A New Approach for Applying Dynamic Exergy Analysis and Exergoeconomics to a Building Envelope

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Abstract:
The aim of this study is to formulate dynamic exergetic and exergoeconomic analyses for the building envelope. An exergy analysis specifies the magnitude, location and causes of the thermodynamic inefficiencies in a system, while an exergoeconomic analysis provides very useful information about monetary losses due to thermodynamic inefficiencies within different components of a system. Such information and insights are not available just from exergy and economic analyses. In other words, a dynamic exergoeconomic analysis of a building envelope reveals the flow of money inside the building. It identifies not only the locations but also the moments, when the system undergoes high monetary losses and offers reliable suggestions to improve the cost-effectiveness of a system. Moreover, the distribution of fuel costs (i.e. cost of heating and cooling energies) among different components of the system can be approximated by using this analysis. In our case study, we first simulate a building envelope of an office located in the E.ON Energy Research Center in Aachen, Germany. A dynamic Simulink-based model has been developed to carry out a simulation for one year and to obtain heating and cooling demands of the office. A novel approach has been demonstrated afterwards to conduct dynamic exergetic and exergoeconomic analyses. In the proposed approach, indoor air is defined as the reference environment and the temperatures at the system boundaries are considered to be: (1) inner and outer surface temperatures for the external wall, roof and window, (2) temperature of human body for the internal heat gains, and (3) supply temperature of heating and cooling energies. The results show that: (1) choosing indoor air temperature as the reference temperature decreases the complexity of the dynamic exergy/exergoeconomic analysis, (2) results from exergy and exergoeconomic analyses differ significantly during the heating period (3) although sometimes the total exergy destruction in the building rises, its monetary value might fall simultaneously, and (4) different results are obtained from the exergy and exergoeconomic points of view for some components.

Keywords:

1. Introduction
The building sector accounts for nearly one-third of the global energy supply. This large contribution is rising every year, due to the world’s growing population and the increasing tendency towards urbanization and modernization. Most of this energy supply is used to satisfy the comfort criteria inside buildings, which means to keep the room temperature around 21°C and 24°C in winter and summer, respectively. As the room temperature is close to the ambient temperature, the building’s energy demand is considered as a low quality (i.e. low exergy) demand, which can be supplied by a low quality energy source as well. In most of the buildings, however, this low exergy demand is satisfied using high quality (i.e. high exergy) sources such as electricity or fossil fuels. It is more appropriate to use such high quality sources for processes with higher energy-quality demands and benefit from the low quality energy sources by using them for heating/cooling purposes in buildings. Heat pumps, solar collectors, waste heat utilization etc. are examples of low exergy (LowEx) systems that have been implemented successfully in some buildings.
As indicated in [1] the major target of using LowEx systems is minimizing the exergy input to the buildings and consequently decreasing the total CO₂ emissions of the building stock, while obtaining a comfortable and healthy built environment. A review on LowEx heating and cooling systems has been carried out in [2], where the exergetic efficiency for LowEx heating/cooling systems is reported between 0.4% – 25.3%, while for the so-called greenhouses this value lies in the range of 0.11% – 11.5%. LowEx systems are described as an alternative perspective from passive house design in [3], with the advantage of higher flexibility and new possibilities for the design of high performance buildings. The authors show the possibility to decrease the temperature-lift in a heat pump, which can increase its performance drastically, by integrating different LowEx systems. According to [4], thermal comfort levels are reported to be higher in houses with low temperature heating systems than in houses with conventional heating systems. Moreover, LowEx systems enjoy the flexibility of using a variety of sources for heating purposes including district heat, bio-fuels, solar energy, gas, oil or electricity, and the user is not limited by the initial choices made in the planning phase unlike the traditional heating systems.

The first law of thermodynamics does not provide insights into the real thermodynamic inefficiencies in a system. The second law of thermodynamics, however, complements and enhances an energy balance by enabling calculation of both the true thermodynamic value of an energy carrier (i.e. exergy) and the real thermodynamic inefficiencies in a process (i.e. exergy destruction). In other words, exergy can be destroyed and the idea that something can be destroyed is useful in the design, analysis and optimization of energy systems [5].

So far the exergy concept has been applied to the built environment through some studies ([6-11]) for analyzing buildings and their Heating, Ventilation and Air Conditioning (HVAC) systems. Moreover, an Excel-based tool to conduct exergy analysis in buildings has been developed during an ongoing work for the International Energy Agency (IEA) formed within the Energy Conservation in Buildings and Community Systems Program (ECBCSP), Annex 37. This tool illustrates energy and exergy flows through different components of a building, from primary energy conversion to the heating system and building envelope [12]. In [13-15] energy and exergy analyses for space heating in buildings have been carried out based on this Excel-based pre-design tool.

