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Managing the Risk of Terrorism to Interdependent Infrastructure Systems Through the Dynamic Inoperability Input–Output Model

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ABSTRACT

This paper discusses the Dynamic Input–Output Inoperability Model (DIIM), which is an extension to the static Inoperability Input–Output Model (IIM). Based on Wassily Leontief's Input–Output (I–O) model, both the IIM and the DIIM analyze how the system of interdependent sectors can be adversely affected as a result of initial perturbations to other sectors through willful attacks or natural disasters. To model the industry/sector interdependencies, the DIIM uses the national and regional commodity-transaction data from the Bureau of Economic Analysis (BEA) and the Regional Input–Output Multiplier System (RIMS II). In contrast to most traditional dynamic I–O models, the DIIM introduces industry resilience

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coefficients to measure the efficacy of sectors' risk management options. The DIIM also incorporates the stochastic properties of a recovery through the Brownian motion, representing short-term uncertainties. The paper uses two metrics to assess the consequences to the economic sectors of attacks. The DIIM methodology is demonstrated in detail through a two-by-two economy system. This is followed by an analysis of a terrorist attack scenario, using the DIIM and the BEA/RIMS II commodity-flow data of the 59 sectors in Virginia. Syst Eng 9: 241–258, 2006

Key words: risks of terrorism; input–output inoperability model; dynamic IIM; dynamic recovery

1. BACKGROUND

Wassily Leontief [1951a; 1951b], who developed and promoted the Input–Output (I–O) Model for economic systems, is remembered as the "father of the I–O model" in economics. In recognition of his outstanding pioneering work, he was awarded the 1973 Nobel Prize in Economics. Leontief modeled the entire economy as a system of industry sectors that are interdependent on each other through their internal input–output commodity flows. Ever since, scholars have been engaged in many aspects of I–O research to extend the theories and applications from the economic system to others, such as environmental, energy, and infrastructure systems. Notable extensions of the I–O model include a nonlinear Leontief model [Krause, 1992], energy I–O analysis [Proops, 1984], environmental and water resource I–O analysis [Lee, 1982; Haimes and Nainis, 1974; Haimes, 1977]. Miller and Blair [1985] published a comprehensive I–O model book marking the maturity of I–O analysis in the 20th century. Recent advances of I–O research are found in Lahr and Dietzenbacher [2001].

Interdependency analysis has been a major building block in systems engineering, built on the widely accepted premise within the engineering community that systems are interdependent. This means that any effect on one part of the system can propagate and affect others within the system and among other external systems. The I–O model appeals to interdependency researchers and analysts because it is capable of capturing among the industry sectors of the economy the indirect ripple effects of a natural disaster or an attack. Rose et al. [1997] and Rose [2004] proposed an I–O model to estimate the regional economic impacts of electricity lifeline disruptions caused by earthquakes. Olsen et al. [1997] developed an I–O model for optimal deployment of flood protection. Leontief and Duchin [1986] applied the I–O models to predict the potential impacts of automation on the workforce. Haimes and Jiang [2001], Crowther and Haimes [2005], and Haimes et al. [2005a; 2005b] extended the principles of the I–O model to develop the Inoperability Input–Output Model (IIM), with broad applications to critical infrastructure systems. Compared with other mainstream I–O research, the IIM focuses on the inoperability of infrastructure systems due to perturbations resulting from terrorist attacks or other interruptions to the industrial sectors of the economy. These adverse consequences are measured in economic loss and inoperability (i.e., percentage of "dysfunctionality" relative to an ideal state). Thus, the IIM aims to utilize I–O analysis to provide a specific framework for the assessment and management of risks to infrastructure protection and to homeland security. The successful recent applications of the IIM include assessing the threat to the United States from high-altitude electromagnetic pulse attacks and many others. In recent years, there have been many other notable quantitative methodologies for interdependent infrastructure protection and modeling terrorist threats. Apostolakis and Lemon [2005] presented a method for identifying and prioritizing vulnerabilities in infrastructures by employing graph theory and multiattribute utility theory. Paté-Cornell and Guikema [2002] described a systems analysis approach to setting priorities among countermeasures using probabilistic risk analysis.

Here the DIIM is proposed to model the recoveries of industry sectors following a disruptive event such as a terrorist attack or natural disaster. Based on the initial perturbations of sectors caused by the disruptive event and on the estimated recovery times, the model calculates the inoperabilities and economic losses of interdependent sectors during the recovery period. However, the DIIM does not model the transient period during the event in which the sectors that are directly attacked become inoperable.

2. INTRODUCTION

The IIM was developed to assess the efficacy of risk management, evaluating the impacts of economic disruption with and without applying risk mitigation measures [Haimes and Chittester, 2005]. However, enhancements were needed for aspects of the model that concern temporal and risk management evaluations. For example, it is imperative for decisionmakers to know: (i) How does a particular system recover over time? (ii) What are the associated economic losses? (iii) What can be done to minimize the losses during the recovery period after the attack? The DIIM addresses these questions. Although some dynamic aspects of the IIM were briefly and partially discussed in Haimes et al. [2005a], a more thorough analysis of the theoretical and methodological dimensions of the DIIM is presented here. The concept of an industry interdependency index is proposed to measure the degree of each sector's dependency on other interconnected sectors of the economy. From the risk management perspective, the interdependency index of a sector can be decreased through hardening, prevention, and redundancy. This leads to a faster recovery of the economy and less economic loss following an attack. This paper further examines the framework for risk management through a multiobjective optimization formulation using the DIIM, which explicitly evaluates various risk management options based on cost and benefit tradeoff analysis.

