A SYSTEM-LEVEL MODEL REDUCTION TECHNIQUE FOR EFFICIENT SIMULATION OF FLEXIBLE MULTIBODY DYNAMICS

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Abstract. In flexible multibody dynamics, body-level model reduction is typically used to decrease the computational load of a simulation. Body-level model reduction is generally performed by means of Component Mode Synthesis. This offers an acceptable solution for many applications, but does not result in significant model reduction for systems with moving connection points, e.g. due to a flexible sliding joint. In this research, Global Modal Parametrization, a model reduction technique initially proposed for real-time control of flexible mechanisms, is further developed to speed up simulation of multibody systems. The reduction is achieved by a system-level modal description, as opposed to the classic body-level modal description. As the dynamics is configuration-dependent, the system-level modal description is chosen configuration-dependent in such a way that the system dynamics are optimally described with a minimal number of degrees of freedom. Moving connection points do not pose a problem to the proposed model reduction methodology. The complexity of simulation of the reduced model equations is estimated. The applicability to systems with moving connection points is highlighted. In a numerical experiment, simulation results for the original model equations are compared with simulation results for the model equations obtained after model reduction, showing a good match. The approximation errors resulting from the model reduction techniques are investigated by comparing results for different mode sets. The mode set affects the approximation error similarly as it does in linear modal synthesis.
1 INTRODUCTION

Body-level model reduction, such as linear modal synthesis, is used extensively in flexible multibody dynamics. A modal description of a body’s flexibility requires considering the contribution of the body’s dominant eigenmodes, and one static deformation pattern per interface degree of freedom (DOF): a DOF which could be loaded during the simulated scenario. This loading can be caused either by a constraint or by an external loading. The DOFs, on which a constraint acts, can vary as a function of the relative position and orientation of the connected bodies. This variability holds for almost all constraints typically encountered in multibody models, but for some types of joints it is more pronounced. Slider joints typically result in a highly variable connection interface between bodies. A classic positioning system typically consists of bodies connected through slider joints (e.g. the system shown in Fig. 1). Systems with many interface DOFs due to constraints, will be further referred to as systems with moving connection points.

Figure 1: The FlexCell pick-and-place machine: an example of a system with moving connection points

If the flexibility of bodies with variable connectivity needs to be taken into account, this poses a problem for the body-level model reduction by modal synthesis. Loads can be expected in many DOFs during the simulated scenario, and thus many static deformation patterns should be included in the modal description of the body flexibility. Ignoring the contribution of static deformation patterns can lead to a bad approximation in case of concentrated loads [1], such as typically encountered in multibody systems. Wasfy and Noor give an overview of modelling techniques for flexible multibody systems [2]. Current approaches to model systems with moving connection points are the aforementioned body-level modal synthesis [3], or using expensive non-reduced models [4]. To the authors’ knowledge, system-level model reduction has not yet been used to tackle this problem.
Even after body-level model reduction, the resulting set of equations is still a rather large set of differential-algebraic equations (DAE), especially for problems with moving connection points. Both the DAE-character of the model equations, and the number of degrees of freedom needed to accurately represent flexibility, prohibit fast simulation of these systems. There is however an increasing demand for real-time and faster-than-real-time simulation of flexible multibody systems, such as in Hardware/Human/Software-in-the-Loop and Model Predictive Control applications. Formulations based on relative generalized coordinates may lead to efficient simulation tools [5] but a number of loop closure kinematic constraints is then required to model complex parallel mechanisms, so that the resulting model is still computationally challenging. Relative coordinates also allow body flexibility to be incorporated, e.g. through modal synthesis, but this approach would also suffer from the multitude of required static deformation patterns in case of moving connection points.

Few techniques exist for system-level model reduction of flexible multibody models. Most system-level model reduction techniques for non-linear models build reduced models based on data obtained from user-defined numerical experiments of the original unreduced model, e.g. techniques based on neural networks [6] and techniques based on projection on a fixed vector space [7]. The resulting reduced models only offer an accurate approximation in scenarios similar to the numerical experiments on which they are based. For these techniques, a good approximation for all possible states of the system requires many numerical experiments and limits the computational efficiency of the reduced model equations.

