$; sample period $=$ 1955:1–1988:4), which has a standard error slightly smaller than equation (9) and a t statistic for $p-p^*$ that is higher. If this equation is inverted to give $\Delta\pi_t$ as an infinite-order-distributed lag of itself and $p-p^*$, the sum of the $p-p^*$ coefficients is $-0.0469/(1-0.772) = -0.2057$. Truncating with the fourth lag, the inverted model is

$$\begin{aligned}\Delta\pi_t = &-0.0469(p_{t-1}-p^*_{t-1})-0.0362(p_{t-2}-p^*_{t-2})\\ &-0.0280(p_{t-3}-p^*_{t-3})-0.0216(p_{t-4}-p^*_{t-4})\\ &-0.772\,\Delta\pi_{t-1}-0.596\,\Delta\pi_{t-2}\\ &-0.460\,\Delta\pi_{t-3}-0.355\,\Delta\pi_{t-4}+\varepsilon_t.\end{aligned}$$

statistics are for the inclusion of lags of the residuals in the estimated equation.[14] This model passes a variety of specification tests as detailed in Hallman et al. (1989).

The negative coefficient on the lagged price-gap term $(p - p^*)$ ensures that the endogenous value of p generated by the model converges to p^* in the long run. Since the change in inflation rather than inflation itself appears on the left side of equation (9), a shock to p^* will lead to overshooting and subsequent damped oscillations of p around its new equilibrium. As a check on whether the dependent variable should be the change in inflation or simply the inflation rate itself, consider a levels version of the model with five lags of π:

$$(10) \quad \pi_t = \alpha(p_{t-1} - p^*_{t-1}) + \sum_{j=1}^{5} \delta_j \pi_{t-j}.$$

This can be rewritten as

$$(11) \quad \Delta\pi_t = \alpha(p_{t-1} - p^*_{t-1}) + \gamma\pi_{t-1} + \sum_{j=1}^{4} \beta_j \Delta\pi_{t-j}$$

where $\gamma = (\Sigma_{j=1}^5 \delta_j - 1)$ and $\beta_j = -\Sigma_{i=j+1}^5 \delta_i$. Equation (9) is simply the special case of equation (11) in which $\gamma = 0$ or, equivalently, the special case of equation (10) in which $\Sigma_{j=1}^5 \delta_j = 1$. Therefore, a test of the restriction implicit in going from the levels specification of equation (10) to the first-difference specification in equation (9) can be based on the t statistic of the parameter γ in an expression like equation (11). This test is conducted below in equation (13).

B. *Alternative Models of Inflation Based on Components of the Price Gap*

Our empirical model of inflation can be compared with either a simple version of an output-gap approach or with a more monetarist-like approach. First, as noted at the outset, we can write $p - p^*$ as the sum of the velocity gap, $v - v^*$, and the output gap, $q^* - q$ [see equation (2)]. Substituting this sum into an equation of the general form of equation (9) and allowing for different coefficients on the two terms yields

$$(12) \quad \Delta\pi_t = \alpha_1(v_{t-1} - v^*) + \alpha_2(q^*_{t-1} - q_{t-1}) + \sum_{j=1}^{4} \beta_j \Delta\pi_{t-j}.$$

The output-gap model of inflation is the special case of equation (12) in which $\alpha_1 = 0$. In this case, inflation tends to accelerate only as the output gap opens up—a standard neoclasical explanation of inflation.

In contrast, monetarist models often determine the long-run price level by multiplying money per unit of output by equilibrium velocity.[15] In such models, an injection of money would temporarily cause v to fall short of v^*, thus initiating a process that would raise spending, and then prices, until equilibrium is reached. This view is more generally consistent with the special case of equation (12) in which $\alpha_2 = 0$.

C. *A Generalized Model*

Equation (13) is an estimated version of a general model that nests both equations (11)

[14]For the quarterly models, the test included lags 1–4 of the residuals, while only the first lag was used in testing the annual and biennial models. Such tests have power against both MA and AR error structures. Under the null hypothesis of no serial correlation, the reported statistics (asymptotically) have a chi-square distribution, with four degrees of freedom for the quarterly models, and with one degree of freedom for the other models. The reported P value is the probability that the chi-square random variable would exceed the value of the test statistic (see Robert F. Engle, 1984; T. S. Breusch and A. R. Pagan, 1980).

