Quantum Paradigms of Psychopathology

Quantum Decision Theory for Computational Psychiatry

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ABSTRACT
Quantum probabilistic models (based on quantum decision theory) have recently been introduced into cognitive and behavioral sciences. However, to date, quantum decision theory has not been utilized in computational psychiatry, although mathematical models from neuroeconomics have already prevailed in neuropsychiatric research. We address, in the present paper, the importance of employing quantum decision theory in psychiatry. As an example, a possible application of quantum decision theory to autism research is focused. Future study directions in computational psychiatry employing quantum decision theory are discussed.

Key Words: quantum decision theory, computational psychiatry, neuroeconomics, probability judgment

1. Introduction
Recently, introducing computational and mathematical formalism into psychiatry research attracts much attention (Hasler, 2012; Montague et al., 2012; Sharp et al., 2012). Among mathematical models, models from neuroeconomics (Glimcher and Rustichini, 2004; Takahashi, 2009) have been employed in psychiatry and psychological medicine. For instance, Takahashi et al., (2008) demonstrated the irrationality observed in decision over time by depressed patients with a neuroeconomic model of intertemporal choice. Takahashi (2011) also proposed a neuroeconomic framework for the assessment of rationality in intertemporal decision-making in addicts, by incorporating nonlinear temporal cognition theory of intertemporal choice (Takahashi, 2005). On the other hand, in mathematical cognitive science, mathematical frameworks of “quantum decision theory” have been proposed to explain human probability judgment errors (Busemeyer et al., 2011; Cheon and Takahashi, 2010; Franco, 2009; Khrennikov, 2010). The basic idea of the quantum decision models is that, because mathematical characteristics of quantum probability (which is a generalization of classical Kolmogorovian probability theory) can capture the various aspects of human probability judgment, mathematical frameworks of quantum probability theory can be utilized to model human probability judgment.

Psychiatric studies reported that several types of psychiatric disorders are associated with probabilistic judgment and decision-making (Langdon et al., 2010; Ligneul et al., 2012; Morsanyi et al., 2010). Among these studies, Morsanyi et al.(2012) focused on the effect of autism on conjunction fallacy, a type of cognitive bias or heuristic judgment (Hertwig and Herzog, 2009), which has extensively been studied in cognitive psychology and behavioral economics (Tversky and Kahneman, 1983).
The conjunction fallacy is people's psychological tendency to ascribe higher probabilities to the conjunction of two events than to one of the single events. The well-known example of the conjunction fallacy problem is the Linda problem. The test is preceded by a brief personality sketch: “Linda was a philosophy major when she was a college student. She is bright and concerned with social justice”. After the personality sketch, the subjects are asked which is more probable (or likely) that “Linda is a bank teller” or “Linda is a feminist and a bank teller”. Most subjects judge more probable that “Linda is a feminist and a bank teller” than “Linda is a bank teller”. This probability judgment by typical subjects is incorrect in terms of standard probability theory in that a conjunctive event is less probable than its constituent events, in standard (classical) Kolmogorovian probability theory. After the discovery of this psychological phenomenon by Tversky and Kahneman (1983), extensive studies have been conducted in cognitive psychology, evolutionary psychology, and behavioral economics.

2. Quantum decision theory of probability judgment

In classical probability theory, the following “law of total probability” (LTP hereafter) can axiomatically be derived (e.g., Khrennikov, 2010):

\[ P(a_1) = P(b_0)P(a_1|b_0) + P(b_1)P(a_1|b_1) \]  
(Eq.1)

or equivalently,

\[ P(b_1) = P(a_0)P(b_1|a_0) + P(a_1)P(b_1|a_1) \]  
(Eq.2)

where conditional probabilities are defined as:

\[ P(b_1|a_1) = P(a_1 \text{ and } b_1)/P(a_1) \]  
(Eq.3)

\[ P(a_1|b_1) = P(a_1 \text{ and } b_1)/P(b_1) \]  
(Eq.4)

where \( P(x_i) (x=a,b, i=0,1) \) denotes event a (or b) occurs or not.

Note that these equations for conditional probability cannot be derived; these are definitions in classical Kolmogorovian probability theory (Khrennikov, 2010). From the definitions of the conditional probabilities, we get the following Bayes' theorem:

\[ P(b_0)P(a_1|b_0) + P(b_1)P(a_1|b_1) \]

or equivalently,

\[ P(b_1) = P(a_0)P(b_1|a_0) + P(a_1)P(b_1|a_1) \]

where \( P(x_i) (x=a,b, i=0,1) \) denotes event a (or b) occurs or not.

Thus, Franco (2009) speculated that conjunction fallacy: \( P(a_1 \text{ and } b_1) \geq P(a_1) \) (a1: bank teller, b1: feminist) occurs due to the violation of LTP (Cheon and Takahashi, 2012) for more complete quantum probability formalism).

