FREQUENCY DOMAIN CLOSED-LOOP ANALYSIS AND SLIDING MODE CONTROL OF A NONMINIMUM PHASE BUCK-BOOST CONVERTER

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Abstract

In this paper, a robust closed-loop cascaded controller is designed and rigidly analyzed for an inverting buck-boost converter. The fast inner current loop uses sliding mode control (SMC). The slow outer voltage loop uses the proportional-integral (PI) control. Analysis of stability and robustness, proof of nonminimum phase structure, and proper selection of PI gains are based on the closed-loop transfer function. The controller design is facilitated by the system frequency responses. The output voltage transients are predictable during step changes of the input voltage and load resistance. The operating range of the reference voltage is discussed. The simulation results show that the reference output voltage is well tracked under modeling uncertainties, disturbances or step changes of parameters, confirming the validity of the proposed method.

Key Words

SMC, frequency response, pole placement, nonlinear, linear, transfer function, closed-loop, nonminimum phase, transient, buck-boost converter

1. Introduction

Power converters are extremely popular in home appliances, medical devices, uninterrupted power supplies, electric vehicles, battery chargers, aerospace equipments, etc because they are highly efficient and have low volume and weight [1, 2]. Due to high nonlinearity of the relation between the output and the input, uncertain input voltage, load resistance, inductance or capacitance, and unpredictable disturbances, there are challenges for controller designs of power converters.

Buck-boost converters are a typical type of power conditioning devices due to its simple structure and practical functionality of dropping or lifting the output voltage with respect to an input voltage [3]. Small signal linearization, state space averaging, and circuit averaging are the conventional design and analysis methods for buck-boost converters [2, 4]. These methods require linearization for every operating point and end up with the local stability. They often fail to perform satisfactorily under uncertain system parameters, varied input voltage, varied load or unpredictable disturbances. A variety of nonlinear feedback controls...
such as feedback state linearization, input output linearization, flatness, passivity based control, dynamic feedback control by input-output linearization, exact tracking, and error passivity feedback have been applied to power converters [3]. However, most of these nonlinear tools require conventional PWM modulators, understanding of them needs professional control expertise and practical implementation of them takes uncertain risks. One-cycle control is successfully developed for Cuk converters [5]. Robust backstepping technique can control interleaved buck converters [6]. Synergetic control is tested in buck and boost converters although the closed-loop analysis for the stability and robustness of these systems is not addressed [7, 8].

In the past three decades, SMC has been widely applied to power converters [9]. The detailed SMC design theory can be referred to [9]. Briefly, SMC can make a system insensitive to disturbances and parametric uncertainties and automatically reduces the order of the system by the number of switching manifolds or controls. SMC works for linear systems, nonlinear systems and discontinuous systems with discontinuous state variables or controls. Although SMC theory is well developed, its applications are far from being exhausted [10]. Open issues exist for combination of SMC and other controls for better performance [9], chattering elimination [9], switching frequency limitation [11], integrated circuit for controller implementation [9], etc.

Open loop SMC has been applied to many types of converters [3, 9]. Indirect control of the current on a switching manifold is used for output voltage regulation. Sliding mode observers are also used to improve system transient responses under open loop SMC [9]. Unfortunately, open loop SMC is not robust. Parametric uncertainties and disturbances will lead to invalidity of the method. Combination of a PI controller for the voltage loop and SMC for the current loop is proposed for a DC-DC boost converter [12]. Although the simulation results are reported, the author fails to provide a scientific guide for the controller design. The author incorrectly concludes that the system transients during step changes of the load resistance and input voltage are due to the nonminimum phase structure. Buck converters are successfully regulated under SMC with special switching manifolds or auxiliary dynamics [13-15]. The limitation of these methods is the impossibility of extending them to more complicated power converters, e.g., a buck-boost converter. Simulation and experimental results are reported for a buck converter under SMC and fuzzy control [16]. A buck-boost converter uses SMC with the switching manifold as a function of linear
combinations of the voltage error and its derivative [17]. However, noise is present in the output voltage if disturbances exist on the input voltage source or load resistance.

It is desirable to provide theoretical analyses for transient phenomena of power converters. No or poor explanations for transients of power converters are provided in [12-14, 17, 18]. A reliable analysis for the operating range of the reference voltage is not seen in these papers, either. In this paper, a PI and sliding mode cascaded controller is designed for a buck-boost converter. This control structure was proposed fifteen years ago but its analytical solution has never been seen in the existing literature [9]. This paper fills this gap, showing that this controller has a solid theoretical foundation, is robust and reliable, and can easily be simulated and implemented. The linear and nonlinear closed-loop dynamics are developed. Stability, robustness, power transients and nonminimum phase structure of the closed-loop system are analyzed. Although the frequency response method is seldom used in SMC, it is shown that it can provide invaluable information for the controller design.