In this research, Global Modal Parametrization, a model reduction technique for flexible multibody systems proposed by Brüls for controller design of flexible mechatronic systems [8], is developed further for the purpose of speeding up simulation of flexible multibody systems. The reduction of the model is achieved by projection on a curvilinear subspace [9] instead of the classically used fixed vector space. Using a fixed vector space for highly non-linear systems requires the inclusion of many vectors in the modal basis to span the state-dependent dominant dynamics of the system, i.e. the system’s dominant eigenmodes and relevant static deformation patterns, for any point of the configuration space. The curvilinear subspace, however, is defined by imposing that the tangent space spans exactly and only, the state-dependent dominant eigenmodes, the rigid body modes and low-frequency elastic eigenmodes, and the relevant static deformation patterns. In this way, the dominant dynamics of a highly non-linear system can be represented by a minimal number of coordinates. The system-level eigenmodes and static deformation patterns automatically satisfy the constraints imposed by the joints; Moving connection points do not induce additional difficulty.

The next section deals with the methodology of GMP. Section 3 explains how this methodology should be applied to minimize computational load during simulation of the system. The complexity of simulation of the reduced model equations is estimated and compared to simulation of the original model equations. Section 4 elaborates on the applicability of GMP to systems with moving connection points. A validation of the methodology is done by a numerical experiment in Section 5. The test case is tailored to show the possibilities of the methodology in systems with moving connection points. Simulation results for the original model equations are compared with simulation results for the model equations obtained after the proposed model reduction methodology. The latter require much less degrees of freedom to represent the studied dynamical phenomena as compared to unreduced and body-level reduced models. The approximation errors resulting from the model reduction techniques are investigated by comparing results for different mode sets.
2 GLOBAL MODAL PARAMETRIZATION

Irrespective of the type of coordinates used, the equations of motion of a (flexible) multibody system can be written as a (non-linear) second-order DAE in terms of its generalized coordinates \( q \):

\[
M(q) \ddot{q} + h(q, \dot{q}) + \nabla_q V + \Phi^T_q \lambda = g_q
\]

\[
\Phi(q) = 0
\]

In these equations:

- \( q \) is a vector of the \( n \) generalized coordinates, the value of \( q \) defines the configuration of the system.
- \( M(q) \) is the configuration-dependent mass matrix.
- \( \nabla_q V \) is the gradient of the potential energy. Only potential energy due to structural deformation is considered in this term. The generalized forces due to other potential energy sources (e.g., gravity) will be taken into account through the source term \( g_q \).
- \( g_q \) denotes the generalized forces due to external loads on a mechanical component.
- \( \Phi(q) = 0 \) expresses \( m \) kinematic holonomic constraints. Its gradient \( \Phi^T_q \lambda \) is assumed to be of full rank for all configurations \( q \).
- \( \lambda \) is a vector of \( m \) Lagrange multipliers associated with the constraints.
- \( \Phi_q \) is the matrix of constraint gradients, \( \Phi^T_q \lambda \) represents the reaction forces and moments enforcing the constraints.
- \( h(q) \) gathers the centrifugal and Coriolis inertia forces which are quadratic in \( \dot{q} \). Using the index summation convention, we have

\[
(h_i) = (\Gamma_{i}^{qq})_{ijk} \dot{q}_j \dot{q}_k
\]

where \( (\Gamma_{i}^{qq})_{ijk} \) is the Christoffel symbol of the first kind:

\[
(\Gamma_{i}^{qq})_{ijk} = \frac{1}{2} \left( \frac{\partial (M_{ij})_{k}}{\partial q_j} + \frac{\partial (M_{jk})_{i}}{\partial q_j} - \frac{\partial (M_{ik})_{j}}{\partial q_j} \right)
\]

The overall motion is decomposed into a large amplitude rigid body motion \( q^r \) and a small amplitude elastic displacement \( q^f \)

\[
q = q^r + q^f
\]

