Reflector Antenna Discrete Distortion Determination: An Iterative-Field-Matrix Solution

José Ángel Martinez-Lorenzo, Carey M. Rappaport
CenSSIS / Department of Electrical and Computer Engineering, Northeastern University, MA 02115

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Abstract

The idealized shapes of satellite reflector antennas are often distorted once they are placed in orbit. The performance of such antennas can be improved by identifying the locations and amount of their surface distortions and then by correcting them using active surface distortion or array feeding. This work presents a method to determine the required discrete surface distortions to correct errors. The algorithm starts by discretizing the entire reflector surface into triangular patches, then by finding a linear relationship between the local distortion and the difference of the distorted and undistorted farfields patterns. A linear system of equations with the distortions as unknowns results when the number of observation points is the same as the number of triangular patches a unique solution is achieved without iteration. The conditioning of the system as a function of the observation direction and the spatial resolution is discussed. The method has been applied to determine thermal, gravitational and random distortions on a reflector antenna.

Introduction

• Satellite **reflector antennas** are often **distorted** from their idealized shapes once they are placed in **orbit**. This distortion may be due to thermal gradients, mechanical stresses, damage from launch, or collisions with foreign objects. Variations from the designed shape lead to antenna performance degradation, compromising the antenna radiation pattern. The shape distortions can often be **dynamically corrected** with on-board actuators or feeding arrays, but it is essential to determine exactly where and how great the distortions are.

• The proposed method **approximates** the nonlinear **association** between the **position** of a patch of reflector **and** the contribution to the **field radiated** by this patch it in the farfield. By assuming a small positional perturbation, the association is **linearized**, and thus the perturbed position can be **inverted** given the perturbed pattern. Repeated application of this small perturbation inversion leads to an **iterative procedure** which quickly approaches the final patch position.

Analysis Technique

• The forward model is based on the **radiation of electric currents** in a free space [1]:

$$\mathbf{E^s}(\mathbf{r}) = -jkZ_0 \int_{\Omega} [g_1 \mathbf{J}(\mathbf{r}') - g_2(\mathbf{J}(\mathbf{r}')\cdot\hat{\mathbf{R}})\hat{\mathbf{R}}] G d\Omega \tag{1}$$

$$g_1 = 1 - 1/(kR)^2 - j/(kR)\ ,\ g_2 = 1 - 3/(kR)^2 - j3/(kR)$$

$$\mathbf{R} = \mathbf{r} - \mathbf{r}'\ ,\ R = |\mathbf{R}|\ ,\ \hat{\mathbf{R}} = \mathbf{R}/R$$

$$G = \frac{exp(-jkR)}{4\pi R}\ ,\ k = \omega\sqrt{\mu_0\epsilon_0}\ ,\ Z_0 = \sqrt{\mu_0/\epsilon_0}$$

• The **Physical Optics** (PO) approximation considers the induced currents to be proportional to the magnetic incident field to the PEC

$$\mathbf{J}(\mathbf{r}') \approx \mathbf{J_{po}}(\mathbf{r}') = 2\ \hat{\mathbf{n}}' \times \mathbf{H^{inc}}(\mathbf{r}') \tag{2}$$

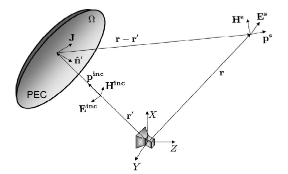
To solve (1), the surface is divided in **triangular facets**, the electric field contribution of the subdomain i in the observation point l can be expressed as

$$\mathbf{E}^{\mathbf{s}}_{l,i} = \mathbf{E^s}(\mathbf{r}_l, \mathbf{r}'_i) =$$
$$-jkZ_0 \frac{exp(-jkr_l)}{4\pi r_l} \int_{\Omega_i} [\mathbf{J}(\mathbf{r}'_i) - \hat{\mathbf{r}}_l(\mathbf{J}(\mathbf{r}'_i)\cdot\hat{\mathbf{r}}_l)] exp(jk\ \mathbf{r}'_i \cdot \hat{\mathbf{r}}_l) d\Omega \tag{3}$$

$$r_l = |\mathbf{r}_l|\ ,\ \hat{\mathbf{r}}_l = \mathbf{r}_l / r_l$$

• The **total scattered** field is determined by the **summation** of the contributions of each **subdomain**