This paper is organized as follows. The buck-boost converter model is studied in Section 2. The controller is developed in Section 3. Simulation results are reported in Section 4. Conclusion is in Section 5. References follow.

2. Buck-boost Converter Model

The buck-boost converter is shown in Fig. 1. It consists of an input voltage source \( E \), a transistor switch \( S_n \), a diode \( D \), a capacitor \( C \), an inductor \( L \) and a load resistor \( R \). The inductor current is \( i \). The output voltage across the resistor is \( v \). The absolute value of \( v \) can be greater or less than \( E \). \( v \) is opposite to \( E \) in polarity. When \( S_n \) is on, the circuit is shown in Fig. 2. The voltage source \( E \) supplies energy to the inductor \( L \) and the capacitor \( C \) supplies energy to the load resistor \( R \). When \( S_n \) is off, the circuit is shown in Fig. 3. The inductor supplies energy to the capacitor \( C \) and the load \( R \). As shown in [10], if the control signal \( u \) is 1 when \( S_n \) is on and 0 when \( S_n \) is off, the ordinary differential equations for the buck-boost converter are represented by:

\[
\begin{align*}
    i' &= \frac{1-u}{L} v + \frac{u}{L} E \\
    v' &= -\frac{1-u}{C} i - \frac{1}{CR} v 
\end{align*}
\]
(1) and (2) have the typical state space format with the discontinuous right hand side. The control and the state variables have a bilinear relation.

![Buck-boost converter](image1)

**Fig. 1.** Buck-boost converter.

![Buck-boost converter when the switch $S_w$ is on.](image2)

**Fig. 2.** Buck-boost converter when the switch $S_w$ is on.

![Buck-boost converter when the switch $S_w$ is off.](image3)

**Fig. 3.** Buck-boost converter when the switch $S_w$ is off.

### 3. Closed-Loop Control And Analysis

#### 3.1 Cascaded Control

In this section, the feedback controller is designed. The highly nonlinear closed-loop dynamics involving the PI voltage controller for the outer loop and the sliding mode current controller for the inner loop is developed and analyzed. The control goal is to track a constant reference voltage with a tolerable error despite of disturbances or uncertainties. The cascaded control structure is shown in Fig. 4 where $i^*$ is the feedback reference current, $v_d$ is the input reference voltage, $E$, $v$ and $u$ are the same as defined previously, and $e$ is the error between $v_d$ and $v$. $i$ is a positive feedback signal due to the structure of the sliding mode controller as shown in (8). The sensors are needed measuring $v$ and $i$. The condition for the stability of the inner current loop is provided.
3.1.1 Outer Voltage Loop With PI Control

The equilibrium point of the buck-boost converter corresponding to a constant value of the average control input is obtained by letting the right hand side of (1) and (2) be zero while the control variable is set to be \( u = U \) where \( U \) is a constant [3, 9]. The equations for the equilibrium inductor current \( i_d \) and the equilibrium voltage which is also equal to \( v_d \) are given by:

\[
0 = \frac{1 - U}{L} v_d + \frac{U}{L} E = 0
\]

\[
0 = \frac{1 - U}{C} i_d - \frac{1}{CR} v_d = 0
\]

Eliminating \( U \) in (3) and (4), \( i_d \) in terms of \( v_d \) is:

\[
i_d = \frac{v_d}{R} \left( \frac{v_d}{E} - 1 \right)
\]

(5) provides a theoretical relation between the inductor current and the output voltage when the converter is in the steady state. \( i_d \) is also the reference input current for the power converter under open loop SMC control as shown in Section 3.2. With \( e = v_d - v \), the reference current for the inner current loop is generated by a PI controller as:

\[
i' = K_i e + K_p \int e dt
\]

3.1.2 Inner Current Loop with Sliding Mode Control

The switching manifold for SMC is designed as:

\[
s = i - i'
\]

For the buck-boost converter, the control scheme is:

\[
u = 0.5(1 - \text{sign}(s)) = 1 \text{ if } s < 0 \text{ or } 0 \text{ if } s > 0
\]

Fig. 4. The structure of the cascaded controller.
This control structure will be verified when the inner loop stability is analyzed. Both the switching manifold \( s \) and its derivative are equal to zero after sliding mode occurs. In Fig. 5, \( t_{\text{sm}} \) is the time instant at which the system reaches \( s = 0 \). Before \( t_{\text{sm}} \), the motion is in the reaching phase. Upon \( t_{\text{sm}} \), the motion is in sliding mode. The existence condition of sliding mode (in other words, selection of appropriate control to guarantee that the system can reach and stay on \( s = 0 \)) can be derived with the candidate Lyapunov function. Let a Lyapunov function for the system represented by (1) and (2) be:

\[
V = 0.5s^2 > 0 \quad \text{if} \; s \neq 0
\]  

(9)

The differentiation of (7) is:

\[
s' = i' - i'^* = \frac{1}{L}v + \frac{u}{L}E - i' = i' - i'^*
\]  