Both \( q \) and \( q^r \) satisfy the constraint equations Eqn. \( \Phi(q) = 0 \). \( q^r \) represents a non-deformed configuration. \( q^r \) will be further referred to as the rigid body motion. The term \( \nabla_q V \) in Eqn. \( M(q) \ddot{q} + h(q, \dot{q}) \) represents generalized forces due to elastic deformation. As only the elastic displacement \( q^f \) results in a change of the potential energy and as the small elastic deformation allow a linearization around the undeformed configuration \( q^r \), this term can be rewritten as:

\[
1^{\text{Non-holonomic constraints are not considered in this work.}}
\]
The number of rigid body degrees of freedom of the multibody system is referred to as $s$. The rigid body motion can be represented by $\theta$, which is a selection of $s$ coordinates out of the set of coordinates $q$. This representation of the system inevitably leads to singularities in the dead points corresponding to the coordinates $\theta$. If the system has dead points corresponding to the coordinates $\theta$, the analysis of the system should be limited to an area in which this representation is valid, i.e. away from the system’s dead points. It will further be assumed that this is the case. The invertible, sufficiently continuous coordinate transformation $\rho$, which maps $\theta$ to the rigid body configuration $q^r$, can be defined:

$$q^r = \rho(\theta)$$ (7)

Its Jacobian $\rho,\theta(\theta)$ can be interpreted as the matrix of rigid-body modes $\Psi^{q\theta}(\theta)$. Note that, for all values of $\theta$ and away from the system dead points, this matrix has maximal rank, its columns (the rigid-body modes) have finite length and are continuous for varying $\theta$. A vector is continuous if all of its elements are continuous.

$$\rho,\theta(\theta) = \Psi^{q\theta}(\theta)$$ (8)

The rigid body modes $\Psi^{q\theta}(\theta)$ respect the constraint equations by definition:

$$\Phi_{q,\theta} \Psi^{q\theta} \equiv 0$$ (9)

The degrees of freedom (DOF) $q$ can be partitioned in:

- $s$ rigid body DOFs $\theta$
- $n^g$ constrained DOFs $q^g$: the DOFs that don’t belong to $\theta$ and on which an external loading will be applied during the intended simulation
- the internal DOFs $q^i$: the remainder of the DOFs

The elastic deformation $q^f$ is considered as a deviation from the undeformed configuration $q^r$, which can be represented by $(n - m) - s$ independent coordinates $\hat{\delta}$ through an invertible, sufficiently continuous coordinate transformation $\sigma$.

$$q^f = \sigma(\theta, \hat{\delta}) \quad \sigma(\theta, 0) \equiv 0$$ (10)

$q$ has to satisfy the constraint equations:

$$\Phi(\rho(\theta) + \sigma(\theta, \hat{\delta})) = 0$$ (11)

Linearizing around an undeformed configuration $(\theta, \hat{\delta}) = (\theta, 0)$ and ignoring second order terms, results in:

$$\Phi_{q,\theta} \sigma_{\hat{\delta}}(\theta, 0) = 0$$ (12)

Under the small deformation assumption, $\sigma(\theta, \hat{\delta})$ can be approximated linearly:

$$q^f = \sigma_{\theta}(\theta, \hat{\delta}) \hat{\delta} = \Psi^{\hat{\delta}}(\theta) \hat{\delta}$$ (13)

$$\mathcal{V}_q = K^{q\theta}(q^f) q^f + O(q^f^2)$$ (6)
Note that, for all values of $\theta$ and away from the dead points, this matrix has maximal rank, its columns (the flexible modes) have finite length and are continuous for varying $\theta$.

Furthermore, $q^f$ can be approximated by projecting it on a (configuration-dependent) vector space $\Psi^{q\delta}(\theta)$. $\Psi^{q\delta}(\theta)$ is constructed as a subspace of $\Psi^{q\delta}(\theta)$ for each configuration. In addition, for all values of $\theta$ and away from the dead points, this matrix has to be of full rank, its columns (the flexible modes) need to have finite length and need to be continuous for varying $\theta$. In order to give a good approximation of the dynamics of the system, $\Psi^{q\delta}(\theta)$ should span the (configuration-dependent) dominant, system-level, flexible eigenmodes and static deformation patterns of the system for all values of $\theta$. Several possibilities for sets of modes are described by Craig [10] and satisfy the demands on the mode set $\Psi^{q\delta}$ stated above [9]. Brüls [8] selected the Hurty mode set. In this work, a slightly different mode set will be used:

$\Psi^{q\delta}(\theta)$ consists of:

- $\Psi^{q\gamma}(\theta)$: $n_g$ constraint modes $\Psi^{q\gamma}$ which are the static deformation patterns for a unit displacement at the constrained DOFs $q^g$ while the rigid body DOFs $\theta$ are kept fixed.