[15]The notion of a long-run equilibrium level of velocity, to which actual velocity returns, is implicit in some of the empirical work of Milton Friedman, who relates actual prices to money per unit of *actual* output. Our approach relates actual prices to money per unit of *potential* output, where our measure of potential output roughly corresponds to the notion of permanent income that Friedman has stressed in much of his theoretical work and in some of his empirical work as well.

and (12):[16]

$$(13)\quad \Delta\pi_t = \underset{(1.3)}{0.395} - \underset{(2.1)}{0.102}(v_{t-1} - v^*) - \underset{(3.8)}{0.182}(q^*_{t-1} - q_{t-1}) - \underset{(1.1)}{0.068}\pi_{t-1} - \underset{(6.5)}{0.630}\,\Delta\pi_{t-1} - \underset{(4.3)}{0.447}\Delta\pi_{t-2} - \underset{(3.4)}{0.336}\,\Delta\pi_{t-3} - \underset{(1.6)}{0.129}\,\Delta\pi_{t-4}$$

($\bar{R}^2 = 0.331$, SE $= 1.57$, serial LM $= 2.63$ [P value $= 0.72$]; sample period = 1955:1–1988:4). Several conclusions can be inferred from these estimates. The t statistic for the coefficients of π_{t-1} is small, which indicates that an acceleration specification is preferable to a specification that employs lagged levels of inflation as regressors.[17] Both the output and velocity gaps have means near zero, as does $\Delta\pi_t$, so the intercept is not needed.[18] Its small t statistic reflects these properties. The coefficients on the velocity and output gaps each have the expected sign and are not significantly different from each other. This suggests that imposing the restriction that α_1 must equal α_2 in equation (12) will not hurt the fit of the model.[19]

The price-gap model (9) can be derived from (13) by removing the intercept and the lagged inflation term (π_{t-1}) and restricting the coefficients of the lagged output and velocity gaps to equal each other. As expected, equation (9) fits rather well, with a smaller regression standard error and higher adjusted coefficient of determination ($\bar{R}^2$) than the general model of equation (13).[20] Finally, the coefficient restriction embedded in the price-gap formulation of equation (9) —that the coefficients on p and p^* are equal in magnitude but of opposite sign—is readily accepted over the sample period.[21]

[16]An additional reason to test whether the velocity gap enters equation (13) significantly concerns the methodology used to construct the values of Q^* which enter in the construction of P^* (see footnote 2). The values of Q^* are tied to a measure of the rate of unemployment consistent with no acceleration in inflation. If this is the source of the explanatory power of the price gap, then in equation (13) the output gap should enter significantly, and the velocity gap should have no statistical significance.

[17]The distribution of the t statistic for this coefficient is nonstandard. Notice that, if the output and velocity gaps are dropped from the equation, this t statistic is precisely the augmented Dickey-Fuller test statistic (ADF) for testing the hypothesis of a unit root in inflation. To reject the null at a 95-percent confidence level would require a t statistic less than -2.89. Adding the stationary output and velocity gaps to the ADF regression will not change the asymptotics of this statistic, since they are driven by the integrated regressor π_{t-1}.

[18]The velocity gap has a zero mean by construction, and the output gap nearly so.

[19]We also tested the effects of this restriction in equation (9) by excluding either $q^*_{t-1} - q_{t-1}$ or $v_{t-1} - v^*$ via an LM test for omitted variables. Our data did not reject the restriction.

[20]The relationship between inflation and $p - p^*$, the price-gap model, may also have been affected by the imposition and lifting of price controls during the Nixon administration and the oil-price shocks of 1973 and 1979. The effects of these events can be checked by testing various dummy variables as potential additional regressors using tests for omitted variables. This extended equation fits the data rather well. With the addition of the dummy variables, the coefficients of other variables in the model and the t statistics have higher absolute values, but α (the coefficient of the lagged price gap) is virtually unchanged.

Adding the dummies improves the fit of the model, but it will not have much effect on the out-of-sample forecasts, because the coefficients of the remaining variables are only moderately changed. The coefficients on the dummy variables do not matter for future forecasts, because the dummies will take values of zero for the foreseeable future. More extensive results are discussed in Hallman et al. (1989).

[21]Kenneth N. Kuttner (1990) starts with a more general version of equation (13) that includes two lags each of $q^* - q$ and $p - \bar{p}$, where $\bar{p}$ is defined as $\bar{p} = m + v^* - q$. The variable $\bar{p}$ is the log of the implicit price deflator (IPD) that would obtain if v returned to v^* and output remained at its current level. If p is log(IPD), then $p = m + v + q$, and $p - \bar{p} = v - v^*$. However, for the same m, v, and q concepts, if p is redefined as the log of another price index, then p does not equal $m + v + q$ and $p - \bar{p}$ does not equal $v - v^*$. In these cases, the stationarity of $v - v^*$ does not imply the stationarity of $p - \bar{p}$.

