❐ 73

Designing Intelligent Variable Structure Controller for HIV Infection

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Article Info	ABSTRACT
	Fuzzy adaptive controller is developed for HIV infection in which functions of the system are unknown. A non-affine nonlinear system is considered for the HIV infection dynamic model. The merits of the proposed method is as the stability of the closed-loop system (HIV + Controller), the convergence of the infected cells concentration rates to zero and the boundedness of the internal signal and infected cell concentration. The simulation results show the promising performance of the proposed method.

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1. INTRODUCTION

Nowadays some class of investigations have been done on HIV, it's biological model and control theory given to the reason that there are a few papers have been written on that lately. Some previous works show that infection in its first steps can be developed using nonlinear geometric control [1]. There are other examples which describe the procedure. In [2], time-delay feedback control using Lyapunov function has been used to stabilize the same HIV1 model in this paper. Other examples such as model based feedback [3], predictive control [4], optimal control [5], adaptive control [6, 7] are discussed. Also simple nonlinear model state space models by adding clinical results have been analyzes [8].

Because of tunable structure and the performance of the FAC, it is superior that of the fuzzy controller and adaptive control.Designing Impulsive control for HIV dynamic system as a class of nonlinear impulsive system is presented in [9]. Feedback linearization control is applied to HIV nonlinear dynamics in [10]. Designing nonlinear state estimator and model predictive controller based on observer is discussed in [11].

Using of the adaptive controller (FAC) has been fully studied as follow: To use the TS fuzzy systems for modelling of nonlinear systems and designing the controllers with guaranteed stability are presented in [12]. To model affine nonlinear systems and to designing stable TS based controllers have been employed in [13]. Designing of the sliding mode fuzzy adaptive controller for a class of multivariable TS fuzzy systems are presented in [14]. In [15], the non-affine nonlinear functions are first approximated by the TS fuzzy systems, and then stable TS fuzzy controller and observer are designed for the obtained model. In these papers, modelling and controller has been designed simply, but the systems must be linearizable around some operating points.

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[16] has considered linguistic fuzzy systems to design stable adaptive controller for affine systems based on feedback linearization and furthermore it has been considered that the zero dynamic is stable. To design designing stable FAC and linear observer for class of affine nonlinear systems are presented in [17]. Fuzzy adaptive sliding mode controller is presented for the class of affine nonlinear time delay systems in [18]. The output feedback FAC for class of affine nonlinear MIMO systems is suggested in [19]. [20] designed FAC for a class of affine nonlinear time delayed systems. The main incompetency of these papers is those restricted conditions.

FAC has been never applied to HIV Treatment. This paper proposes a new method to design an adaptive controller based on fuzzy systems for a class of non-affine nonlinear systems with the following properties: guaranteed stability, robustness against uncertainties and external disturbances and approximation errors, to avoid chattering, and finally convergence of the output error to zero.

The rest of the paper is organized as follows. Section 2 gives problem statement. To design Fuzzy adaptive controller is proposed in section 3. Section 4 shows simulation results of the proposed controller and finally section 5 concludes the paper.

2. PROBLEM STATEMENT

Consider the following non-affine nonlinear system

$$
\begin{cases}
\dot{x}_l = f_l(x) + g_l(x)u + d_l(t) & l = 1, 2, \ldots, n \\
y = x_n
\end{cases}
\tag{1}
$$

where $x = [x_1, \ldots, x_n]^T \in R^n$ is the state vector of the system which is assumed available for measurement, $u \in R$ is the control input, $y \in R$ is the system output, $f_l(x)$'s are unknown smooth nonlinear functions, $d_l(t)$ for $\ell = 1, 2, \ldots, n$ are bounded disturbance.

The control objective is to design an adaptive fuzzy controller for system (1) such that the system output $y(t)$ follows a desired trajectory $y_d(t)$ while all signals in the closed-loop system remain bounded.

Consider the following assumption in this paper.