An exergy analysis provides information about the magnitude, location and causes of inefficiencies in a system. This information is very useful from the thermodynamic point of view and is not available from energy analysis. Nevertheless, a successful completion of a thermal design requires an estimation of the major costs. Since exergy measures the true thermodynamic values of irreversibilities and due to the fact that the costs should only be assigned to commodities of value, it is meaningful to consider exergy as a basis for assigning costs in thermal systems [16]. Such analysis is called exergoeconomic analysis; it has been successfully applied to power plants, cogeneration plants, energy-intensive chemical plants and cryogenic plants. Exergoeconomic analysis has been also applied to the built environment by relatively few researchers during the recent years [17-23], but all studies are based on a static calculation.

Definition of the reference state is a principal issue in an exergy analysis. According to [24], when the state of a system is extremely different from that of the reference state, variations in properties of the reference state do not influence the results of an exergy analysis significantly (e.g., in power plants). On the contrary, when properties of a system are close to that of the reference state, results of an exergy analysis undergo strong variations depending on the definition of the reference environment [25]. This corresponds to the case of applying an exergy analysis to the heating and cooling systems in buildings, especially if their temperature levels are close to the ambient temperature (i.e. LowEx systems).

The majority of studies in the literature follow a steady-state approach to conduct an exergy analysis. The reference temperature, therefore, can be chosen as a fixed temperature such as the seasonal mean temperature, the annual mean temperature, the design temperature, etc. On the other hand, accurate estimations for energy and exergy flows in buildings are obtained only through a
dynamic analysis. In [26] the difference between steady-state analysis and dynamic exergy analysis for three different climates has been presented. Deviations of exergy flows obtained from a steady-state model as compared to a dynamic annual simulation are 9%, 93% and 44% for a cold climate, a hot and humid climate, and a temperate sea climate, respectively. Several possible reference temperatures are proposed in [27] for a dynamic exergy analysis: (1) the universe temperature, (2) the indoor air temperature, (3) the undisturbed ground temperature, and (4) the outdoor temperature. The authors of [27] recommend to use the outdoor air as the reference environment.

In the present paper, however, we selected the indoor air as the reference state and formulate dynamic exergy and exergoeconomic analyses for a building envelope based on this assumption. In section 2 we introduce our case study, which is one office in the main building of the E.ON Energy Research Center in Aachen, Germany. Section 3 represents a dynamic heat transfer model formulated for this case study. Detailed frameworks of dynamic exergy and exergoeconomic analyses are given in sections 3 and 4, respectively. Results and discussions on the advantages of a dynamic exergoeconomic analysis are shown in section 5, and finally, conclusions are drawn in section 6. To the best of the authors’ knowledge based on the open literature, this is the first study of a dynamic exergoeconomic analysis under the assumption of indoor air as the reference state.

2. Case Study

The main building of the E.ON Energy Research Center (E.ON ERC) has a net floor area of 7222 m² located in the Campus Melaten of RWTH Aachen University in Germany. Its state-of-the-art building technologies, multi-level usage and complex HVAC equipment makes it an ideal case study for various control and energy related researches. Detailed information about this building and its energy systems is given in [28,29].

The present study is part of preliminary work for developing an exergy-based control strategy to optimize the operation of HVAC systems in the above-mentioned building. In fact, the major target of this study is to develop a detailed procedure for conducting a dynamic exergoeconomic analysis, which can be implemented in the control system of the building in future. For this reason, we consider one office in this building in our case study and simulate it using measured data from Aachen (ambient temperature and solar radiation). As types of the offices are very similar in E.ON ERC, the same approach can be applied to the rest of the offices without a major modification. The only difference would be the orientation of external walls and windows, size of the offices and their occupancy density. Fig. 1 illustrates the layout of the E.ON ERC main building and location of our case study.

![Fig. 1. Layout of the 2nd floor of the E.ON ERC main building and location of our case study.](image)
3. Mathematical Model

3.1. Dynamic heat transfer model

In order to calculate the annual energy demand, a dynamic heat transfer model for thermal elements of the office has been formulated. This model is based on the lumped-capacitance method, where the walls are assumed to have uniform temperatures across their volumes. The model takes following terms into account: (1) conduction through the windows and walls, (2) convection due to the air movement inside and outside the building, (3) ventilation, (4) solar radiation through the windows, (5) absorption of solar radiation into the external walls and roof, (6) internal heat gains, and (7) heat storage capacity of the walls. In the proposed model, the room air temperature is assumed to be constant (21°C in winter and 24°C in summer), which means that heat storage capacity of the room is neglected in our study.

In real applications, however, the room air temperature fluctuates around a set point within an acceptable range and does not always remain constant. Subsequently, energy can be accumulated/dispersed in the room with respect to the rate of changes in its temperature. Deviation of the actual room temperature from the set point is strongly dependent on the control strategy, and on account of the fact that this study is one step before the analysis and comparison of different control systems in a building, we can ignore changes of the room air temperature at this step and assume a constant value for it. On the other hand, thanks to the selection of the indoor air as the reference environment, the storage effect of the room does not appear in an exergy analysis as the room has zero exergy content (see section 4). Therefore, neglecting the heat storage capacity of the room leads to a simplification of the problem and does not introduce any error into our study.