When p is log(IPD), equation (9) is obtained from Kuttner's model by restricting the coefficients on π_{t-1},

D. *The Effects of Time Aggregation*

As a final aspect of the econometric specification, Table 2 shows the effects on the model estimates of lengthening the time interval between successive observations. The quarterly change in inflation is a fairly volatile series: the value of $\bar{R}^2$ for that frequency indicates that the price-gap model explains roughly a third of the variation in the change in inflation from one quarter to the next. If the variables in the model are observed annually (say, every fourth quarter) or biennially (every eight quarter), lagged dependent variables are not required to fit the data, simplifying equation (9) to

$$(14) \qquad \Delta\pi_t = \alpha(p_{t-1} - p^*_{t-1}).$$

With this equation, $\bar{R}^2$ increases dramatically, to 43 percent at the annual frequency and to 75 percent at the two-year frequency. The fit of the price-gap model at both of these frequencies is clearly superior to that of the output- and velocity-gap models and is slightly better than the fit of the model with unrestricted coefficients on the output and velocity gaps. Even with the somewhat rigid view embodied in these specifications (that there is a fixed one-year or two-year lag between the level of M2 and the price level), the primacy of the monetary explanation of changes in inflation is evident.

$p_{t-2} - \bar{p}_{t-2}$, and $q^*_{t-2} - q_{t-2}$ to be zero and constraining the coefficient of $p_{t-1} - \bar{p}_{t-1}$ and $q^*_{t-1} - q_{t-1}$ to be equal. Kuttner finds that, while he cannot reject the restrictions when the IPD is used, he can reject them when the alternative price indexes (and their inflation rates) are used. He interprets this as evidence against equation (9). This inference is incorrect, because $\bar{p}$ is the equilibrium value only for IPD, and therefore there is no reason to expect the consumer price index or any other alternative indexes to move toward the IPD's equilibrium level. However, it is possible to replace the lagged $p - \bar{p}$ terms in Kuttner's model with the corresponding lags of $v - v^*$. When this is done, the p^* restrictions cannot be rejected for any of the alternative price indexes he considers.

III. Out-of-Sample Forecasts

The above tests of the general model [equation (13)] indicate that the price-gap model [equation (9)] is preferable to specifications with either the output gap or velocity gap alone. However, these tests were based on within-sample data; a more strenuous test is to use the model to forecast inflation out of sample. In this spirit, we compare the performance of our model and several competitors with rolling-horizon forecasts.

In commenting on an earlier version of this paper, Lawrence J. Christiano (1989) pointed out that our measure of potential output is constructed using information that would not have been available to a forecaster operating in real time.[22] A measure Q^{**}_t that is available in real time can be constructed by setting $Q^{**}_t = Q_t$ for $t = 0, 1, 2, \ldots, 10$ (Q_t is real GNP) and recursively setting

$$(15) \quad Q^{**}_t = 0.96(Q^{**}_{t-1}) + a_0 \sum_{i=1}^{11} b_i Q_{t-i}$$

for $t > 10$. The weights $\{b_i\}$ are

$$(0, -0.0004, 0.0013, 0.0023, 0.0028, 0.0028, 0.0025, 0.0020, 0.0022, 0.0008, 0.0002).$$

When forecasting from period t, a new Q^{**} series is first formed by choosing a_0 to minimize $\Sigma^t_{s=11}[\log(Q_s) - \log(Q^{**}_s)]^2$. The values for $\{b_i\}$ and the form of equation (15) are taken from the Federal Reserve Board MPS quarterly econometric model's equation for real business potential output XBC_t:

$$(16) \quad \text{XBC}^*_t = 0.96(\text{XBC}_{t-1}) + \alpha_0 \sum_{i=1}^{11} b_i \text{XB}_{t-i} \qquad t > 10$$

[22] While the forecast at time t does not use values of potential output beyond Q^*_{t-1}, estimates of Q^*_t are based on a nonlinear least-squares procedure that uses all the sample information. Therefore, by construction, Q^*_t incorporates future shifts in labor-force participation and productivity.