Understanding the subject of randomness is critical for any attempt to apply probability theory to catastrophic attacks. Probability theory [Gnedenko, 1963] has been very successfully applied in the natural sciences because of the *regularities* in the randomness of natural phenomena, whether in physics, astronomy, chemistry, or physiology. Unlike precipitation, with a vast database, terrorism scenarios do not seem to belong to a random process, and thus no single probability density function can be assigned to represent credible knowledge of the likelihood of such attacks. The inherent lack of regularity in the randomness of catastrophic attacks can be characterized as *unknown nonstationary random events* (not a process), because they are not only unknown and without historical precedence, but they are also changing in time. In terms of risk analysis, the question is how to deal with a not-unlikely extreme and possibly catastrophic event. Note: A not-unlikely event has an entirely different connotation from an unlikely one. The term *unlikely* implies that the observer has a sufficient level of belief or confidence that the event has a low probability of occurrence. On the other hand, the term *not-unlikely* implies that the observer has a sufficient level of belief or confidence that the event may occur, but without knowledge of its probability [Haimes, 2004].

Based on the above discussion, risk, which is a measure of the probability and severity of adverse effects [Lowrance, 1976], will be represented in this paper only in terms of consequences, assuming a not-unlikely probability of terrorist attacks. Once credible intelligence becomes available, then the expected value of risk supplemented by the conditional expected value of risk for extreme events [Asbeck and Haimes, 1984; Haimes 2004] can be used for the risk metric.

After the above introduction of the DIIM, the paper is organized as follows. Two basic applications of the dynamic model are described in Section 3, which emphasizes the concept of industry resilience coefficients. Section 4 discusses the implications of the DIIM for risk management during the recovery of an economic system. Section 5 formulates the risk management as a multiobjective problem. To illustrate the model, Section 6 demonstrates the DIIM using data from the two-industry economic system discussed in Miller and Blair [1985]. Section 7 presents an in-depth case study of a terrorist attack scenario using I–O data from the BEA/RIMS II databases to apply the DIIM to actual Virginia economic sectors. The paper concludes in Section 8.

3. DIIM FORMULATION

The formulation of the original Leontief I–O model is given in Eq. (1), where x_i represents the total production output of Industry i. The Leontief technical coefficient a_{ij} indicates the ratio of the input from Industry i to Industry j, with respect to the overall production requirements of Industry j. In Eq. (1), c_i represents the final demand of the ith industry, defined as the portion of Industry i's total output for final consumption by end-users.

$$\mathbf{x} = \mathbf{Ax} + \mathbf{c} \Leftrightarrow \left\{ x_i = \sum_j a_{ij}x_j + c_i \right\} \qquad \forall i. \tag{1}$$

Based on the classic Leontief I–O model, Santos and Haimes [2004] proposed the demand-reduction IIM. This enables quantitatively assessing the reduction of each sector's economic output resulting from initial final-demand perturbations to a set of economic sectors. The concept of inoperability of an economic sector is defined as *the percentage of output reduced from the ideal output*, which is triggered by demand reduction. Formally, if $\hat{\mathbf{x}}$ is defined as the as-planned total production vector of the economy, and $\tilde{\mathbf{x}}$ is the degraded total production vector, the demand-based inoperability $\mathbf{q}$ is defined in

$$\mathbf{q} = [(diag(\hat{\mathbf{x}}))^{-1}(\hat{\mathbf{x}} - \tilde{\mathbf{x}})], \tag{2}$$

where the operator $diag(\hat{\mathbf{x}})$ is the resulting diagonal matrix constructed from the given output vector $\hat{\mathbf{x}}$ as illustrated in Eq. (3) for a dimension of n (note that this notation will also be used later):

$$diag(\hat{\mathbf{x}}) = diag\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\\ \vdots\\ \hat{x}_n\end{bmatrix} = \begin{bmatrix}\hat{x}_1 & 0 & \cdots & 0\\ 0 & \hat{x}_2 & \ddots & \vdots\\ \vdots & \ddots & \ddots & 0\\ 0 & \cdots & 0 & \hat{x}_n\end{bmatrix}. \quad (3)$$

The demand-reduction IIM is given in

$$\mathbf{q} = [\mathbf{I} - \mathbf{A}^*]^{-1}\mathbf{c}^*, \quad (4)$$

where the notation $\mathbf{c}^*$, defined in Eq. (5), is the percentage vector of reduced final demand from the total nominal output and the notation $\mathbf{A}^*$, defined in Eq. (6), is the normalized interdependency square matrix:

$$\mathbf{c}^* = [(diag(\hat{\mathbf{x}}))^{-1}\ (\hat{\mathbf{c}} - \tilde{\mathbf{c}})], \quad (5)$$

$$\mathbf{A}^* = [(diag(\hat{\mathbf{x}}))^{-1}\ \mathbf{A}(diag(\hat{\mathbf{x}}))]. \quad (6)$$