A mathematical model of dynamic heat transfers in different elements of the office has been formulated as seen in (1) – (3). An energy balance for the room is demonstrated in (1). Since the storage capacity of the room is neglected, the summation of all energy streams equals zero in this equation. Dynamic energy balances for the external wall and roof are formulated in (2) and (3), respectively. In these equations, the heat storage capacities are also taken into account, as denoted by $C_{ew}$ and $C_{rf}$.

\[
\frac{T_{ew}^m - T_r}{R_{ew} + R_{i,ew}} + \frac{T_{rf}^m - T_r}{R_{rf} + R_{i,rf}} + \left(\frac{T_{amb} - T_r}{R_{ew} + R_{i,ew} + R_{j,ew}}\right) + \dot{Q}_{\text{vent}} + \dot{Q}_{i,G} + \dot{Q}_{H/C} = 0
\]  

(1)

\[
C_{ew} \frac{dT_{ew}^m}{dt} = \left(\frac{T_r - T_{ew}^m}{R_{i,ew} + R_{ew}}\right) + \left(\frac{T_{amb} - T_{ew}^m}{R_{ew}}\right)
\]  

(2)

\[
C_{rf} \frac{dT_{rf}^m}{dt} = \left(\frac{T_r - T_{rf}^m}{R_{i,rf} + R_{rf}}\right) + \left(\frac{T_{amb} - T_{rf}^m}{R_{rf}}\right)
\]  

(3)

There are five unknown parameters ($T_{ew}^m, T_{rf}^m, \dot{Q}_{H/C}, T_{ew}^{\text{sky}}, T_{rf}^{\text{sky}}$) and only three equations. Therefore, two more equations are needed to calculate the surface temperatures of the external wall and the roof as shown below:

\[
\frac{T_{amb} - T_{ew}^{\text{sky}}}{R_{i,ew}} + \left(\frac{T_{ew}^m - T_{ew}^{\text{sky}}}{R_{ew}}\right) + \dot{Q}_{ew} - \dot{Q}_{\text{sky,ew}} = 0
\]  

(4)

\[
\frac{T_{amb} - T_{rf}^{\text{sky}}}{R_{i,rf}} + \left(\frac{T_{rf}^m - T_{rf}^{\text{sky}}}{R_{rf}}\right) + \dot{Q}_{rf} - \dot{Q}_{\text{sky,rf}} = 0
\]  

(5)
Here \( \dot{Q}^{\text{rad}}_{\text{sky,ew}} \) and \( \dot{Q}^{\text{rad}}_{\text{sky,rf}} \) correspond to heat flows from the surface of the building to the environment due to the thermal radiation, which can be obtained from (6) and (7).

\[
\dot{Q}^{\text{rad}}_{\text{sky,ew}} = \sigma \varepsilon_{\text{ew}} A_{\text{ew}} \left( T_{\text{ew}}^4 + 273.15 \right)^4 - T_{\text{sky}}^4 \tag{6}
\]

\[
\dot{Q}^{\text{rad}}_{\text{sky,rf}} = \sigma \varepsilon_{\text{rf}} A_{\text{rf}} \left( T_{\text{rf}}^4 + 273.15 \right)^4 - T_{\text{sky}}^4 \tag{7}
\]

As seen in the above equations, radiative heat transfer between the surface of the building and the environment occurs between two different temperatures: (a) surface temperature of the building and (b) sky temperature. The latter \( T_{\text{sky}} \) is, in fact, a fictitious parameter that we used in order to take into account the nighttime radiative cooling (i.e. sky thermal radiation to the building). In reality, some parts of the radiation from surface of the buildings to the environment turn back to the building depending on the clouds coverage and height and also the ambient relative humidity as explained in [30]. In our simulation, to avoid unnecessary complexities, instead of calculating the exact value of the sky downward radiation, an equivalent temperature for the sky is considered. This temperature is assumed to be 0°C in our case study during the entire period of simulation. Finally, a dynamic Simulink-based model has been developed to run a simulation for one year and to obtain the heating and cooling demands of the office.

For a better visualization of the heat transfer phenomena in our model, a thermal equivalent circuit is illustrated in Fig. 2.