(10)

With (10), the derivative of \( V \) is

\[
V' = ss' = s\left(\frac{1}{L}v + \frac{u}{L}E - i'^*\right) = \frac{1}{L}s((E - v)u + v - Li'^*)
\]

\[
= \frac{1}{L}s((E - v)\frac{1}{2}(1 - \text{sign}(s)) + v - Li'^*)
\]

\[
= \frac{1}{2L}s(E + v - 2Li'^*) - (E - v)\text{sign}(s)
\]

\[
= \frac{1}{2L}s(E + v - 2Li'^*) - |s|(E - v)
\]

\[
\leq \frac{1}{2L}|s||E + v - 2Li'^*| - |s|(E - v)
\]

\[
\leq \frac{1}{2L}|s||E + v - 2Li'^*| - |s|(E - v)
\]

(11)

Therefore, the sufficient condition for \( V' < 0 \) is:

\[
|s||E + v - 2Li'^*| - |s|(E - v) < 0
\]

(12)

The inequality (12) leads to:

\[
|E + v - 2Li'^*| < E - v
\]

(13)

If sliding mode is reached and \( v_d \) as a constant is tracked due to the voltage controller, \( i' \) must be close to \( i_d \) and \( i'^* \) must be small. Since \( L \) is very small, then \( Li'^* \) must be close to 0. With \( Li'^* \approx 0 \) and due to \( E > 0 \), \( v \) must be negative to guarantee the inequality (13). The inequality (13) defines an attraction domain of the sliding manifold. Because the control in (8) contains no control gains to be adjusted, the domain of attraction (the inequality (13)) is predetermined by the system architecture. In the steady state, the
inequality (13) is fulfilled by the definition of an inverting buck-boost converter: the polarities of the input source voltage $E$ and the output voltage $v$ are opposite to each other regardless of their magnitudes. In the reaching phase, with normal and practical initial conditions of $i(0)$ and $v(0)$, and with the well-designed voltage controller for generating appropriate $i^*$ such that the inequality (13) holds, the motion will eventually go through the reaching phase and reaches the sliding mode. The derivation of (11) implicitly validates (8) since it results in a stable system.

![Diagram](image)

Fig. 5. The system reaches sliding mode after the reaching phase.

### 3.1.3 Closed-Loop Dynamics

Closed-loop analysis is applicable to the system after sliding mode occurs. In sliding mode, the equivalent control method can be explored [19]. Once the system is in sliding mode, $s = 0$ and $\dot{s} = 0$ hold. The discontinuous control $u$ in $\dot{s} = 0$ can be replaced by a continuous equivalent control $u_{eq}$ and $\dot{s} = 0$ is solved for $u_{eq}$. Under $u_{eq}$, the state velocity lies in the tangential manifold. $u_{eq}$ is close to the slow component of $u$ which contains both low and high frequency signals. After sliding mode occurs, one has:

\[ s = i - i^* = 0 \]  
\[ s^* = i^* - i^{**} = \frac{1-u_{eq}}{L} v + \frac{u_{eq} E}{L} - i^* = 0 \]

Solving (14) for $i$ renders:

\[ i = i^* \]  
\[ i = i^* \]

Solving (15) for $u_{eq}$ renders:

\[ u_{eq} = \frac{v - L i^*}{v - E} \]

To show that $u_{eq}$ takes the values only from 0 to 1 in sliding mode, the inequality (13) is solved as:

\[ -(E - v) < E + v - 2L i^* < E - v \]
Adding \(v-E\) to the inequality (18) renders:

\[
2(v - E) < 2(v - Li^{i'}) < 0
\]  

(19)

With \(v-E < 0\), dividing the inequality (19) by \(2(v-E)\) renders:

\[
0 < u_
u = \frac{v-Li^{i'}}{v-E} < 1
\]  

(20)

Since there is one switching manifold that is related to current control and \(\dot{s} = 0\) contains the current dynamics as shown in (1) that is controlled by SMC in sliding mode, (1) can be naturally dropped from the original system and only the voltage dynamics (2) is left for study. Plugging \(i\) in (16) and \(u_{eq}\) in (17) into (2) yields:

\[
v' = -\frac{1 - u_{eq}}{C}i' - \frac{1}{CR}v = -\frac{1}{C}(1 - \frac{v - Li^{i'}}{v-E})i' - \frac{1}{CR}v
\]  

(21)

(21) is rearranged as:

\[
Ri' = \frac{(CvE + v)(v - E)}{E - Li^{i'}}
\]  

(22)

\(i\) contains the integral term of the voltage error. It is inconvenient to analyze a dynamics system with integral terms. The practical way is to rearrange the dynamics equations and eliminate the integral term by differentiation. By the calculus quotient rule, both sides of (22) are derived with respect to time, resulting in:

\[
Ri'' = \frac{((CvE + v)'(v - E))(E - Li^{i'})}{(E - Li^{i'})^2} - \\
\frac{((CvE + v)(v - E))(E - Li^{i''})}{(E - Li^{i'})^2}
\]  