TABLE 2—ESTIMATES OF EQUATIONS FOR ALTERNATIVE INFLATION INDICATORS OVER VARIOUS TIME AGGREGATIONS

Equations	Frequency									Serial LM
	$p_{t-1}-p^*_{t-1}$	$q^*_{t-1}-q_{t-1}$	$v_{t-1}-v^*_{t-1}$	$\Delta\pi_{t-1}$	$\Delta\pi_{t-2}$	$\Delta\pi_{t-3}$	$\Delta\pi_{t-4}$	$\bar{R}^2$	SE	(P value)
Price gap:										
Quarterly	−0.148 (4.4)			−0.667 (8.0)	−0.463 (4.9)	−0.338 (3.6)	−0.124 (1.6)	0.336	1.563	3.12 (0.56)
Annual	−0.217 (4.9)							0.425	1.092	1.70 (0.19)
Biennial	−0.340 (6.9)							0.751	0.888	0.18 (0.67)
Output gap:										
Quarterly		−0.163 (3.3)		−0.657 (7.6)	−0.466 (4.7)	−0.356 (3.6)	−0.136 (1.7)	0.297	1.608	2.94 (0.57)
Annual		−0.206 (2.9)						0.208	1.280	0.70 (0.40)
Biennial		−0.310 3.2						0.411	1.369	0.56 (0.45)
Velocity gap:										
Quarterly			−0.111 (2.5)	−0.591 (7.0)	−0.360 (3.7)	−0.241 (2.5)	−0.059 (0.7)	0.269	1.640	2.24 (0.69)
Annual			−0.199 (2.8)					0.193	1.293	0.05 (0.83)
Biennial			−0.271 (2.4)					0.259	1.533	0.005 (0.94)
Unconstrained velocity and output gaps:										
Quarterly		−0.176 (3.7)	−0.126 (2.9)	−0.678 (8.0)	−0.480 (4.9)	−0.358 (3.7)	−0.137 (1.8)	0.335	1.564	2.90 (0.58)
Annual		−0.221 (3.6)	−0.214 (3.5)					0.407	1.108	1.80 (0.18)
Biennial		−0.355 (5.4)	−0.323 (4.8)					0.740	0.908	0.19 (0.61)

Notes: The sample periods for the time aggregations were as follows: quarterly, 1955:1–1988:4; one-year average, 1955–1988; two-year average, 1956–1988.

where XB_t is real gross private business domestic product and α_0 is fixed at 3.72298 (see Flint Brayton and Eileen Mauskopf, 1985). The specification of this particular equation in the MPS model and its coefficients have not changed at least since Anne Williams (1970) and may be even older. Equation (15) thus represents a model for generating estimates of current and past potential output using only information that was available in real time, abstracting from data revisions.

TABLE 3—ROLLING-HORIZON FORECASTS OF INFLATION (Q4–Q4 INFLATION RATES)

Year	Actual	P^*	V^*	Q^*	Unrst	ARIMA	MLynch	ConfBd	Chase	DRI	WEFA	P^*_{rt}	T-bill
1971	6.1	5.0	5.1	5.1	5.0	5.0	NA	NA	4.1	3.2	3.7	4.9	3.9
1972	4.4	6.4	6.8	6.0	6.8	5.6	NA	3.3	4.1	3.8	4.3	6.2	5.5
1973	8.2	5.3	5.0	4.6	5.4	4.9	NA	4.2	4.6	3.8	3.9	4.9	4.7
1974	10.0	8.8	8.1	8.7	8.6	7.6	NA	8.2	5.9	7.5	NA	8.4	7.4
1975	8.3	9.1	9.5	9.6	9.1	10.4	NA	7.2	6.4	7.8	NA	9.0	9.6
1976	5.7	7.6	8.2	7.9	7.7	8.6	NA	6.0	5.5	6.0	NA	7.4	7.8
1977	6.8	5.9	6.4	5.4	6.1	6.6	5.9	7.0	6.2	6.2	NA	5.4	5.9
1978	8.0	7.3	7.3	6.9	7.4	6.7	6.3	6.4	6.2	6.3	NA	6.8	7.2
1979	8.9	7.8	6.8	8.7	7.5	7.9	7.8	7.7	7.2	8.2	7.6	7.5	9.2
1980	9.9	7.9	7.6	9.1	7.7	8.5	9.7	8.7	8.9	10.2	9.1	7.8	10.5
1981	8.7	8.2	8.8	9.6	8.2	10.0	8.7	9.7	10.4	10.2	10.4	8.3	11.7
1982	5.2	6.6	7.5	7.9	6.6	8.7	6.1	6.3	4.2	6.8	7.7	6.8	8.1
1983	3.6	3.2	5.2	3.1	3.2	5.3	5.9	4.9	4.9	4.6	3.1	3.3	4.9
1984	3.4	2.9	4.0	2.6	3.0	4.1	5.4	5.8	4.7	4.6	5.0	2.7	3.7
1985	2.9	3.0	3.6	2.9	3.1	3.3	4.5	3.7	4.3	3.3	4.1	2.9	3.0
1986	2.6	3.2	3.6	2.6	3.2	3.1	2.3	3.4	3.6	2.3	3.1	2.9	3.0
1987	3.0	3.6	4.1	2.1	3.7	2.8	4.1	3.6	3.1	3.0	3.6	3.1	2.8
1988	4.0	3.7	3.6	3.1	3.7	2.8	3.7	3.0	NA	3.2	3.8	3.3	2.7
1989	3.8	4.3	4.1	4.2	4.3	4.2	4.2	4.8	NA	3.4	4.5	3.9	4.3
Mean error:		−0.19	0.08	−0.18	−0.19	0.14	0.31	−0.19	−0.68	−0.48	−0.05	0.43	−0.11
RMSFE:		1.25	1.61	1.44	1.31	1.71	1.21	1.51	1.80	1.57	1.72	1.35	1.69