![Thermal equivalent circuit of the heat transfer model of the case study.](image)

### 3.2. Known parameters of the model

The ventilation load of the office is calculated from (8). Note that in this equation, the mass flow rate of incoming fresh air to the office is obtained based on an air change ratio of 2 [h⁻¹].

\[
\dot{Q}_{\text{vent}} = \dot{m}_{\text{vent}} c_{\text{Air}} \left( T_{\text{amb}} - T_{r} \right) \tag{8}
\]

The amount of solar radiative heat fluxes, which are absorbed by the external wall and roof, and also transmitted to the room via the window are given in (9) – (11).

\[
\dot{Q}_{\text{rad}}^{\text{ew}} = \alpha_{\text{ew}} A_{\text{ew}} q_{\text{rad},w} \tag{9}
\]

\[
\dot{Q}_{\text{rad}}^{\text{rf}} = \alpha_{\text{rf}} A_{\text{rf}} q_{\text{rad},h} \tag{10}
\]
\[ \dot{Q}_{\text{win}}^{\text{rad}} = \tau_{\text{win}} A^{\text{win}} q_{\text{rad}, \tilde{t}}^{\text{}} \]  

(11)

Here \( q_{\text{rad}, h}^{\text{}} \) and \( q_{\text{rad}, t}^{\text{}} \) are the solar radiation per unit of area on horizontal and tilted surfaces, respectively. In our case study \( q_{\text{rad}, h}^{\text{}} \) is measured locally in the location of the building and \( q_{\text{rad}, t}^{\text{}} \) is determined based on \( q_{\text{rad}, h}^{\text{}} \) and equations given in [31].

The internal heat gains are estimated based on the working hours of the office (7:00 – 19:00), occupancy density, operation of electronic devices and lighting. Internal heat gains, as well as solar radiation and ambient temperature are illustrated for an entire period of the year 2015 (our simulation period) in Fig. 3. Other parameters used in the heat transfer model are given in Table 1.

Table 1. Known parameters of the dynamic heat transfer model.

<table>
<thead>
<tr>
<th>Thermal resistance ( mK/W )</th>
<th>Thermal capacity ( MJ/K )</th>
<th>Surface area ( m^2 )</th>
<th>Other Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{i,ew} )</td>
<td>13.7</td>
<td>( C_{ew} )</td>
<td>4.943</td>
</tr>
<tr>
<td>( R_{o,ew} )</td>
<td>6.9</td>
<td>( C_{rf} )</td>
<td>5.931</td>
</tr>
<tr>
<td>( R_{ew} )</td>
<td>303.7</td>
<td>( A_{\text{win}} )</td>
<td>2.51</td>
</tr>
<tr>
<td>( R_{rf} )</td>
<td>7.5</td>
<td>( A_{ew} )</td>
<td>8.60</td>
</tr>
<tr>
<td>( R_{o,rf} )</td>
<td>2.1</td>
<td>( A_{rf} )</td>
<td>18.75</td>
</tr>
<tr>
<td>( R_{\text{win}} )</td>
<td>97.9</td>
<td>( m_{\text{vent}} )</td>
<td>45 kg/h</td>
</tr>
<tr>
<td>( R_{rf} )</td>
<td>249.4</td>
<td>( c_{\text{air}} )</td>
<td>1005 J/kg K</td>
</tr>
<tr>
<td>( R_{i,rf} )</td>
<td>7.5</td>
<td>( \tau_{\text{win}} )</td>
<td>0.80</td>
</tr>
<tr>
<td>( R_{o,rf} )</td>
<td>2.1</td>
<td>( \alpha_{ew} )</td>
<td>0.75</td>
</tr>
<tr>
<td>( R_{\text{win}} )</td>
<td>97.9</td>
<td>( \alpha_{rf} )</td>
<td>0.75</td>
</tr>
<tr>
<td>( R_{rf} )</td>
<td>249.4</td>
<td>( \varepsilon_{ew} )</td>
<td>0.75</td>
</tr>
<tr>
<td>( R_{\text{win}} )</td>
<td>249.4</td>
<td>( \varepsilon_{rf} )</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Fig. 3. Solar radiation per unit of area over horizontal and tilted surfaces (\( q_{h}^{\text{}} \) and \( q_{t}^{\text{}} \)), ambient temperature (\( T_{\text{amb}} \)) and internal heat gains (\( Q_{i,G} \)) for our case study during the simulation period – year 2015. Time intervals of 1 min. are used in the simulation.
4. Exergy Analysis

4.1. Reference state

In this paper, the water content of indoor and outdoor air is assumed to be equal (i.e. humidity is neglected). As a consequence, the chemical exergy of air does not appear in an exergy analysis. Moreover, as the process of air conditioning in buildings takes place under atmospheric pressure, mechanical part of the physical exergy will also be the same for both indoor and outdoor environments. Therefore, only the thermal part of the physical exergy is needed to formulate an exergy analysis. In other words, selection of an appropriate reference state in this case corresponds to the selection of an appropriate reference temperature.

As already stated in section 1, there is no common agreement on a proper definition of the reference environment for a steady-state analysis. For a dynamic exergy analysis, nevertheless, ambient air temperature has been widely used as the reference temperature in a number of studies. Due to the fluctuations in the ambient air temperature, the exergy of each stream in a system, as well as the exergy content of components with heat storage capacity (e.g., walls) will change now and then, even if the thermodynamic state of the system does not undergo a change. Results of such an exergy analysis are strongly under the influence of outdoor air temperature and cannot be compared from time to time, because each exergy has been calculated based on a different reference temperature at each time step.