- $\Psi^{q\iota}(\theta)$: all normal eigenmodes with an eigenfrequency below a certain threshold with as boundary conditions: rigid body DOFs $\theta$ kept fixed and constrained DOFs $q^g$ kept free. The frequency threshold should be sufficiently higher than the maximal frequency in the spectrum of the excitation. In linear dynamics problems, this threshold is usually defined as 1.5 to 2 times the highest frequency in the spectrum of the excitation. Note that due to the variable dynamics, the eigenfrequencies are configuration-dependent. To consider inclusion of a certain eigenmode in the mode set, one should look at the lowest value of the eigenvalue for the range of configurations encountered in the intended simulation.

Both deformation patterns are evaluated for an undeformed system at rest.

This results in the total mode set:

$$\Psi^{q\delta}(\theta) = \begin{bmatrix} \Psi^{q\gamma} & \Psi^{q\iota} \end{bmatrix} \quad (14)$$

$q^f$ is thus approximated by:

$$q^f = \Psi^{q\delta}(\theta) \delta \quad (15)$$

Using Eq. (5), Eq. (7) and Eq. (15) the parametrization can be written as:

$$q = \rho(\theta) + \Psi^{q\delta}(\theta) \delta \quad (16)$$

For the sake of overview, the modal degrees of freedom are bundled in a vector of length $n_\eta$:

$$\eta = \begin{bmatrix} \theta \\ \delta \end{bmatrix} \quad (17)$$

The differentiation of the parametrization leads to

$$q_{\eta}(\theta, \delta) = \begin{bmatrix} \Psi^{q\theta}(\theta) + \frac{\partial \Psi^{q\delta}(\theta)}{\partial \theta} \delta & \Psi^{q\delta}(\theta) \end{bmatrix} \quad (18)$$

Introducing the parametrization in the model equations Eq. (1-4) and bearing Eq. (12) in mind result in [8]:

$$M^{\eta}(\eta) \ddot{\eta} + h^{q}(\eta, \dot{\eta}) + K^{\eta}(\eta) \eta = g^{\eta} \quad (19)$$
with

\[ M^{\eta \eta} = q^T \eta M^{qq} q_{\eta} \]  
(20)

\[ K^{\eta \eta} = q^T \eta K^{qq} \begin{bmatrix} 0 & \Psi^{q \delta} \end{bmatrix} \]  
(21)

\[ g^{\eta} = q^T \eta g^q \]  
(22)

\[ (h^\eta)_i = (\Gamma^{\eta \eta \eta})_{ijk} \eta_j \dot{\eta}_k \]  
(23)

The components of the new Christoffel symbol are easily obtained from Eq. (4) and Eq. (20):

\[ (\Gamma^{\eta \eta \eta})_{ijk} = \frac{\partial q_u}{\partial \eta_i} \frac{\partial q_v}{\partial \eta_j} \frac{\partial q_w}{\partial \eta_k} (\Gamma^{qqq})_{uvw} + \frac{\partial^2 q_u}{\partial \eta_i \partial \eta_j} \frac{\partial q_v}{\partial \eta_k} (M^{qq})_{uv} \]  
(24)

In conclusion, one could say the essence of the model reduction technique is twofold:

1. The rigid body dynamics are represented by a minimal number of coordinates \( \theta \).
2. The flexibility of the system is approximated by only considering the dominant (configuration-dependent) elastic eigenmodes and static deformation patterns.

### 3 USE OF GMP FOR SIMULATION

As both the parametrization of the rigid body position and the superimposed flexible deviation respect the constraint equations by definition (Eq. (9) and (12)), the model equations become a set of ordinary differential equations instead of the original set of differential-algebraic equations.

Furthermore, as the elastic deformation is approximated by a limited set of configuration-dependent modes, the number of degrees of freedom \( \eta \) is much smaller than the number of initial coordinates \( q \) and Lagrange multipliers \( \lambda \). Adding an additional vector to the configuration-dependent vector set \( \Psi^{q \delta}(\theta) \), and thus an additional DOF to \( \delta \), 1) increases the cost for computing the vector set \( \Psi^{q \delta}(\theta) \) and the assembly of the reduced model equations Eq. (19) and (24), 2) increases the accuracy of the reduced model equations as an approximation of the original model equations and 3) increases the computational load for simulation of the reduced model equations. A trade-off has to be defined by the user.