Mnemonics for the Various Forecasts

P^*:	price-gap model [equation (14)]
V^*:	velocity-gap model
Q^*:	output-gap model
Unrst:	unrestricted model with separate coefficient on output and velocity gaps
ARIMA:	an ARIMA(0, 2, 1) model
MLynch:	Merrill Lynch
ConfBd:	Conference Board
Chase:	Chase Econometrics
DRI:	Data Resources, Inc.
WEFA:	Wharton Econometric Forecasting Associates
P^*_{rt}:	Price-gap model using real-time estimates of Q^* and P^*
T-bill:	Christiano's quarterly Treasury-bill model

Christiano also maintains that a quarterly model of the form

$$(17)\qquad \Delta\pi_t = \alpha\,\Delta r_{t-1} + \sum_{i=1}^{4} \beta_i\,\Delta\pi_{t-i} \qquad \alpha > 0$$

where r_t is the yield on 90-day Treasury bills, forecasts about as well as the price-gap model.

Table 3 shows actual and forecasted Q4–Q4 (fourth quarter) inflation rates for a number of forecasts, including those from annual versions of the price-gap, velocity-gap, and output-gap models, an unrestricted (includes both velocity and output gaps) model, and Christiano's model (17). Also included are forecasts from a quarterly ARIMA(0, 2, 1) model of the log(price level) and from several private forecasters.

In Table 3, the model-based forecasts for each year were found by estimating the appropriate equation(s) using data dated up to Q4 of the previous year. The figures for private forecasters were computed from implicit price deflator forecasts listed each year in the March issue of the *Statistical Bulletin*, a monthly publication of the Conference Board. Some of the private forecasters did not appear in all years of the *Statistical Bulletin*; the forecasts for these years are marked as NA. The private forecasts were generally made in February, after the first revision of the data for Q4 of the previous year became available.

The entries in the mean error and root-mean-squared forecast error (RMSFE) rows at the bottom of the Table 3 are not directly comparable across columns, since some of the columns do not contain forecasts for all years. The Merrill Lynch forecast, for example, appears to have the smallest RMSFE, but this is because it did not attempt to forecast the very difficult 1971–1976 period. To form a basis for comparison, Table 4 shows the ratio of the RMSFE of the individual forecasts to the RMSFE of the P^* forecasts. Here, the denominator of each ratio (the P^* RMSFE) is calculated using observations for only those years during which the numerator forecast was available. The MLynch entry, for example, is the ratio of the MLynch RMSFE to the P^* RMSFE over the 1977–1989 period. By this metric, the P^* forecasts are superior to the alternatives.

TABLE 4—COMPARISON OF ALTERNATIVE INFLATION FORECASTS TO THAT OF THE P^* MODEL

Forecaster	RMSFE ratio[a]
V^*	1.29
Q^*	1.15
Unrestricted	1.05
ARIMA(0, 2, 1)	1.37
Merrill Lynch	1.37
Conference Board	1.20
Chase	1.38
DRI	1.26
Wharton	1.35
P^*_{rt}	1.08
T-bill	1.35

[a]Ratio of RMSFE of the forecaster to that of the price-gap model [equation (14)].