In this paper, the indoor air is defined as the reference state and a methodology to apply dynamic exergy and exergoeconomic analyses with regard to the proposed reference temperature has been demonstrated. Based on our proposed approach, energy streams have zero exergy at indoor air temperature, and non-zero (and positive) exergies at any other temperature level, either colder or warmer than the room. In other words, as the exergy content of indoor air equals zero, the directions of all exergies crossing room boundaries (see Fig. 4) are always towards the room, which makes the analysis much easier. In addition, as the exergy of outdoor air is always larger than the exergy of indoor air, no exergy loss can be defined for the system.

4.2. System boundaries

The system boundaries for the external walls, roof, windows and room are depicted in Fig. 4. The temperatures of these boundaries correspond to the:

- Inner and outer surface temperatures of the external walls, roof and windows.
- Supply temperature of heating and cooling energies ($T_{H/C}$), which is, in our case study, 34°C and 13°C for heating and cooling modes, respectively.
- Temperature of human body for internal heat gains ($T_{i.G}$). Although internal heat gains have other sources than the presence of people, its temperature is assumed to be 37°C for the sake of simplicity.

Fig. 4. System boundaries for the exergy analysis.
The advantage of selecting such system boundaries is that radiative and convective heat transfers on outer surfaces of the external walls and roof do not appear in an exergy analysis. In fact, the calculation of temperatures on these surfaces, from (4) and (5), already involves the influence of radiative and convective heat transfers.

\[ \dot{Q}_{\text{win}} = \frac{T_{\text{win}} - T_{\text{r}}} {R_{\text{win}}} \]
\[ \dot{E}_{\text{win}} = \dot{Q}_{\text{win}} \left( 1 - \frac{T_r + 273.15} {T_{\text{win}} + 273.15} \right) \]
\[ \dot{E}_{D,\text{win}} = \dot{E}_{\text{win}} - \dot{E}_{\text{win}}^{\text{out}} \]
\[ \dot{E}_{\text{win}}^{\text{out}} = \max \{ \dot{E}_{\text{win}}, \dot{E}_{\text{win}}^{\text{out}} \} \]
\[ \dot{E}_{\text{win}}^{\text{in}} = \min \{ \dot{E}_{\text{win}}, \dot{E}_{\text{win}}^{\text{in}} \} \]

\[ \dot{Q}_{\text{ext}} = \frac{T_{\text{ext}} - T_{\text{ext}}^{\text{m}}} {R_{\text{ext}}} \]
\[ \dot{E}_{\text{ext}} = \dot{Q}_{\text{ext}} \left( 1 - \frac{T_r + 273.15} {T_{\text{ext}}^{\text{m}} + 273.15} \right) \]
\[ \dot{E}_{D,\text{ext}} = \dot{E}_{\text{ext}} + \dot{E}_{\text{ext}}^{\text{out}} - \frac{dE_w}{dt} \]
\[ - \dot{E}_{\text{win}}^{\text{in}} = \max \{ \dot{E}_{\text{ext}}, \dot{E}_{\text{ext}}^{\text{out}} \} \]
\[ \dot{E}_{\text{ext}}^{\text{out}} = \min \{ \dot{E}_{\text{ext}}, \dot{E}_{\text{ext}}^{\text{in}} \} \]

\[ \frac{dE_w}{dt} = \frac{C_w}{\Delta t} \left( T_{\text{m}}^{k+1} - T_{\text{m}}^k \right) \]
\[ - (T_r + 273.15) \ln \left( \frac{T_{\text{m}}^{k+1} + 273.15} {T_{\text{m}}^k + 273.15} \right) \]

\[ \dot{Q}_{\text{room}} = \frac{T_{\text{room}} - T_{\text{r}}} {R_{\text{room}}} \]
\[ \dot{E}_{\text{room}} = \dot{Q}_{\text{room}} \left( 1 - \frac{T_r + 273.15} {T_{\text{room}} + 273.15} \right) \]
\[ \dot{E}_{D,\text{room}} = \dot{E}_{\text{room}} + \dot{E}_{\text{room}}^{\text{out}} + \dot{E}_{\text{vent}} \]
\[ \dot{E}_{\text{vent}}^{\text{vent}} = \dot{m}_{\text{vent}} c_p \left( T_{\text{vent}} - T_{\text{vent},\text{in}} \right) \]

\[ - (T_r + 273.15) \ln \left( \frac{T_{\text{vent}} + 273.15} {T_{\text{vent},\text{in}} + 273.15} \right) \]
4.3. Formulation of exergy balances

As explained in section 4.1, exergy always enters the room from its boundaries, because the exergy of indoor air is zero and all boundaries have higher exergies than it. This simple interpretation cannot be put on other components of a system, as their boundaries have non-zero exergies. Indeed, it is not clear which side of each component has a higher and which side has a lower exergy in this case. Therefore, the exergy values of heat streams at different boundaries of a component must be calculated at first. Then, according to values of calculated exergies, the direction of exergy flow in a component can be obtained, which is always from higher to lower exergies.