The reduced dimension of the problem, as well as the switch from DAE to ODE, make the projected system equations much cheaper to solve. However, assembling the reduced system equations requires a considerable effort and can generally only be done numerically. The cost of this assembly can outweigh the advantage of the small resulting set of equations. However, as all elements of the projected model equations are defined by smooth functions using inputs that vary smoothly with \( \theta \), the elements of the projected model equations are continuous themselves. If these elements are calculated for a discrete set of configurations \( \theta^{(k)} \) in a preparation phase, they can be calculated by interpolation for any value of \( \theta \) encountered during simulation. Increasing the number of discrete configurations composing the database for interpolation 1) increases the computational load during the preparation phase, 2) increases memory requirements to store the database and 3) decreases the interpolation error on the projected model equation elements and the backtransformation. Again, a trade-off has to be defined by the user. Linear
interpolation was used in this work for the sake of simplicity, though other more advanced inter-
polation methods are likely to give more accurate results. In real-time applications, at the
end of each time step the coordinates $\eta$ need to be transformed back to the coordinates $q$. This
phase will further be referred to as the back-transformation.

The interpolation ($O(n_\eta^3)$ operations), solving of the projected model equations ($O(n_\eta^3)$ operations) and back-transformation to the initial coordinates $q$ ($O(n_\eta n)$ operations) is significantly
cheaper per time step than simulation of the original model equations ($O((n+m)^3)$ operations).

Remarks:

- The back-transformation does not need to be performed for all coordinates $q$, in most
  applications only a limited number of coordinates are of interest.

- Solving a linear matrix equation of size $N$ and bandwidth $D$ is assumed to have a $O(N D^2)$
  complexity. Iterative solvers, with better computational complexity, are only advanta-
geous for large matrix equations [11]. Current multibody models are small from a numer-
ical linear algebra point of view (typically up to some thousands of DOFs).

The splitting up of the process in a preparation and simulation phase is described in [9]. In
the applications envisioned in this research, such as real-time simulation, gain in simulation
speed during actual simulation justifies an expensive preparation phase.

4 APPLICABILITY TO SYSTEMS WITH MOVING CONNECTION POINTS

To model a system with moving connection points using modal synthesis for body-level
model reduction, a static deformation pattern should be included in the modal description of the
beam for each DOF receiving loads due to constraints. Depending on the degree of variability
of the body-body connection this will be a large number of DOFs.

Both body-level modal synthesis as GMP require a static deformation pattern for each DOF
that is externally loaded. Note that in case of GMP 'external' means external to the system,
whereas in body-level modal synthesis it means external to the respective body. Interaction
between bodies due to connections is internal from GMP’s system-level point of view, thus
not requiring additional static deformation patterns. It is however external from the viewpoint
of each individual body, thus requiring an additional static deformation pattern for the modal
description of the bodies’ flexibility in body-level modal synthesis. The total number of degrees
of freedom of the system after body-level model reduction will be significantly higher than
the number of degrees of freedom needed to represent the system with system-level GMP-
reduction.

In the GMP-approach a limited number of modes are calculated for many configurations
in the preparation step, allowing the use of a limited number of degrees of freedom during
simulation. In the classic body-level model reduction many modes need to be calculated for a
single configuration: the flexible body at rest. GMP requires an expensive preparation, which
allows a cheap simulation of the resulting reduced model equations. In comparison to GMP,
the classic approach requires a cheaper preparation, but results in a more expensive simulation.
GMP will only result in net gain in computation time if the gain during the simulation phase
outweighs the additional computational load during the preparation phase: This will be true
if projected models computed during the preparation phase can be reused many times during
simulation, e.g. in repetitive scenarios for the rigid body movement $q^r$. For some applications
however, e.g. real-time simulation, cheap simulation during the simulation phase is vital and an
expensive preparation can be afforded, favoring GMP.
5 NUMERICAL EXPERIMENT

The model reduction technique is applied to a system consisting of a flexible beam undergoing an imposed motion through a fixed slider joint (see Fig. 2). This is a simplification of the flexible beam of the pick-and-place machine shown in Fig. 1. As the flexible beam moves through the slider joint, the slider joint constraint is imposed on a changing location on the beam: the connectivity and the dynamic properties of the system change. The system properties of this 2-dimensional problem are summarized in table 1. The slider joint is imposed by enforcing zero displacement and rotation at the slider location.