An alternative method of comparing competing forecasts, as elucidated by Yock Y. Chong and David F. Hendry (1986), is to ask whether the forecast error from a given model can be explained ("encompassed") by the forecasts of another model. To make this comparison, let f_t^i and f_t^j denote the forecasts made by models i and j, and let the model-i forecast error be denoted by e_t^i. Define $t(i,j)$ as the t statistic for β in the regression

$$e_t^i = \beta(f_t^j - f_t^i) + \eta_t. \tag{18}$$

Model i is said to forecast-encompass model j if $t(i,j)$ is not significantly different from zero but $t(j,i)$ is. Tables 5 and 6 show some $t(i,j)$ statistics computed from the forecasts of Table 3, where the statistic $t(i,j)$ for models i and j is shown in row i and column j of the tables.

To interpret Table 5, notice that all of the numbers in the column labelled "P^*" are larger than 2, indicating that the forecast from the price-gap model contains useful information beyond that contained in the forecasts from the other three models in the table. Reading across the P^* row, it appears that only the unrestricted (Unrst) model forecast contains any information beyond that contained in the price-gap forecast. Thus, the price-gap model forecast-encompasses both the velocity- and output-gap models but does not encompass the unrestricted model, though the value 2.06 is not very large considering that there are only 19 observations on which to base the comparison. On the other hand, the velocity-gap forecast does not help explain the forecast errors of any of the other models and is encompassed by all of them.

With the exception of the first row and column, all of the forecasts compared in Table 6 either were or could have been made in real time. The price-gap model using Q_t^* encompasses all of the other models. The real-time version of the price-gap model encompasses all but one of the other real-time forecasts and is not itself encompassed by any of them.[23]

[23]We have also conducted cross-validation tests of the price-gap, velocity-gap, and output-gap models and the unrestricted model with both velocity and output gaps. The price-gap model generally outperforms the velocity- and output-gap models, and the restrictions on the output and velocity gaps that produce the price-gap model improve the forecasting performance of the model. These results are in Hallman et al. (1989).

TABLE 5—FORECAST-ENCOMPASSING STATISTICS FOR MODELS RELATED TO P^*

	Model			
Model	P^*	V^*	Q^*	Unrst
P^*		−1.32	0.09	−2.06
V^*	3.86		2.33	3.32
Q^*	2.44	0.83		2.04
Unrst	2.56	−1.10	0.60	

Note: The i,jth element is the statistic $t(i,j)$ for the models in row i and column j. See the discussion of equation (18) for the definition of the statistic $t(i,j)$.

TABLE 6—FORECAST ENCOMPASSING STATISTICS FOR REAL-TIME FORECASTS

	Forecast								
Forecast	P^*	P^*_{rt}	ARIMA	T-bill	MLynch	ConfBd	Chase	DRI	WEFA
P^*		−0.97	−0.39	0.59	1.73	1.61	1.27	1.29	0.20
P^*_{rt}	2.10		−0.12	0.73	2.09	1.77	1.26	1.27	0.36
ARIMA	4.03	3.28		1.16	2.32	2.74	2.29	2.32	0.69
T-bill	3.99	3.32	0.98		2.73	2.55	2.01	1.95	1.16
MLynch	4.01	3.20	1.23	1.91		1.17	1.50	2.71	1.59
ConfBd	3.35	2.74	1.15	1.51	1.10		0.19	1.33	0.62
Chase	4.17	3.52	2.40	2.26	1.30	2.21		1.96	0.53
DRI	3.63	2.88	1.27	0.82	0.39	0.84	0.65		0.44
WEFA	3.28	2.84	1.58	1.26	1.91	1.10	1.92	1.30	

Note: The i,jth element is the statistic $t(i,j)$ for the models in row i and column j. See the discussion of equation (18) for the definition of the statistic $t(i,j)$.

IV. Examining the 1870–1954 Period

As shown in Figure 3, over the period from after the Civil War to World War II, the velocity of M2 exhibited a downtrend, which was at first pronounced but which gradually moderated. This general period plus the years through 1970 were used by Gould and Nelson (1974) and by Nelson and Plosser (1982) to examine the behavior of velocity, from which they concluded that V2 has a unit root. In contrast, over the period 1954–1988, we have used the sample mean of V2 as the measure of long-run velocity.