Based on the above-mentioned explanation, exergy balance for each component of a building envelope is formulated in Table 2. Note that the storage term (dEw/dt) appears only in exergy balances of the external walls and roof, as the heat storage capacity of the windows and rooms are neglected in our study. After all, the total exergy destruction in a building envelope is calculated from (12) through the summation of all exergy destructions in different components.

\[
\hat{E}_{D,\text{tot}} = \hat{E}_{D,r} + \hat{E}_{D,\text{ew}} + \hat{E}_{D,\text{rf}} + \hat{E}_{D,\text{win}} = \hat{E}_{\text{hi}} + \hat{E}_{\text{lg}} + \hat{E}_{H/C} + \hat{E}_{\text{wind}} + \hat{E}_{\text{ven}} + \hat{E}_{D,\text{ew}} + \hat{E}_{D,\text{rf}} + \hat{E}_{D,\text{win}}
\] (12)

5. Exergoeconomic Analysis

An exergy analysis provides desired information about the thermodynamic performance of a system and its components. Therefore, a system can be analyzed/improved from the thermodynamic point of view through an exergy analysis. However, there is a lack of information about monetary losses associated with exergy destruction (i.e. the thermodynamic inefficiencies). An exergoeconomic analysis provides effective assistance in identifying, evaluating, and reducing the thermodynamic inefficiencies and the costs in a thermal system. It improves our understanding of the interactions among system components and reveals opportunities for design improvements that might not be detected by other methods [32].

A complete exergoeconomic analysis consists of an exergy analysis, an economic analysis and an exergy costing. For buildings application, since an exergoeconomic analysis in most of the cases will be applied to an already-existing building and its HVAC systems, there will be no need to take equipment costs into account. This means only an exergy analysis and exergy costing would be sufficient, without an economic analysis. This is not true when the design of a building and the selection of its HVAC components are considered for a new system. In this situation, an economic analysis is necessary to find the optimal size and type of system components, as well as the optimal building design.

In the present paper, the building and its energy systems already exist. Thus, the capital investment and operating and maintenance (O&M) expenses are not included in the analysis. The intention of conducting an exergoeconomic analysis for a building envelope in this study is, in fact, not to change the system configurations, but to improve its operation.

5.1. Exergy costing

Exergoeconomic analysis is founded on the fact that exergy is the only rational basis for assigning costs to the irreversibilities within components of a system. Consequently, in exergy costing a cost is assigned to each exergy stream as shown in (13).

\[
\hat{C}_i = c_i \cdot \hat{E}_i
\] (13)

Here, \(c_i\) denotes the average cost per unit of exergy. In the present work, an exergoeconomic analysis has been applied to a building envelope, not to a complete energy chain. Therefore, the average costs per unit of cooling and heating exergies are not calculated in the present work and 1 $/MJ is assumed for both. Note that in reality the average costs per unit of cooling and heating exergies are not the same and differ from 1 $/MJ. An actual value of \(c_i\) can be obtained through application of an exergoeconomic analysis to the entire energy supply chain of a building.
One of the most important aspects of exergy costing is calculating the cost of exergy destruction in each component of a system as given in (14). The cost rate associated with exergy destruction is a hidden cost that can be revealed only through an exergoeconomic analysis.

$$\dot{C}_{D,k} = c_{D,k} \cdot \dot{E}_{D,k}$$  \hspace{1cm} (14)

Using (12) and (14) a so-called cost balance is formulated to obtain the total monetary losses associated with exergy destructions within each component of the system.

$$\dot{C}_{D,k} = c_{D,ht} \cdot \dot{E}_{ht} + c_{D,H/C} \cdot \dot{E}_{H/C} + c_{D,rad,w} \cdot \dot{E}_{rad,w} + c_{D,vent} \cdot \dot{E}_{vent}$$

$$+ c_{D,ew} \cdot \beta_{ew} \cdot \dot{E}_{ew} + c_{D,rf} \cdot \beta_{rf} \cdot \dot{E}_{rf} + c_{D,win} \cdot \dot{E}_{win}$$  \hspace{1cm} (15)

In buildings application, the average cost per unit of exergy destruction in a component \((c_{D,k})\) can be approximated by a share of fuel costs (i.e. the average cost per unit of heating or cooling exergies that is assumed 1 $/MJ in this study), which is supplied to this component in order to cover the exergy destruction within it. Equation (16) shows calculation of this average cost.

$$c_{D,k} = c_{H/C} \cdot \frac{(\beta_k) \cdot \dot{E}_{D,k}}{\dot{E}_{tot}}$$  \hspace{1cm} (16)

As seen in (17), during a heating period, exergy destructions due to solar radiation and internal heat gains are not considered as parts of \(\dot{E}_{tot}\). The reason is that these two exergies are in favor of the heating process and both are for free. In other words, exergy destructions caused by these two sources do not impose costs on the system. Therefore, \(c_{D,k}\) for solar radiation and internal heat gains is assumed to be zero during the heating period.