In quantum probability theory, LTP is generalized into:

\[ P(a_1) = P(b_0)P(a_1|b_0) + P(b_1)P(a_1|b_1) + 2\sqrt{P(a_1)P(b_0)P(a_1|b_0)P(a_1|b_1)\cos \theta} \]  
(Eq.8)

\[ P(b_1) = P(a_0)P(b_1|a_0) + P(a_1)P(b_1|a_1) + 2\sqrt{P(a_1)P(b_0)P(a_1|b_0)P(a_1|b_1)\cos \theta}. \]  
(Eq.9)

where the newly appearing parameter \( \theta \) is a “quantum phase”, showing the quantum-like “interference effect” in human probability judgment (the interference effect is due to Born’s rule in quantum theory, see Franco (2009) for details). According to Franco (2009) and Cheon and Takahashi (2012), the quantum phase parameter can be utilized to parameterize the degree of irrationality in human probability judgment. It is to be noted that Cheon and Takahashi (2010) and Cheon and Takahashi (2012) generalized the quantum probability formalism (Equation 8-9) to include two quantum phases (which can also be characterized by a single effective quantum phase):

\[ P(a_1) = \frac{P(b_0)P(a_1|b_0) + P(b_1)P(a_1|b_1)}{1 + 2\sqrt{P(b_0)P(b_1|a_0)P(b_1|a_0)\cos \theta} + 2\sqrt{P(b_0)P(b_1)P(a_1|b_0)P(a_1|b_1)\cos \theta}} \]  
(Eq.10)

which may be useful for more complete investigations into neuropsychological processes underlying probability judgment errors.

3. Probability judgment in psychiatry research

Studies in psychological medicine and psychiatry revealed that some psychiatric patients or subjects with developmental disorders have altered probability judgment. Morsanyi et al (2010) reported that autistic subjects are less susceptible to conjunction constituent probability (see Franco, 2009 for the derivation):
fallacy. Morsanyi and colleagues speculated that people with autistic tendency may be less sensitive to the contextual information in the conjunction fallacy task. In this respect, the quantum phase parameter may be an indicator of psychological sensitivity to contextual information. Note that, in analogy with quantum physics where electron’s quantum phase is physically sensitive to the magnetic field, human probability judgment error in the context-information sensitive conjunction fallacy is identical to the well-known Aharonov-Bohm effect, at least in terms of a mathematical aspect, but not in terms of physics (Sakurai 1994, for a brief explanation of the Aharonov-Bohm effect). Consistent with interpretation, Morsanyi et al. (2010) argued that autistic subjects may have “decontextualized mind”, rather than more logical mind in comparison to non-autistic subjects. Regarding other psychiatric problems, schizophrenia (Langdon et al., 2010) and pathological gambling (Ligneul et al., 2012) were demonstrated to be associated with probabilistic judgment and decision making. Future computational psychiatric studies should examine the usefulness of quantum probability formalism in the investigations into reasoning processes by psychiatric patients (Selesnick and Owen, 2012, for the application of quantum logic in schizophrenia research).

4. Possible extension of quantum decision theory

Although the combination of accepting the definition of the conditional probability and rejecting the law of total probability is popular in quantum decision theory (Khrennikov, 2010; Franco, 2009), there is evidence of the violation of the formula of the conditional probability (Eq.3-4) (Zhao et al., 2009). In their experimental study, Zhao and colleagues (2009) observed that the subjective conditional probability judgment violates the definition of Kolmogorovian probability theory. Therefore, future studies should examine the validity of the following quantum-generalized conditional probability formula:

$$P(b_1|a_1):= P(a_1 \text{ and } b_1)/\left( P(a_1 | b_0) + P(b_1) P(a_1 | b_1) + 2\sqrt{P(a_1)P(b_0)P(a_1 | b_0)P(a_1 | b_1)(\cos \theta)} \right)$$

(Eq.11)

where the denominator of Eq. 3 is re-expressed by employing the quantum LTP (Eq. 8). This examination can help to answer whether the violation of conditional probability formula (and resulting violation of Bayes' theorem) can be explained by quantum decision theory.

5. Conclusions

In the present paper, we introduced the recently-developed quantum decision theoretical models and addressed the usefulness of future studies in computational psychiatry. An immediate application may conceivably be the examination of autistic subjects’ contextual information processing captured by the reduced susceptibility to conjunction fallacy. Quantum phase parameters could characterize autistic subjects’ altered neural processing or disconnection between higher-order association areas and the frontal lobe (Geschwind and Levitt, 2007). Furthermore, how male hormones such as testosterone (Takagishi et al., 2010) regulates quantum phases during probabilistic judgment and decision making should be examined to elucidate neuroendocrinological correlates of quantum decision making by autistic people. These types of investigation also help establish “molecular quantum decision theory”. By employing the extended quantum decision theory proposed above, we will be able to elucidate neuropsychological processes underlying maladaptive judgment and decision by psychiatric patients.
References


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