Assumption: the function $g_n(x)$ satisfies the following condition:

$$
g_n(x) \geq g_{min} > 0 , \forall (x, u) \in R^n \times R
$$
$$
\frac{dg_n(x)}{dt} \geq g_{dm}
\tag{2}
$$

$g_{dm}, g_{min} \in \square$ is known and define later.

The desired trajectory and its time derivatives are all smooth and bounded. The external disturbance is bounded.

Define the tracking error as follow.

$$
e = y_d - y
\tag{3}
$$

Taking derivative of both sides of the equation (3) we have

$$
\dot{e} = \dot{y}_d - \left(f_n(x) + g_n(x)u + d_n(t) \right)
\tag{4}
$$

To construct controller, let v be defined as

$$
v = \dot{y}_d + ke + v'
\tag{5}
$$

By adding and subtracting the term $ke + v'$ from the right-hand side of equation (4), we obtain

$$\dot{e} = v - ke - v' - (f_n(x) + g_n(x)u + d_n(t)) \tag{6}$$

Invoking the implicit function theorem, it is obvious that the nonlinear algebraic equation $f_n(x) + g_n(x)u - v = 0$ is locally soluble for the input u for an arbitrary (x,v). Thus, there exists some ideal controller $u^*(x,v)$ satisfying the following equality for a given $(x,v) \in R^n \times R$:

$$f_n(x) + g_n(x)u - v = 0 \tag{7}$$

As a result of the mean value theorem, the nonlinear function $f_n(x) + g_n(x)u$ can be expressed around u^* as:

$$f_n(x) + g_n(x)u = f_n(x) + g_n(x)u^* + (u-u^*)g_n(x) = f_n(x,u^*) + e_u g_n(x) \tag{8}$$

Substituting equation (7) into the error equation (6) and using (8), we get

$$\dot{e} = -ke - e_u g_n(x) - d_n(t) - v' \tag{9}$$

In the following, a fuzzy system and classic controller will be used to obtain the unknown ideal controller.

3. FUZZY ADAPTIVE CONTROLLER DESIGN

In Section 2, it has been shown that there exists an ideal control for achieving control objectives. In this section, we show how to develop a fuzzy system to adaptively approximate the unknown ideal controller.

The ideal controller can be represented as:

$$u^* = f(z) + \varepsilon_u \tag{10}$$

Where $f(z) = \theta^* w(z)$, and $\theta^* = \begin{bmatrix} y^1 & y^2 & ... & y^M \end{bmatrix}$ and $w(x) = [w_1(x) \ w_2(x) \ ... \ w_M(x)]^T$ are consequent parameters and a set of fuzzy basis functions, respectively.

where $\mu_{A_i^l}(x_i)$ is the membership degree of the input xi to those fuzzy set and y^l is the point at which the membership function of fuzzy set of the output achieves its maximum value. Here, the sum-product inference and the center-average defuzzifier are used for fuzzy system.

ε_u is an approximation error that satisfies $|\varepsilon_u| \leq \varepsilon_{max}$ and $\varepsilon_{max} > 0$. The parameters θ_1^* are determined through the following optimization.

$$\theta_1^* = \arg \min_{\theta_1} \left[\sup \left| \theta_1^T w_1(z) - f(z) \right| \right] \tag{11}$$

Denote the estimate of θ_1^* as θ_1 and u_{rob} as a robust controller to compensate approximation error, uncertainties, and disturbance to rewrite the controller given in (10) as:

$$u = \theta^T w(z) + u_{rob} \tag{12}$$

In which u_{rob} is defined below.

$$u_{rob} = \frac{sign(pe)}{g_{min}}(u_r + g_{min}u_{ic} + \hat{v}') \tag{13}$$

In the above, u_{ic} compensates for approximation errors and uncertainties, u_r is designed to compensate for bounded external disturbances, and $\hat{v}'$ is estimation of v'. Define error vector $\tilde{\theta} = \theta - \theta^*$ and use (12) and (13) to rewrite the error equation (10) as:

$$\dot{e} = -ke - g_n(x)\left(\tilde{\theta}^T w + u_{rob} - \varepsilon\right) - d_n(t) - v' \tag{14}$$