Storage effects of the walls that (sometimes) play a positive role and lead to less energy consumption (i.e. less exergy destruction), are taken into account as \(\beta\) in (15) – (17), which varies between 0 and 1, depending on the storage capacity of the wall.

Equation (15) is used in the case, where the goal of an exergoeconomic analysis is to find monetary losses in different components and processes of a building envelope. In other cases, where the total exergy destruction and the total monetary losses are required, an equivalent average cost per unit of total exergy destruction \((c_{D,tot})\) can be defined as shown in (18). Under such circumstances there is no need to find each contributor to the total exergy destruction.

$$\dot{C}_{D,tot} = c_{D,tot} \cdot \dot{E}_{D,tot}$$  \hspace{1cm} (18)

### 6. Results

#### 6.1. Results from the energy analysis

The results of solving dynamic heat transfer equations are illustrated in Fig. 5. The first and second diagrams display heating and cooling demands of the office, respectively. The third diagram shows temperatures at different parts of the external wall: (a) room side, (b) external side, and (c) average wall temperature in the middle. The last diagram is the same as the third one, but shows temperatures at different parts of the roof.

It is expected from a building with relatively high thermal resistances (i.e. low overall heat transfer coefficients) for the external wall, roof and windows, as given in Table 1, to have lower heating and cooling demands than what is shown in Fig. 5. The reasons that justify these somewhat higher...
demands of the simulated building than the already-existing building are: (a) considering a constant room temperature during the working hours as well as the nights and weekends, (b) excluding a possibility to recover heat through ventilation process, (c) assuming rather high values for air change ratio, especially during the nights and weekends, when no one is in the office, and (d) neglecting the heat storage capacity of the room. Although these assumptions lead to a deviation from expected heating and cooling demands of the building, they do not affect the methodology proposed for applying dynamic exergetic and exergoeconomic analyses to a building envelope.

Fig. 5. Results from energy analysis.

6.2. Results from the exergy analysis
Final results of a dynamic exergy analysis under the assumption of indoor air as the reference environment are depicted in Fig. 6. The first four diagrams illustrate time-varying exergy destructions as well as cumulative exergy destructions within different components of the system: (a) the room, (b) the window, (c) the external wall, and (d) the roof. The fifth and sixth diagrams represent the rate of changes in exergy content of the external wall and roof. Finally, the total exergy destruction and cumulative exergy destruction for the entire system are illustrated in the last diagram of this figure.

As seen in Fig. 6, the rate of changes in exergy content of the external wall and roof are not identical. One reason is that the thermal capacity of the roof is almost 20% larger compared to the thermal capacity of the external wall (see Table 2). Besides, the thermal resistance between the outer surface of the external wall and ambient air is around 3.3 times larger than that of the roof. Thus, the exergy level of the external wall will not be affected as much as the exergy level of the roof will be.
6.3. Results from the exergoeconomic analysis

Selected results from a dynamic exergoeconomic analysis for a building envelope are displayed in Figs. 7–10. Fig. 7 demonstrates one of the main results of the application of an exergoeconomic analysis to our case study. It shows different contributors to the sum of total exergy destruction in year 2015 (left diagrams) as well as the costs associated with each contributor (right diagrams). This figure also compares the results from exergy analysis and exergoeconomic analysis for both heating and cooling modes. As seen in this figure, the results of exergoeconomic analysis differ extremely from the results of exergy analysis during the heating period, while the difference in the cooling period is not significant. The reason lies in the fact that internal and solar heat gains, which are both in favor of the heating process, provide additional free sources of heat. As a result, the cost rates associated with exergy destruction due to solar radiation and internal heat gains become zero and other exergy destructions become more expensive. This does not happen to the system, when it is in the cooling mode, because none of the exergy destructions is in favor of the cooling process and all of them impose costs to the system.

Fig. 8 shows the ratio between the average cost per unit of total exergy destruction ($c_{D,tot}$), calculated from (18), and the average cost per unit of heating/cooling exergies ($c_{H/C}$). In the left diagram, variations of this parameter over the entire period of one year are presented, while the right-side diagram depicts changes within a period of one day during the heating mode (07-Dec-2015). As seen in the right diagram, the average cost per unit of total exergy destruction suddenly drop right after 8:00 AM, when occupancy density and solar radiation increase. Ultimately, a meaningful conclusion from Fig. 8 can be drawn: The cost rates associated with exergy destruction do not remain constant during a time period and fluctuate with time.
Fig. 7. Comparison of the sum of exergy destructions (for the entire one-year simulation) and cost rates associated with them for heating and cooling modes.