Length (m) | 0.53
--- | ---
Cross-section (m²) | 4.53E-4
Second moment of area (m⁴) | 1.427E-8
Density (kg/m³) | 7800
E (GPa) | 210

Table 1: System properties.

The beam has uniform properties and is modeled with 30 identical 2-noded beam elements, using cubic shape functions to represent lateral displacement due to bending. The rigid body coordinate \( \theta \) is defined as the position of the beam relative to the non-moving sliding joint (see Fig. 2). The imposed motion is a harmonic function, shown in Fig. 3. The flexible beam is loaded at its end by a lateral force (see Fig. 2). The loading is a smoothed step function, shown in Fig. 4. The loading increases harmonically up to 0.2 s, and then levels out at its maximal amplitude. Both imposed motion and loading were selected such that high-frequency components in the input are limited. At \( t = 0 \) the system is undeformed. Simulation results are calculated for the original set of DAE’s Eq. (1-2) and for the set of ODE’s obtained after projection Eq. (19), both describing the configuration-dependent lateral dynamics.

The algebraic equations Eq. (2) of the initial DAE Eq. (1-2) impose the use of an implicit solver which iterates within each time step until all equations are satisfied up to a user-defined tolerance. Although the ODE-character of the projected model equations also allows explicit solvers, the same implicit solver was used for simulation of both model equations: the generalized-\( \alpha \) method [12]. The solver parameters were chosen as \( \rho_\infty = 0.95 \) (a small amount of numerical damping at high frequencies), timestep \( h = 2e-5 \) s.

The convergence criteria for the original model equations were defined as:

\[
\begin{align*}
\text{abs} \left( M^{qq}(q) \ddot{q} + h^q(q, \dot{q}) + V_q + \Phi^T \lambda - g^q \right) &< \epsilon^q \quad (25) \\
\text{abs} (\Phi(q)) &< \epsilon^\Phi \quad (26)
\end{align*}
\]
All elements of $\epsilon^q$ were set to $1e-8$ N (resp. Nm), whereas those of $\epsilon^\Phi$ were set to $1e-10$ m (resp. rad). For a meaningful comparison of the simulation results of the original DAE and the projected model equations, the tolerance vector $\epsilon^q$ should be premultiplied analogously as the the original DAE is, resulting in the following convergence criterion:

$$ \text{abs} \left( \mathbf{M}^{\eta \eta}(\eta) \ddot{\eta} + \mathbf{h}^n(\eta, \dot{\eta}) + \mathbf{K}^{\eta \eta}(\eta) \eta g^{\eta} \right) < \text{abs} \left( \mathbf{q}^T_{\eta \eta} \epsilon^q \right) $$

(27)

With these settings the convergence criterion on the original dynamic equations Eq. (25) proved to be the more critical one of both Eq. (25-26). One should conclude that the simulation results of the projected model equations offer the same precision as the simulation results of the original DAE for those modal degrees of freedom which the reduced model can represent. Note that the satisfaction of the constraint equation of simulation results of the projected model equations, after back-transformation, depends in the general case on the precision of 1) the back-transformation calculated for a discrete set of configurations in the preparation phase, 2) the interpolation technique used in the back-transformation, and 3) the calculated and interpolated modes satisfying the constraint equations Eq. (12). In this case study, only the latter will
result in an error because the back-transformation is trivial.

Different configuration-dependent mode sets were investigated. In a first case, the mode set \( \Psi^{q\delta}(\theta) \) consisted of only the first eigenmode: the first bending mode in which the loaded part of the beam participates. This very limited mode set was selected to clarify the approximation errors of the method. Fig. 5 shows the time history of the lateral displacement of the loaded end for both simulations: 1) the unreduced model, i.e. the original set of DAE’s Eq. (1-2) and 2) the GMP-reduced model, i.e. the set of ODE’s obtained after projection on the configuration-dependent mode set Eq. (19). The approximation error of the reduced model simulation results is shown in Fig. 6. Two effects can be observed:

- The approximation error clearly shows a low-frequency component indicating an overestimation of the static stiffness. This is to be expected because the mode set is not statically complete [10]. The modal description lacks a static deformation pattern for the loaded DOFs, for this case the loaded tip. A static input cannot be fully represented by this modal description \( \Psi^{q\delta}(\theta) \).