(23)

\[
= \frac{((CvE + v)'(v - E) + (CvE + v)v')(E - Li^{i'})}{(E - Li^{i'})^2} - \\
\frac{((CvE + v)(v - E))(-Li^{i''})}{(E - Li^{i'})^2}
\]

Rearranging (23) yields:

\[
Ri''(E - Li^{i'})^2 = \\
((CvE + v)'(v - E) + (CvE + v)v')(E - Li^{i'}) + L(CvE + v)(v - E)i^{i''}
\]  

(24)
3.2 Open Loop Transfer Function

With the open loop control, the reference input current is:

\[ i' = i_d = \frac{v_e}{E} \left( \frac{v_d}{E} - 1 \right) \]  

(25)

Plugging (25) into (22) results in:

\[ v_e \left( \frac{1}{E} v_d - 1 \right) \left( E - \frac{2L}{RE} v_e v_e' + \frac{L}{R} v_e' \right) = (C R v_e + v_e)(v - E) \]  

(26)

Let \( v_e \) be the equilibrium point of \( v_d \) and \( v \). Let \( v_d \) and \( v_{dd} \) be the perturbations of \( v \) and \( v_d \). Consequently, \( v = v_e + v_d \), \( v_e = v_{ee} + v_e \), \( \dot{v} = \dot{v}_e \) and \( \ddot{v} = \ddot{v}_{ee} \). Plugging these variables into (26) renders:

\[ \left( v_{dd} + v_e \right) \left( \frac{1}{E} v_{dd} - 1 \right) \left( E - \frac{2L}{RE} v_e + v_e \right) \dot{v}_{ee} + \frac{L}{R} \ddot{v}_e = (C R \dot{v}_e + v_e)(v_e + v_e - E) \]  

(27)

Dropping the high orders of \( v_d \), \( v_{dd} \), \( \dot{v}_d \), and \( \ddot{v}_d \) and any product of some of them, a linear equation is obtained as:

\[ g_1 \dot{v}_e + g_2 v_e = h_1 \ddot{v}_d + h_2 v_{dd} \]  

(28)

where \( g_1 = \left( \frac{2v_e}{E} - 1 \right) E \), \( g_2 = v_e \left( \frac{1}{E} v_e - 1 \right) \left( L - \frac{2L}{E} \right) \), \( h_1 = 2v_e - E \) and \( h_2 = C R (v_e - E) \). The transfer function of (28) is:

\[ H(s) = \frac{v_e(s)}{v_{dd}(s)} = \frac{g_1 s + g_2}{h_1 s + h_2} \]  

(29)

With \( E = 15 \text{ V} \), \( R = 30 \Omega \), \( L = 0.02 \text{ mH} \), \( C = 20 \mu \text{F} \), and \( v_e = -22.5 \text{ V} \) as the nominal parameters, (29) is written as \( H(s) = (-0.15s+60)/(0.0225s+60) \) whose frequency response is shown in Fig. 6. This open loop system passes the low frequency signal and amplifies the high frequency signal substantially. The open loop system has the undesirable positive gain margin. Thus, the system is sensitive to disturbances or parametric uncertainties. A closed-loop transfer function must be sought and analyzed.
3.3 Closed-Loop Transfer Function

Differentiating (6) with respect to time renders:

\[ i' = K_e \dot{e} + K_i e \]  

(30)

Differentiating (30) with respect to time renders:

\[ i'' = K_e \ddot{e} + K_i \dot{e} \]  

(31)

With \( e = v_e - v \), \( \dot{e} = v_e' - v' \) and \( \dot{e}' = v_e'' - v'' \), (30) in terms of \( v_d \) and \( v \) is written as:

\[ i' = K_p \dot{v}_d - K_p \dot{v} + K_i v_d - K_i v \]  

(32)

Rewriting (31) in terms of \( v_d \) and \( v \) renders:

\[ i'' = K_p \ddot{v}_d - K_p \ddot{v} + K_i \dot{v}_d - K_i \dot{v} \]  

(33)

Plugging (32) and (33) into (24) renders:

\[
\begin{align*}
R(K_p \dot{v}_d - K_p \dot{v} + K_i v_d - K_i v)(E - LK_p \dot{v}_d + \\
LK_p \dot{v} - LK_p v_d + LK_p v) &= \\
((CRv' + v')(v - E) + (CRv' + v')(E - LK_p \dot{v}_d + \\
LK_p \dot{v} - LK_p v_d + LK_p v) + L(CRv' + v)(v - E)(K_p \dot{v}_d \\
- K_p \dot{v} + K_i \dot{v}_d - K_i \dot{v})
\end{align*}
\]

(34)

Let \( v_e \) be the equilibrium point of \( v_d \) and \( v \). Let \( v_\delta \) and \( v_{d\delta} \) be the perturbations of \( v \) and \( v_d \).