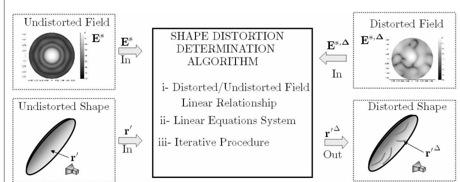
$$\mathbf{E^s}_l = \mathbf{E^s}(\mathbf{r}_l) = \sum_{i=1}^{N_\Omega} \mathbf{E}^{\mathbf{s}}_{l,i} \tag{4}$$



Shape Distortion Determination Algorithm

• The ideal **undistorted shape**, given by position vector $\mathbf{r}'$, produces the **scattered field** $\mathbf{E^s}$. Distortions to this ideal shape produce **distorted scattered field** $\mathbf{E^{s,\Delta}}$.

• **The problem:** Determine the **distorted surface**, given by a position vector $\mathbf{r}'_\Delta$, that produces the distorted scattered field $\mathbf{E^{s,\Delta}}$.



Distorted/Undistorted Fields Linear Relationship

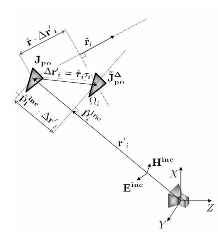
• A local displacement produces **new induced currents** that can be approximated as

$$\tilde{\mathbf{J}}^{\mathbf{\Delta}}_{\mathbf{po}}(\mathbf{r}'_i + \Delta\mathbf{r}'_i) = \mathbf{J_{po}}(\mathbf{r}'_i)\ exp(-jk\ \hat{\mathbf{p}}^{\mathbf{inc}}_i \cdot \hat{\boldsymbol{\tau}}_i \tau_i) \tag{5}$$

• The **electric farfield** produced by this **new induced currents** can be written as

$$\tilde{\mathbf{E}}^{\mathbf{s,\Delta}}_{l,i} = \tilde{\mathbf{E}}^{\mathbf{s,\Delta}}(\mathbf{r}_l, \mathbf{r}'_i + \Delta\mathbf{r}'_i) = \mathbf{E}^{\mathbf{s}}_{l,i}\ exp(j\phi_{l,i}\tau_i) \tag{6}$$

$$\phi_{l,i} = k\ (\hat{\mathbf{r}}_l - \hat{\mathbf{p}}^{\mathbf{inc}}_i) \cdot \hat{\boldsymbol{\tau}}_i$$



• The **total scattered field** can be written as the **summation** of the **contribution** of each **facet**

$$\tilde{\mathbf{E}}^{\mathbf{s,\Delta}}_l = \tilde{\mathbf{E}}^{\mathbf{s,\Delta}}(\mathbf{r}_l) = \sum_{i=1}^{N_\Omega} \tilde{\mathbf{E}}^{\mathbf{s,\Delta}}_{l,i} \tag{7}$$

• The **relationship** between the **distorted** and **undistorted** fields is **linearized** by approximating the exponential term in (6) by the first two terms of a series expansion

$$\tilde{\mathbf{E}}^{\mathbf{s,\Delta}}_{l,i} = \mathbf{E}^{\mathbf{s}}_{l,i}(1 + j\phi_{l,i}\tau_i) \tag{8}$$

• The **total scattered field** can be written as the **summation** of the **contribution** of each **facet**

$$\tilde{\mathbf{E}}^{\mathbf{s,\Delta}}_l = \sum_{i=1}^{N_\Omega} \tilde{\mathbf{E}}^{\mathbf{s,\Delta}}_{l,i} \tag{9}$$

Linear System of Equations

• When the number of **observation points** is coincident with the number of **distorted subdomains**, a **linear system of equations** is derived

$$[A]\ [x] = [b] \tag{10}$$

$$A(l,i) = \hat{\boldsymbol{\zeta}} \cdot \mathbf{E^s}_{l,i}\ j\phi_{l,i} \tag{11}$$

$$x(i) = \tau_i \tag{12}$$

$$b(l) = \hat{\boldsymbol{\zeta}} \cdot (\mathbf{E}^{\mathbf{s,\Delta}}_l - \mathbf{E}^{\mathbf{s}}_l) \tag{13}$$

Iterative Procedure

• The **error** introduced in the **linearization** of the problem imposes that an **iterative scheme** must be adopted for accurate results

$$\mathbf{r}'^{(m)}_i = \mathbf{r}'^{\Delta\ (m-1)}_i = \mathbf{r}'^{(m-1)}_i + \tau_i^{(m-1)} \boldsymbol{\tau}_i \tag{14}$$

$$\mathbf{r}'^{(0)}_i = \mathbf{r}'_i \tag{15}$$

$$\tau_i^{(0)} = 0 \tag{16}$$

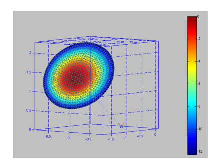
Application Example: Thermal and Random Errors Determination

• The **method** is applied to determine Thermal/Gravitational and Random errors.