A resolution of these conflicting findings starts with the results given above in Table 1. As noted above, this test, which is the same as that used by Nelson and Plosser, does not reject a unit root for velocity for the 1870–1954 period. However, over the 1954–1988 period, the unit root is rejected, and the autocorrelations of V2 do not look like those of a random walk. The reason for the instability of these test results can be seen in Figure 3. In their tests, Nelson and Plosser use a linear time trend for the log of velocity, and such a trend line is persistently below the actual V2 series at the beginning and end of the sample period for either the 1870–1954 or the 1869–1970 periods. While such observations are consistent with a unit root about that particular trend, they may merely reflect the ad hoc choice of a linear time trend.[24] An alternative approach is to examine whether velocity is cointegrated with variables of economic interest. For ex-

[24]Gould and Nelson used the period 1869–1960 to model velocity and used the years 1961–1970 for post-sample predictions. In testing mean reversion for velocity, they checked over the 1961–1970 period for the reversion of velocity to its mean calculated over 1869–1960 and also to the mean over 1891–1960. Expressing these means in terms conformable to Figure 3, the log of the mean over the former period is 0.884, and for the later period it is 0.663. Over the 1955:1–1988:4 period for which we find a stationary velocity series, log(V2) never reaches either of these values.

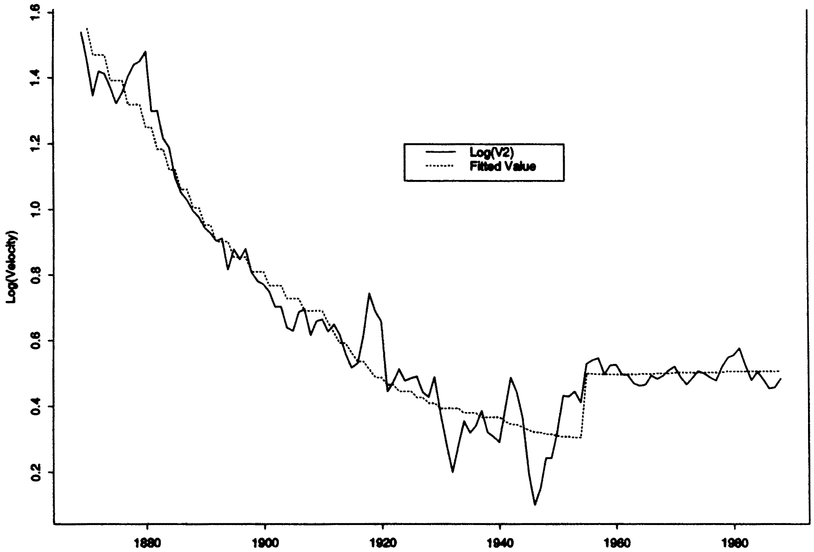

FIGURE 3. ACTUAL AND FITTED VALUES OF M2 VELOCITY

ample, Milton Friedman and Anna J. Schwartz (1982) attribute the strong secular decline in V2 from 1880 until World War I to the monetization of the economy as evidenced by the growth of financial institutions and the industrialization of the economy.[25] As a proxy for the industrialization of the U.S. economy, Bordo and Jonung (1987) use the variable nal_t, the fraction of the labor force not employed in agriculture. The following model illustrates how the trend in velocity might be explained by such considerations:[26]

$$(19)\quad v_t = \underset{(7.5)}{0.33} + \underset{(2.3)}{0.61}(\text{nal}_t) + \underset{(11.3)}{3.80}(\text{nal}_t^2) + \underset{(5.6)}{0.19}(\text{shift}_t) + \varepsilon_t$$

($\bar{R}^2 = 0.947$, SE $= 0.076$; sample period $=$ 1870–1988).

The much-noted increase in velocity after World War II is captured in this model by the dummy-variable shift, which is 0 until 1954 and 1 thereafter. Of course, adding the dummy variable to equation (19) does not explain why the shift took place.[27] The ADF

[25] In support of this explanation, they note on page 146 that from "1880 to 1910, United States population nearly doubled, but the number of banks multiplied more than sevenfold. The fraction of the population residing in rural areas had declined from over two-thirds to one-half; the fraction of the work force in agriculture had declined from one-half to less than one-third."

[26] Bordo and Jonung use several other variables which we omit since including them would impair interpreting the fitted value of the regression as the long-run value of velocity. Two variables are included to explain short-run variations in velocity: the ratio of measured per capita permanent income to permanent real per capita income and a six-year moving standard deviation of the annual percentage change in real income per capita. Two other variables, which are financial in nature, have the similar problem that there is little reason to interpret their values in any given year as the long-run values. They are the ratio of currency to M2 and the ratio of total private nonbank financial assets to total private financial assets. We exclude them also since they include M2 in their construction and, therefore, may be correlated with velocity by construction. Finally, Bordo and Jonung also use a short-term nominal interest rate. We exclude this variable to conform to our notion that V^* is a long-run measure and should not depend on short-run variations in monetary policy.