- The approximation error also has a high-frequency component. The combination of input and non-linear effects clearly dynamically excites eigenmodes of higher eigenfrequency, which are not included in the modal description \( \Psi^{q\delta}(\theta) \).

![Figure 5: Simulation results for the lateral displacement of the loaded end](image)

Adding a static deformation pattern to \( \Psi^{q\delta}(\theta) \) for the loaded DOF gives a better approximation (see Fig. 7 and 8). The mode set \( \Psi^{q\delta}(\theta) \) becomes statically complete; The low-frequency content of the approximation error is reduced dramatically. The high-frequency component
however remains similar both in frequency content and magnitude; higher-frequency dynamics are still not ‘captured’.

In a last case, the configuration-dependent mode set $\Psi^\delta(\theta)$ consists of the first 2 eigenmodes in which the loaded part of the beam participates. These are actually eigenmodes 1 and 3 for the configurations encountered during the specified imposed rigid body motion. The second eigenmode is a bending mode in which the unloaded part of the beam participates. This mode does not contribute significantly to the total system response. The simulation results for this case can be seen in Fig. 9 and 10. The effect of an additional eigenmode is 1) a decrease of the low-frequency component of the error, and 2) a better representation of the higher-frequency dynamics. Adding more modes to the modal set increases the precision, but also results in a higher computational load. A trade-off has to be specified by the user. One can conclude that the GMP-description gives a good precision, while only requiring a very limited number of degrees of freedom.

6 CONCLUSIONS AND FUTURE WORK

A system-level model reduction methodology has been proposed which significantly reduces the number of degrees of freedom needed to accurately describe and simulate flexible multibody systems, and more in general sets of second-order differential-algebraic equations. The original degrees of freedom are projected on a configuration-dependent mode set. The proposed model reduction methodology is thus a system-level model reduction technique, as opposed to the classic body-level model reduction. The resulting projected set of equations is a low-dimensional set of ODE’s, which is considerably cheaper to solve than the original high-dimensional set of DAE’s. The assembly of the projected model equations is done in an off-line preparation.
phase for a discrete number of configurations of the system. During the actual simulation, the elements of the projected model equations are then obtained by cheap interpolation. The combined interpolation and solving of the projected model equations is significantly cheaper than simulation of the original model equations. In the applications envisioned in this research, such as real-time simulation, gain in simulation speed justifies an expensive preparation.

The proposed model reduction methodology is especially suited for systems with moving connection points. For such systems, constraints result in body loading in a multitude of DOFs. The classic body-level model reduction based on modal synthesis does not result in a significant reduction, as it requires a static deformation pattern for each of the possibly loaded DOFs. A modal description requires a static deformation pattern for each externally loaded DOF. Body-body interaction is external from a body-level point of view, whereas it is internal from a system-level point of view. The proposed model reduction methodology is based on a system-level modal description, and thus moving connection points do not pose a problem. The methodology is applied on a simplified system with a moving connection point. This numerical experiment shows the validity and the applicability of the methodology to systems with configuration-dependent dynamics, such as flexible multibody systems. The numerical experiment also indicates the applicability to systems with moving connection points: the proposed description gives a good precision, while only requiring a very limited number of degrees of freedom. Different system-level mode sets are investigated. The effect of the mode set on the approximation error is explained and is similar to the effects of the mode set on the approximation error in linear modal synthesis.

In future research, this model reduction methodology will be applied to systems in which the
configuration-dependency of the dynamics results from large relative rotations of bodies. The methodology will be implemented in a compiled programming language to allow comparison with existing computer packages. Both in the proposed model reduction procedure, as in the time integration solver, trade-offs between computational speed and accuracy have to be defined by the user. These trade-offs were not fully investigated in this research and will be addressed in the future.

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REFERENCES


Figure 9: Simulation results for the lateral displacement of the loaded end
tip: full: original high-dimensional DAE, dashed: low-dimensional ODE obtained after GMP-reduction (2 eigen-modes)


Figure 10: Approximation error for the lateral displacement of the loaded end (2 eigenmodes)