Another result of an exergoeconomic analysis is presented in Fig. 9, where the total exergy destruction (blue-solid line) is compared with its monetary value (red-dotted line). It can be concluded that the rate of changes in exergy destruction does not always have the same tendency as the rate of changes in its monetary values. For instance, between 9:00 AM and 11:00 AM the exergy destruction increases, while the cost rates associated with exergy destructions decrease. Moreover, according to this diagram, the exergy destruction is not an appropriate criterion to compare performance of a system at different times. For example, around 11:30 AM and 3:30 PM the system has almost the same exergy destruction, but the former imposes more costs as compared to the latter.

Fig. 8. Changes in the average cost per unit of total exergy destruction.
Fig. 9. A comparison between the total exergy destruction and its monetary value.

Another interesting result of this study is demonstrated in Fig 10. Here, performance of two components of the system (the roof and the window) has been compared from the exergetic and exergoeconomic points of view. The exergy destruction in the roof is always higher than the exergy destruction in the window during one day (23-Mar-2015), as shown in Fig. 10. This means that from the thermodynamic point of view the roof is less efficient. From the exergoeconomic point of view, however, the performance of the roof is not always worse than that of the window, which means that the monetary losses associated with exergy destruction in the roof are sometimes more and sometimes less than the monetary losses associated with the exergy destruction in the window.

Fig. 10. Comparison of the performance of the roof and the window from exergetic and exergoeconomic points of view.

7. Conclusions

In summary, we have demonstrated a novel framework for applying dynamic exergetic and exergoeconomic analyses to a building envelope. The advantage of exergoeconomic analysis is providing insights into the monetary losses associated with exergy destruction (i.e. thermodynamic inefficiencies) within different components and processes involved in a heating, cooling and air conditioning system of a building.

As the temperature levels of all heat streams inside a building envelope are very close to (or the same as) the reference temperature (either ambient temperature, as suggested by [27], or indoor air temperature, as proposed in this work), a correct formulation of exergetic and exergoeconomic analyses, which can lead to a rational interpretation of results, is a challenging problem. In the present study, indoor air appears to be a promising alternative to previously used reference environments (e.g., outdoor air) for doing a dynamic exergy analysis in a built environment. The authors believe that the selection of ambient air as the reference state does not appear to be a proper choice in buildings applications, as it results in a fluctuating exergy content of heat streams with constant temperature levels. Indoor air, on the contrary, offers a constant temperature as the reference temperature, which makes the analysis much easier and provides more reasonable results from exergy-based methods.

One essential conclusion that can be drawn from this paper is that the results from the exergetic and exergoeconomic analyses are significantly different during the heating period, as the monetary losses associated with exergy destructions due to solar and internal heat gains are zero. In other
words, although sometimes the exergy destruction rises, its monetary value can fall simultaneously (depending on the source of exergy destruction). Moreover, a comparison of different components might have different outcomes from the exergetic and exergoeconomic points of view.

This analysis, as mentioned earlier in section 2, is a preliminary work to prepare an exergy-based control strategy to optimize the operation of buildings energy systems. When a set of different offices, with different heating and cooling requirements are connected together and various sources of heating and cooling (such as a heat pump, a boiler, a chiller etc.) fulfill their demands, an exergoeconomic analysis becomes more interesting, as it provides a good platform to compare performance of different energy conversion units, heating/cooling devices, construction elements and heat and mass transfer processes inside each office. In this case, a model predictive controller can be proposed with the objective function of minimizing the sum of all monetary losses due to exergy destruction in a complete energy supply chain of the building.

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Nomenclature

\( A \)  surface area, m\(^2\)  
\( c \) average cost per unit of exergy, $/MJ  
\( C \) thermal capacity, MJ/K  
\( c_p \) specific heat at constant pressure, J/(kg K)  
\( \dot{C} \) cost rates associated with an exergy stream, $/h  
\( \dot{E} \) exergy flow rate, W  
\( \dot{m} \) mass flow rate, kg/s  
\( q^* \) radiative heat flux density per unit of area, W/m\(^2\)  
\( \dot{Q} \) heat flow rate, W  
\( R \) thermal resistance, mK/W  
\( t \) time, s  
\( T \) temperature, °C  

Greek symbols

\( \alpha \) absorption coefficient of the wall  
\( \beta \) coefficient corresponding to the heat storage capacity of the walls  
\( \varepsilon \) emissivity  
\( \sigma \) Stefan-Boltzmann constant, 5.67×10\(^{-8}\) W/(m\(^2\) K\(^4\))  
\( \tau \) transmissivity of glass of the window  

Subscripts and superscripts

\( \text{amb} \) ambient  
\( \text{D} \) destruction  
\( \text{ew} \) external wall  
\( \text{h} \) horizontal (surface)  
\( \text{ht} \) heat transfer  
\( \text{H/C} \) heating / cooling
i     internal, subscript for exergy and cost streams
in    incoming
i.G   internal heat gains
m     middle part (of the external wall and roof)
o     outside
out   outgoing
r     room
rad   radiation (or radiative)
rf    roof
s     surface
t     tilted (surface)
tot   total
vent  ventilation
win   window

References