[27] Bordo and Jonung provide a brief survey of the explanations of the post-World War II rise in velocity; see page 12 of their work for explanations focusing on the development of money substitutes and page 19 for references that appeal to technological advances in the payment mechanism.

statistic for the residuals of equation (19) is 5.06, which is well above even the 99-percent critical value given by Robert F. Engle and Byung Sam Yoo (1987), so that a hypothesis of no cointegration is clearly rejected. As shown in Figure 3, the series for long-run velocity implied by equation (19) captures the general trend in velocity and approximately levels out after 1955 (as nal_t does).[28]

For the pre-1955 period, we use the fitted values from equation (19) as our estimates of the log of long-run velocity V_t^*. We first estimated a general model of the form of equation (6) which includes the velocity and output gaps separately, but specification tests indicated the need to add second lags of the output and velocity gaps.[29] That equation with the extra lags simplified to

$$(20)\quad \pi_t = \underset{(2.7)}{-0.15}(v_{t-1} - v_{t-1}^*) \underset{(3.5)}{-0.26}\,\Delta(p_{t-1} - p_{t-1}^*) \underset{(5.9)}{+0.51}\pi_{t-1} + \text{WW1}$$

($\bar{R}^2 = 0.61$, SE $= 3.81$, serial LM $= 1.89$, [P value $= 0.17$]; sample period $= 1871$–1954), where WW1 denotes the contributions of separate dummy variables for the years 1917–1921. The coefficients on the dummy variables for the first four years are positive and sum to 44, while the coefficient for 1921 is -23.

While the price gap $(p - p^*)$ appears as a first difference and, therefore, does not tie the price level to M2, the velocity gap does enter with a negative coefficient. If, in addition, output equals its potential level in the long run, then in such a state $v - v^*$ equals $p - p^*$, and equation (20) suffices to drive P to P^*. The dynamics of this model are different from those of the simple price-gap model as represented by equation (9) above, but both models share the feature that the level of the money supply eventually ties down the price level. However, the parameter estimates, including which lags of the variables enter, are quite sensitive to the sample period used. Therefore, the evidence for modeling the relation between money and inflation through P^* as we have implemented it, and possibly for P^* in general, over the pre-1954 period has not been firmly established.

[28]Given the intercept shift allowed for in 1955 and the nearly constant value of nal after 1955, the long-run values of velocity after 1955 nearly equal the sample mean of 1.65 that we used in constructing P^* after 1955. The values range from 1.64 in the years just after 1955 to 1.66 at the end of the sample period.

[29]In the model without second lags, neither the output nor the velocity gap entered significantly.

V. Conclusion

The P^* model of inflation arose from an attempt to identify the inflationary potential of the economy. Our approach is to identify the long-run price level through the construction of P^* and then to estimate reduced-form short-run dynamics that drive actual prices to P^*. While a structural approach to handling the short-run dynamics is required for a thorough analysis of the effects of money and other factors on inflation, we have demonstrated that embedding the long-run relations between the levels of money and prices in the model has a significant payoff in terms of the tractability of the model and its forecasting performance for the period since the Korean War.

In the long-run, inflation does seem to be a monetary phenomenon, as evidenced by the close relation of the aggregate price level and P^*. While this conclusion is hardly novel, we have demonstrated a stronger form of this result than is usually considered. First, P^* ties together the level of money and prices, not merely their growth rates. Models that relate inflation to the growth rates of velocity or money face several shortcomings. The growth rates are typically calculated over a period of several quarters and then are lagged a number of quarters. The freedom to choose the time horizons over which to calculate the growth rates and the length of the lags may lead to an overfitting of the relationship. Consequently, the closeness with which money growth and inflation are related may well be overstated. By incorporating only the stock of money, P^* does not afford these choices: it provides an absolute reference point that takes into account all past money growth.

The P^* models estimated over the post-Korean War period indicated that the actual price level adjusts to P^* at a rather modest rate and that there is considerable inertia in the inflation rate. This finding is consistent with structural models in which monetary policy has real effects in the short run. However, as a guide to monetary policy, keeping track of P^* exerts a certain discipline on the conduct of the monetary authority. By demonstrating that the accumulative effects on the money stock will eventually show up in the price level, it shows that, for example, even if there is excess productive capacity in the economy, an increase in the money supply will affect future inflation. The danger of conducting monetary policy to stabilize the real economy is that the price level then gets determined as a by-product of such monetary policy actions. What is needed is a framework in which the determination of prices is the primary long-run focus and in which short-run stabilization actions can be evaluated and monitored for their consistency with long-term objectives for price developments. P^* is a possible contribution to the development of such a framework since, through its dependence on long-run values of velocity and output, it can be used to indicate long-term price developments.

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