Antenna configuration

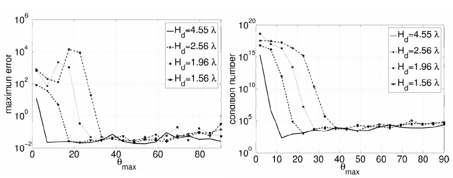
CHARACTERISTIC PARAMETERS OF THE SIMULATED EXAMPLE

Main Reflector	
Reflector diameter	$D = 1.68$ m
Focal length	$F = 1.832$ m
Offset height	$H = 1.45$ m
Feed	
Frequency	$f = 8.45$ GHz
Electric field polarization	$\mathbf{p} = \hat{\mathbf{x}}_\mathbf{f}$
Offset angle	$\theta_f = 43.18$
Subtended angle	$\theta_s = 45.11$
Taper edge E-plane	$t_E = 12$ dB
Taper edge H-plane	$t_H = 12$ dB



Conditioning of the field matrix

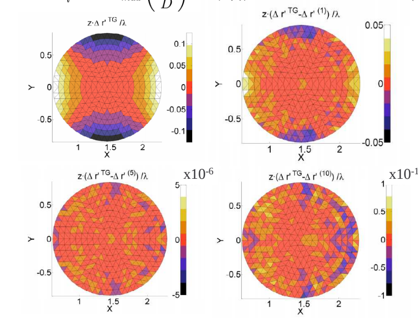
• The **condition number** of the **matrix** depends on the **spatial resolution**, described in terms of the averaged size of the subdomains, as well as maximum **observation angle** existing between an observation point and the antenna boresight direction.



Shape variation

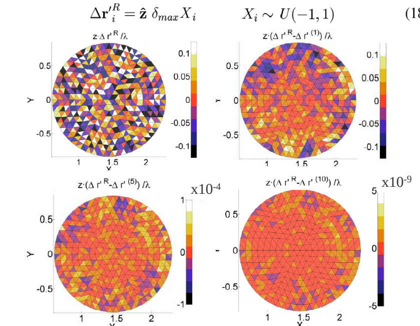
• The **Thermal/Gravitational** errors can be expressed analytically by

$$\Delta\mathbf{r}'^{TG}_i = \hat{\mathbf{z}}\ \delta_{max} \left(\frac{2\rho_i}{D}\right)^3 cos(2\varphi_i) \tag{17}$$

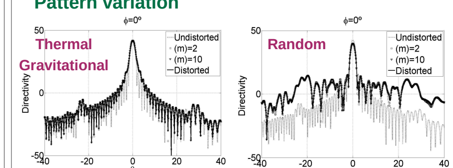


• The **Random** errors are characterized by an **uniform** distribution

$$\Delta\mathbf{r}'^{R}_i = \hat{\mathbf{z}}\ \delta_{max} X_i \qquad X_i \sim U(-1,1) \tag{18}$$



Pattern variation



Conclusions and technology transfer

• A new method to determine the **discrete distortions** on reflector antennas **have been presented**. The method is based on the discretization of the reflector surface into triangular patches. By linearizing the difference between the distorted and undistorted far filed patterns a linear system of equations is constructed. The **application** of the method for correcting **thermal/gravitational** and **random** surfaces yielding accurate results has been presented. The method can be **applied** to improve the **radiation performance** of **on-board satellite** reflector antennas by using active surfaces or phased arrays [2].

References

[1] J. A. Martinez-Lorenzo, A. G. Pino, I. Vega, M. Arias, and O. Rubios, "Icara: Induced-current analysis of reflector antennas," *IEEE Antennas and Propagation Magazine*, vol. 47, no. 2, pp. 92–100, 2005.

[2] A. G. Pino, J. A. Martinez-Lorenzo, M. A. Arias, and C. Compostizo, "Design and analysis of an adjustable subreflector for the hybrid mechanical-electronic pointing system at the satellite q/v band." in *Proc. IEEE Antennas and Propagation Society International Symposium (AP-S'06)*, Albuquerque, N.M., USA, July 2006, pp. 2451 – 2454.