Consequently, \( v = v_\delta + v_e \), \( v_d = v_{d\delta} + v_e \), \( \dot{v} = \dot{v}_\delta \), \( \dot{v}_d = \dot{v}_{d\delta} \), \( \ddot{v} = \ddot{v}_\delta \) and \( \ddot{v}_d = \ddot{v}_{d\delta} \). Plugging these variables into (34) renders:

Fig. 6. The magnitude and phase of the open loop transfer function \( H(s) \) with respect to the angular frequency under \( v_e = -22.5 V \).
\[ R(K_y \dot{y}_d - K_y y_d + K_y y_d - K_y y_d)(E - LK_y \dot{y}_d) + \\
L_C K_y y_d - LK_y y_d + LK_y y_d = \\
((CRv_y' + v_y')y_y + v_y - E) + (CRv_y' + v_y)v_y' \\
(E - LK_y \dot{y}_d + LK_y y_d - LK_y y_d + LK_y y_d) + \\
L(CRv_y' + v_y)(v_y + v_y - E)(K_y y_d \\
- K_y \dot{y}_d + K_y y_d - K_y y_d) \]

(35)

Dropping the high orders of \( v_d, v_d, v_d, v_d \), and \( v_d \), and any product of some of them renders a

linear equation as:

\[ a_2 v_d'''' + a_1 v_d'' + a_0 v_d = b_2 \dddot{v}_d'''' + b_1 \dddot{v}_d'' + b_0 \dddot{v}_d \]

(36)

where \( a_0 = RE^2 K_y \), \( a_1 = RE^2 K_y + E(2v_y - E) - Lv_y (v_y - E)K_y \), \( a_2 = CRE(v_y - E) - Lv_y (v_y - E)K_y \), 

\[ b_0 = RE^2 K_y, b_1 = -Lv_y (v_y - E)K_y + RE^2 K_y \quad \text{and} \quad b_2 = -Lv_y (v_y - E)K_y. \]

The transfer function of (36) is:

\[ G(s) = \frac{v_y(s)}{v_d(s)} = \frac{b_2 s^4 + b_1 s^3 + b_0 s^2}{a_2 s^4 + a_1 s^3 + a_0 s^2} = \frac{b_2}{a_2} + F(s) \]

(37)

where \( F(s) = b_2 \left( \frac{b_1 / b_2 - a_1 / a_0}{a_2} \right) s + \left( \frac{b_0 / b_2 - a_0 / a_1}{a_2} \right) \).

(37) is rewritten as:

\[ G(s) = \frac{1}{a_2} \frac{b_2 s^4 + b_1 s^3 + b_0}{s^4 + \left( \frac{a_1}{a_2} \right) s + \left( \frac{a_0}{a_2} \right) } \]

(38)

The denominator of (38) is:

\[ s^4 + \left( \frac{a_1}{a_2} \right) s + \left( \frac{a_0}{a_2} \right) = 0 \]

(39)

Let the desired denominator with the poles \( \lambda_1^* \) and \( \lambda_2^* \) be:

\[ s^4 - \left( \lambda_1^* + \lambda_2^* \right) s + \lambda_1^* \lambda_2^* = 0 \]

(40)

(39) and (40) are desired to be the same. Thus, one has:

\[ a_1 / a_2 = \left( \lambda_1^* + \lambda_2^* \right) \]

(41)

\[ a_0 / a_2 = \lambda_1^* \lambda_2^* \]

(42)

(41) and (42) result in a matrix equation for \( K_y \) and \( K_y \) as:
where $B = \begin{bmatrix} RE^2 + L v_e (E-v_e)(\lambda_1' + \lambda_2') & L v_e (E-v_e) \\ L v_e (v_e - E)\lambda_1'' \lambda_2'' & RE^2 \end{bmatrix}$ and $D = \begin{bmatrix} E(E-2v_e) + CRE(E-v_e)(\lambda_1' + \lambda_1') \\ CRE(v_e - E)\lambda_1'' \lambda_2'' \end{bmatrix}$.

The values for $K_p$ and $K_i$ are computed with the inverse matrix of $B$ as:

$$\begin{bmatrix} K_p \\ K_i \end{bmatrix} = B^{-1} D$$

(44)

If $K_p$ and $K_i$ are selected in the procedure as shown above, the system will be stable and robust, which will be demonstrated in Section 4 by simulations. With $E = 15 \, \text{V}$, $R = 30 \, \Omega$, $L = 0.02 \, \text{mH}$, $C = 20 \, \mu\text{F}$, and $v_e = -22.5 \, \text{V}$ as the nominal parameters, some pair of real negative poles are placed and it is noticed that the poles $\lambda_1' = -900$ and $\lambda_2' = -500$ are desirable. The related PI gains are $K_p = 0.00096866$ and $K_i = -23.5897$. Then one has:

$$G(s) = \frac{0.01635s^2 - 404.6s + 159200}{0.3538s^2 + 495.4s + 159200}$$

(45)

The frequency response of $G(s)$ is plotted in Fig. 7. The closed-loop system passes the low frequency signals but is capable of significantly filtering out the high frequency signals. Its phase margin is about 63 degrees and its gain margin is about -2 decibels. This stable closed-loop system is robust against disturbances and parametric uncertainties.