Consider the following update laws.

$$\dot{u}_r = \frac{\gamma_{u_r}}{g_{\min}}\|pe\|, \qquad \dot{\hat{v}} = \frac{\gamma_v}{g_{\min}}\|pe\|, \qquad \dot{\theta} = \frac{\gamma_\theta}{g_{\min}}pew \;, \qquad \dot{u}_{ic} = \gamma_{uic}\|pe\| \tag{15}$$

where $\gamma_{u_r}, \gamma_v, \gamma_\theta, \gamma_{uic} > 0$ are constant parameters.

In following equation, $\lambda_{\max}(.)$ and $svd_{\max}(.)$ are maximum eigenvalue and maximum singular value decomposition, respectively.

<u>Lemma</u>: The following inequality holds if $\lambda_{\max}(Q) \geq -\dfrac{f_{dm}}{f_{\min}}\lambda_{\max}(P)$.

$$\frac{1}{g_n}e^T Qe + \frac{\dot{g}_n}{g_n^2}e^T Pe \geq 0 \tag{16}$$

For proof of the above lemma refer to [21].

Theorem: consider the error of the dynamical system given in (16) for the large scale system (1) satisfying assumption (1), the external disturbances satisfying assumption (3), and a desired trajectory satisfying assumption (2), then the controller structure given in (14), (15) with adaptation laws (17) makes the tracking error converge asymptotically to origin and all signals in the closed loop system be bounded.

Proof: Based on this assumption it has been considered the following function is Lyapunov function.

$$V = \frac{1}{2}\left(\frac{1}{g_n}e^T Pe + \frac{\tilde{u}_r^2}{\gamma_{u_r}} + \frac{\tilde{\theta}^T \tilde{\theta}}{\gamma_\theta} + \frac{\tilde{u}_{ic}^2}{\gamma_{u_{ic}}} + \frac{\tilde{v}^2}{\gamma_{\tilde{v}}}\right) \tag{17}$$

where $\tilde{\theta} = \theta - \theta^*$, $\tilde{u}_r = u_r - d_{\max}$, $\tilde{u}_{ic} = u_{ic} - \varepsilon_{\max}$, and $\tilde{v}' = \hat{v}' - |v'|$. The time derivative of the Lyapunov function becomes.

$$\dot{V} = \frac{1}{2}\left(\frac{1}{g_n}\dot{e}^T Pe + \frac{1}{g_n}e^T P\dot{e} + \frac{\dot{g}_n}{g_n^2}e^T Pe\right) + \left(\frac{\tilde{u}_r \dot{\tilde{u}}_r}{\gamma_{u_r}} + \frac{\tilde{\theta}^T \dot{\tilde{\theta}}}{\gamma_\theta} + \frac{\tilde{u}_{ic}\dot{\tilde{u}}_{ic}}{\gamma_{u_{ic}}} + \frac{\tilde{v}\dot{\tilde{v}}}{\gamma_{\tilde{v}}}\right) \tag{18}$$

Use (14), to rewrite above equation as:

$$\dot{V} = \frac{1}{2}\left(\frac{1}{g_n}e^T(k^T P + Pk)e + \frac{\dot{g}_n}{g_n^2}e^T Pe\right) + \frac{Pe}{g_n}\left(-e_u g_n - d_n(t) - v'\right) + \left(\frac{\tilde{u}_r \dot{\tilde{u}}_r}{\gamma_{u_r}} + \frac{\tilde{\theta}^T \dot{\tilde{\theta}}}{\gamma_\theta} + \frac{\tilde{u}_{ic}\dot{\tilde{u}}_{ic}}{\gamma_{u_{ic}}} + \frac{\tilde{v}\dot{\tilde{v}}}{\gamma_{\tilde{v}}}\right) \tag{19}$$

Use (13), to rewrite (19) as follow.

$$\dot{V} \le -\frac{1}{2g_n}e^T Q e - \frac{\dot{g}_n}{2g_n^2}e^T pe + \frac{\|pe\|\|v'\|}{g_{min}} + \frac{\|pe\|}{g_{min}}d_{max} - pe\tilde{\theta}_1^T w_1 + pe\varepsilon$$

$$-\frac{\|pe\|}{g_{min}}(u_r + g_{min}u_{ic} + \hat{v}') + \left(\frac{\tilde{u}_r\dot{\hat{u}}_r}{\gamma_{u_r}} + \frac{\tilde{\theta}^T\dot{\hat{\theta}}}{\gamma_\theta} + \frac{\tilde{u}_{ic}\dot{\hat{u}}_{ic}}{\gamma_{u_{ic}}} + \frac{\tilde{v}\dot{\hat{v}}}{\gamma_{\tilde{v}}}\right) \tag{20}$$

Using assumption (1) yields $1/g_n \le 1/g_{min}$, to rewrite (20) as follow.

$$\dot{V} \le -\frac{1}{2g_n}e^T Q e - \frac{g_n}{2g_n^2}e^T pe - \frac{\|pe\|}{g_{min}}\underbrace{(u_r - d_{max})}_{\tilde{u}_r} - \frac{\|pe\|}{g_{min}}\underbrace{(\hat{v}' - |v'|)}_{\tilde{v}}$$

$$-\|pe\|\underbrace{(u_{ic} - \varepsilon_{max})}_{\tilde{u}_{ic}} - pe\tilde{\theta}_1^T w_1 + \left(\frac{\tilde{u}_r\dot{\hat{u}}_r}{\gamma_{u_r}} + \frac{\tilde{\theta}^T\dot{\hat{\theta}}}{\gamma_\theta} + \frac{\tilde{u}_{ic}\dot{\hat{u}}_{ic}}{\gamma_{u_{ic}}} + \frac{\tilde{v}\dot{\hat{v}}}{\gamma_{\tilde{v}}}\right) \tag{21}$$

Using (15), the above inequality rewrites as:

$$\dot{V} \le -\frac{1}{2}\left(\frac{1}{g_n}e^T Q e + \frac{\dot{g}_n}{g_n^2}e^T Pe\right) \tag{22}$$

Use the lemma, $\dot{V} \le 0$ are satisfied. Using Barbalet's lemma, it is guaranteed the tracking error asymptotically to the neighborhood of the origin. Furthermore, boundedness of the coefficient parameters is guaranteed. It completes the proof.

4. SIMULATION RESULTS

In this section, we apply the proposed fuzzy model reference adaptive controller to HIV infection model [33].

$$\begin{cases} \dot{x}_1 = 0.001(1-u)x_1x_2 - 0.24x_1 + d(t) \\ \dot{x}_2 = s - 0.02x_2 - 0.001(1-u)x_1x_2 \\ y = x_2 \end{cases} \tag{23}$$

where x_2 is healthy cells concentration, x_1 shows infected cells concentration, and $d(t) = 0.5\sin(20\pi t)$ is bounded external disturbance. The control objective is to make the output of the system track the desire trajectory.