Fig. 7. The magnitude and phase of the closed-loop transfer function $G(s)$ with respect to the angular frequency under $v_e = -22.5 \, \text{V}$.

Next the closed-loop system is shown to have the nonminimum phase structure. Let a stable and strictly proper system be considered (i.e., the number of zeros is less than the number of poles). The step response
of the system is said to have an “undershoot” if it initially “starts off in the wrong direction” [20]. This is the nonminimum phase phenomenon. Such a system is in the nonminimum phase if and only if its transfer function has an odd number of real right-half plane zeros. Analysis of the zeros of the closed-loop transfer function $G(s)$ shows that the buck-boost converter has a nonminimum phase voltage output. The trajectory of the zeros of $F(s)$ as $v_e$ increases from -30 V to -5 V is plotted in the top pane of Fig. 8. All the zeros are positive. Thus $F(s)$ is a stable and strictly proper system and $F(s)$ is in the nonminimum phase. For example, with $v_e = -22.5$ V, one has $b_2/a_2 = 0.0462$ and $F(s) = (-427.5s + 151875)/(0.3538s^2 + 495.4s + 159230)$. The zero of $F(s)$ is 355.2632 and is in the right-half phase plane. For a unit step input, if the undershoot of $F(s)$ is less than -0.0462, the step response of $G(s)$ shown in the second pane of Fig. 8 will start from 0.0462, cross the zero line and then reach the smallest negative value before it converges to the positive unit value. In the bottom pane of Fig. 8, the response of $F(s)$ starts off in the wrong direction from zero. Finally, it is predicted that the output voltage of the buck-boost converter with the proposed cascaded controller has a nonminimum phase phenomenon if the reference voltage experiences a step change.

Fig. 8. The trajectory of the zeros of $F(s)$ and the step responses of $v_e(s)/v_e(s)$ and $F(s)$ under $v_e = -22.5$ V.
3.4 Transients with a Constant Reference Voltage

When the reference voltage is a constant, the transients of the output voltage can still happen if there is a step change for a parameter such as the input voltage, the resistance, the inductance or the capacitance. To ascribe these transients to the nonminimum phase structure of a converter as argued in the reference [12] is incorrect since using small perturbation theory, there is no way to develop a transfer function with a nonminimum phase structure. However, it is found out that the output voltage has an undershoot when the input voltage or the load resistance steps to a smaller value and that the output voltage has an overshoot when the input voltage or the load resistance steps to a larger value.

3.5 Operating Range of Reference Voltage

It is noticed that the reference voltage cannot be too small or too large. For example, if $v_d$ is too close to 0 and if the output voltage $v$ successfully tracks $v_d$, $v$ will also be too close to 0. Then, the inequality (13) could be easily violated due to any transients (e.g., nonzero initial error) since it has been proven that it is more ideal for $v$ to be a smaller negative value. Actually, if $v_d$ is less than about -8.0 V, the existence condition for sliding mode could be destroyed. Meanwhile, if $v_d$ is too small, the frequency response of $G(s)$ deteriorates. Fig. 9 shows the frequency response of $G(s)$ with $v_d = -30$ V (Note: $v_e = v_d = -30$ V since $v_{dd}$ is arbitrarily small but the values of $K_p$ and $K_i$ are based on $v_e = -22.5$ V). The phase margin is about 35 degrees and the gain margin is about 2.45 decibels. Neither is unacceptable. The closed-loop system is unstable. Finally, for the controller designed with the nominal parameters, the desired reference voltage is roughly from -29.0 V to -8.0 V.

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Fig. 9. The magnitude and phase of the closed-loop transfer function $G(s)$ with respect to the angular frequency under $v_d = -30$ V.
4. Simulation

The nominal values of the buck-boost circuit parameters for all the following simulations are $E = 15 \, V$, $R = 30 \, \Omega$, $L = 0.02 \, mH$, $C = 20 \, \mu F$, $v_d = -22.5 \, V$ and $i_d = 1.875 \, A$. The desired eigenvalues are $\lambda_1 = -900$ and $\lambda_2 = -500$. Thus, the nominal linear system is over damped. The corresponding PI gains are $K_p = 0.00096866$ and $K_i = -23.5897$ which are used in all the following simulations. The goal to be accomplished is that in the steady state, the actual output voltage $v$ tracks the reference voltage $v_d$ with a $\pm 0.1 \, V$ error band and the inductor current $i$ is up to $5 \, A$. The system responses under the following conditions are shown: 1) inductance and capacitance modeling uncertainties; 2) step change of input voltage; 3) step change of load resistance; 4) input voltage and load resistance with disturbances; 5) step change of reference voltage. The first four cases are to boost the input voltage and the reference voltage is always the same. The last case is to test the ability of the proposed controller for tracking a reference voltage when it experiences a step change. The transient behaviors for all the simulation results will be discussed according to the theories developed in Sections 3.3 and 3.4. The integration time step length to be used is 0.000001 seconds. Since this paper deals with only simulation and there is no A/D converter, 0.000001 seconds is also the sampling period. Hence, the minimum sliding mode pulse width is 0.000001 seconds or the maximum sliding mode switching frequency is 100 kHz. Such switching or sampling frequency is well achievable with modern technologies. Thus physical implementation of the proposed controller is practical. After sliding mode occurs and in the steady state, the tracking error caused by chattering is within $\pm 0.1 \, V$ for the output voltage. This tracking precision is also achievable with an integration time step length of 0.00001 seconds. A lower maximum switching frequency (e.g., 10 kHz or even less) can be used if a larger voltage tracking error is permitted [3, 21]. There are many advanced methods for reducing or eliminating chattering [9]. However, to pursue them is beyond the scope of this paper.

4.1 Inductance and Capacitance Modeling Uncertainties
In this simulation, modeling uncertainties are considered. With \( L = 0.024 \, mH \) and \( C = 40 \, \mu F \), the inductance varies by 20\% and the capacitance varies by 200\%. The top pane of Fig. 10 shows that the inductor current \( i \) converges to \( i_d = 1.875 \, A \). The mid pane of Fig. 10 shows that the output voltage \( v \) converges to \( v_d = -22.5 \, V \) after a little bit of oscillation. With the actual values of \( L \) and \( C \), the poles of the system (37) have changed to be a complex-conjugate pair with a negative real part, which are \( \lambda_1 = -299.28 + 373.72i \) and \( \lambda_2 = -299.28 - 373.72i \). The linear system (37) and thus the nonlinear system are under-damped. The output voltage goes less than -22.5 \( V \) and converges back to -22.5 \( V \) as shown in the mid pane of Fig. 10. The bottom pane of Fig. 10 shows the 0 or 1 sliding mode control signal. Sliding mode is reached at the time point of about 0.005 seconds and before that, it is the reaching phase. This simulation shows that the proposed controller is robust against modeling uncertainties.

### 4.2 Step Change of Input Voltage

In this simulation, in the first 0.025 seconds, \( E \) takes the nominal value 15 \( V \). At the time point of 0.025 seconds, \( E \) steps to 12 \( V \). At the time point of 0.05 seconds, \( E \) steps to 18 \( V \). The top pane of Fig. 11 shows that the inductor current \( i \) converges to \( i_d = 1.875 \, A \) in the first 0.025 seconds, \( i_d = 2.156 \, A \) in the second 0.025 seconds and \( i_d = 1.6875 \, A \) in the remaining time. The mid pane of Fig. 11 shows that the output voltage \( v \) smoothly converges to \( v_d = -22.5 \, V \) in the first 0.025 seconds since the system is over damped. At the time point of 0.025 seconds, it starts to goes greater than \( v_d = -22.5 \, V \) but soon converges back to \( v_d = -22.5 \, V \). When \( E \) steps from 15 \( V \) to 12 \( V \), the output voltage has an undershoot. At the time point of 0.05 seconds, it starts to goes less than \( v_d = -22.5 \, V \) but soon converges back to \( v_d = -22.5 \, V \). When \( E \) steps from 12 \( V \) to 18 \( V \), the output voltage has an overshoot. At the time point of 0.05 seconds, it starts to goes greater than \( v_d = -22.5 \, V \) but soon converges back to \( v_d = -22.5 \, V \).
12 V to 18 V, the output voltage has an overshoot. The bottom pane of Fig. 11 shows that the switching control signal is 1 during the reaching phase.

**4.3 Step Change of Load Resistance**

In this simulation, the load resistance is 30 Ω in the first 0.025 seconds, 20 Ω in the second 0.025 seconds and 40 Ω in the remaining time. The top pane of Fig. 12 shows that the inductor current $i$ converges to $i_d = 1.875 \, A$ in the first 0.025 seconds, $i_d = 2.8125 \, A$ in the second 0.025 seconds and $i_d = 1.4063 \, A$ in the remaining time. The mid pane of Fig. 12 shows that the output voltage $v$ smoothly converges to $v_d = -22.5 \, V$ in the first 0.025 seconds since the system is over damped. At the time point of 0.025 seconds, it starts to goes greater than $v_d = -22.5 \, V$ but soon converges back to $v_d = -22.5 \, V$. When $R$ steps from 30 Ω to 20 Ω, the output voltage experiences an undershoot. At the time point of 0.05 seconds, it
starts to go less than \( v_d = -22.5 \) \( V \) but soon converges back to \( v_d = -22.5 \) \( V \). When \( R \) steps from 20 \( \Omega \) to 40 \( \Omega \), the output voltage has an overshoot. The switching control signal is shown in the bottom pane of Fig. 12.