We consider the desired value of the output is $y_d = \exp(-\beta t)$. Now we applied the proposed controller defined in (12), (13). Let $x = [x_1, x_2]^T$, $z = [x_1, x_2, v]^T$ and v are defined over $[-40, 40]$. Each fuzzy system has 6 Gaussian membership functions over the defined sets. We set all the initial value of $\theta(0)$, $\hat{u}_r(0)$, $u_{ic}(0)$, and $\hat{v}'(0)$ at zero. Furthermore, it has been considered that $f_{min} = 0.1$, $\gamma_{u_r} = 30$, $\gamma_\theta = 4$, $\gamma_{uic} = 10$, $\gamma_{\hat{v}'} = 2$. The first state of the system has been shown in figure (1).
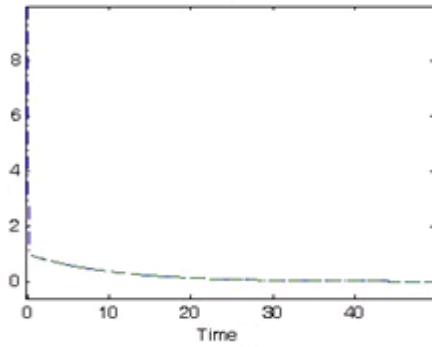
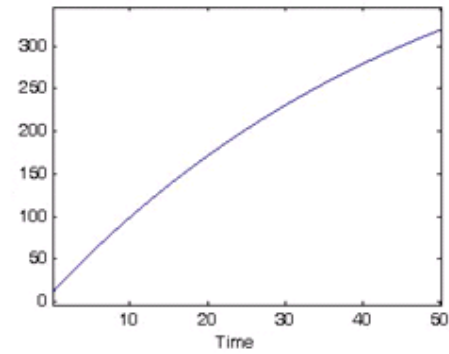Figure 1. The first state of the system

Figure 2. Response of the output

The output (second state) of the system is shown in figure (2). As shown in figure (2), it is obvious that the performance of the proposed controller is promising. The error between the output of the system and the desired value is shown in figure (3).
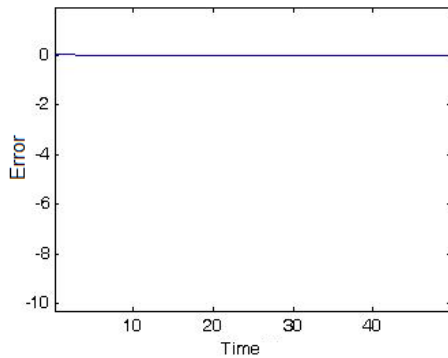

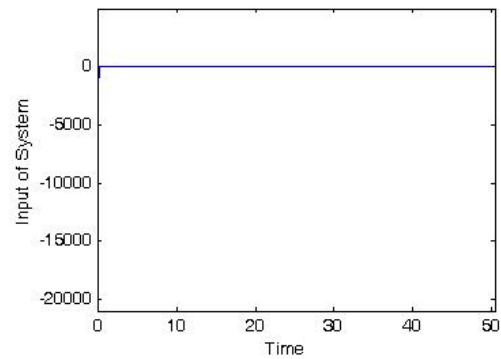Figure 3. The error between the output and the desired value

Figure 4. Control input u

Figure (4) shows the input of system. The input of the system mentioned in figure (4) after the transient time is shown in the following figure.
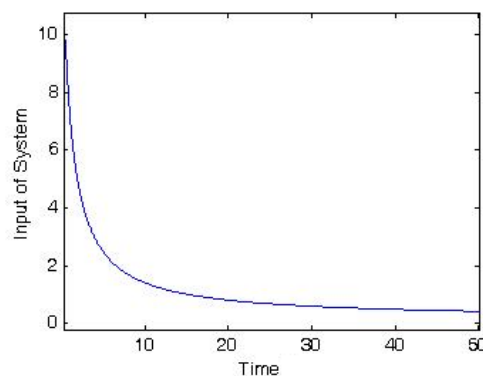

Figure 5. Control input after transient time

Till now, the fuzzy adaptive controller has been never applied to HIV dynamic model for treatment.

5. CONCLUSION

This paper proposes a model reference adaptive controller for HIV treatment. Fuzzy systems are one of the most powerful tools in the controller design procedure. It has been shown that the derived adaptation laws guaranty the stability of closed-loop system, and asymptotic convergence of the infected cell concentration to zero. Robustness against external disturbances and approximation errors and relaxing the conditions are the merits of the proposed controller.

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