### 4.4 Input Voltage And Load Resistance With Disturbances

In this simulation, the input voltage source is mixed with a Gaussian distributed random signal with the 0 mean and the 1 variance. The load resistance is mixed with a Gaussian distributed random signal with the 0 mean and the 2 variance. These noisy signals are shown in the first two panes of Fig. 13. The inductor current, the output voltage and the sliding mode control are shown in the next three panes of Fig. 13. The inductor current smoothly converges to \( i_d = 1.875 \) \( A \). The output voltage smoothly converges to \( v_d = -22.5 \) \( V \). The proposed controller is strongly against disturbances and can filter high frequency noise signals, which is not achieved in [17]. Based on the frequency response of the closed-loop system as shown in Fig. 7, this performance is well expected because the closed-loop system can filter out high frequency noises.

**Fig. 13.** The inductor current, the output voltage and the sliding mode control for the input voltage and load resistance mixed with Gaussian noises.

### 4.5 Step Change of Reference Voltage
In this simulation, the reference voltage is -22.5 V in the first 0.025 seconds, -27.5 V in the second 0.025 seconds and -9.0 V in the remaining time. The top pane of Fig. 14 shows that the inductor current $i$ converges to $i_d = 1.875 \, A$ in the first 0.025 seconds, $i_d = 2.5972 \, A$ in the second 0.025 seconds and $i_d = 0.4800 \, A$ in the remaining time. The mid pane of Fig. 14 shows that the output voltage $v$ smoothly converges to $v_d = -22.5 \, V$ in the first 0.025 seconds since the system is over damped. At the time point of 0.025 seconds, it goes off in the wrong direction before it converges to $v_d = -27.5 \, V$. At the time point of 0.05 seconds, it starts to go less than $v_d = -27.5 \, V$ before it converges to $v_d = -9.0 \, V$. These undershoot phenomena are due to the nonminimum structure of the buck-boost converter as is proven in Section 3.3. The switching control signal is shown in the bottom pane of Fig. 14. One may wonder why this nonminimum phase phenomenon does not happen at the initial time of the simulation. The fact is: due to the structure of the buck-boost converter, regardless of the status of the switches, if the capacitor voltage starts from 0 V, the capacitor voltage (also equal to the output voltage on the load) cannot go greater than 0 V because it is impossible to change the polarity of the capacitor. However, when the capacitor voltage is at a certain negative level, indeed the capacitor can gain or lose some charges due to appropriate control of switching and thus the output voltage can go less or greater than that certain negative value correspondingly. Thus the nonminimum phase phenomenon can happen when the output voltage has been a certain negative value and it is changed to another level. It is noticed that in the bottom pane of Fig. 14, SMC switching frequency decreases substantially after $v_d$ jumps from -22.5 V to -27.5 V at the time point of 0.025 seconds. This demonstrates a benefit of SMC. Different from the conventional PWM method with

![Fig. 14. The inductor current, the output voltage and the sliding mode control for the step change of the reference voltage.](image_url)
the fixed switching frequency, sliding mode control can adjust its switching frequency dynamically according to its load conditions. For high power applications in which a significant amount of energy can be lost due to fast switching [1], sliding mode control may be desirable because its switching frequency can be significantly low.

5. Conclusion

This paper deals with the cascaded controller with PI control and SMC for a buck-boost converter. The physical model and the operating procedure of the buck-boost converter are studied. The nonlinear and linear closed-loop system equations are developed. Stability and robustness of the system are analyzed and demonstrated through the pole placement and frequency response. The PI parameters are scientifically selected. The nonminimum phase structure of the buck-boost converter is mathematically proved. “Undershoot” transients for step changes of the reference voltage are successfully predicted and simulated. The inner current loop stability is proved through the candidate Lyapunov function method. SMC in the inner current loop replaces the conventional PWM and is easy for implementation. Transients for step changes of the input voltage and load resistance are predicted when the reference voltage is constant. The operating range of the reference voltage is analyzed. A variety of operating conditions of the buck-boost converter are simulated. The simulation results show that the system with the proposed controller is stable and robust. The system can perform well under variation of input voltage and load resistance, disturbances of input voltage and load resistance, modeling uncertainties or step change of the reference voltage. The proposed approach can be easily generalized to other types of power converters. The future work includes extension of the proposed method to multiple input multiple output power converters and microprocessor implementation of the controller on a buck-boost converter.

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References


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