Note that the concepts and definitions in the demand-reduction IIM are all applicable to the DIIM, which extends the IIM with additional dynamic and stochastic elements. Since the early development of the Leontief I–O model, researchers have been investigating various forms of a dynamic Leontief I–O model [Sage, 1977]. The dynamic model given in Eq. (7) is one of the most widely accepted [Miller and Blair, 1985]:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{c}(t) + \mathbf{B}\dot{\mathbf{x}}(t) \quad (7)$$

In this basic dynamic form, the notation $\mathbf{x}(t)$ refers to the output vector of the economy sectors at time t. The vector $\mathbf{c}(t)$ represents the final demands of the sectors at time t. The square matrix $\mathbf{A}$ is the *interdependency matrix,* representing the interdependencies among all the economic sectors, which is analogous to matrix $\mathbf{A}$ in the static I–O model. The square matrix $\mathbf{B}$ is introduced in the dynamic model as the *capital coefficient matrix,* which measures the willingness of the economy to invest in capital resources (such as machines, land, structures, and software). When equilibrium is reached in the dynamic I–O model, Eq. (7) takes the same form as the static model in Eq. (1), where $\dot{\mathbf{x}}(t) = 0$ is assumed at equilibrium.

Revising the traditional dynamic I–O model and its interpretations, Blanc and Ramos [2002] showed that the elements of $\mathbf{B}$ in Eq. (7) must be either zero or negative for the model of an economy system to be stable, assuming that $\mathbf{A}$ and $\mathbf{B}$ are constant, and only such condition can produce an economic behavior consistent with the static model, regardless of the initial conditions or final demands. Therefore, according to Blanc and Ramos, the capital coefficient matrix $\mathbf{B}$ can be interpreted as an expression of short-term counter-cyclical policy, instead of long-term growth which was supported by the mainstream I–O research community for years. Blanc and Ramos considered a case where $\mathbf{B} = -\mathbf{I}$ in Eq. (7), as represented in Eq. (8). From this they interpreted the dynamic model as describing the adjustment level of the economy's production following information about an imbalance in supply and demand at time t.

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{c}(t) - \mathbf{x}(t). \quad (8)$$

For a two-industry-economy, Eq. (8) can be specified as follows, where the left-hand-sides represents the adjustments of sectors and the right-hand-sides are their imbalances in supply and demand as discussed previously:

$$\begin{cases} dx_1(t) = (a_{11}x_1(t) + a_{12}x_2(t) + c_1(t) - x_1(t))dt, \\ dx_2(t) = (a_{21}x_1(t) + a_{22}x_2(t) + c_2(t) - x_2(t))dt. \end{cases} \quad (9)$$

For different sectors of the economy, the pace of adjustment can vary greatly, depending on the characteristics of the sectors as well as the specific situations that arise. Therefore, for the two industries in Eq. (9), coefficients k_1 and k_2 can be added to describe the production adjustment rates respectively, as in

$$\begin{cases} dx_1(t) = (a_{11}x_1(t) + a_{12}x_2(t) + c_1(t) - x_1(t))k_1dt, \\ dx_2(t) = (a_{21}x_1(t) + a_{22}x_2(t) + c_2(t) - x_2(t))k_2dt. \end{cases} \quad (10)$$

In general, given that the dynamic process is not completely deterministic, myriad factors can randomly affect the short-term behaviors of the sectors. Therefore, by adding a stochastic component and extending the Blanc and Ramos [2002] model, the dynamic extension of the IIM is formally introduced in Eq. (11), followed by detailed explanations of the parameters:

$$\frac{d\mathbf{x}(t)}{\mathbf{A}\mathbf{x}(t) + \mathbf{c}(t) - \mathbf{x}(t)} = \mathbf{K}dt + \boldsymbol{\sigma}dz, \quad (11)$$

$$\mathbf{K} = Diag(k_1, \ldots, k_n), \quad (12)$$

$$\boldsymbol{\sigma} = Diag(\sigma_1, \ldots, \sigma_n). \quad (13)$$

The symbol dz in Eq. (11), when seen as $dz = \varepsilon(t)\sqrt{dt}$, is the *Wiener process* or the *Brownian motion.* This is used widely in the literature (see Luenberger [1998]) to represent the "random walk" of the variables. The term $\varepsilon(t)$ is a standardized normal distributed random variable with mean zero and variance one. The diagonal matrix **K** in Eq. (12) is the *industry resilience coefficient matrix*, with its *i*th nonzero diagonal element k_i defined as the *industry resilience coefficient,* representing the resilience [Haimes et al., 1998] of the Sector *i*. The industry resilience coefficient matrix dictates an exponential dynamic process for the industrial sectors. The greater the k_i value, the faster the economic system responds to an imbalance in supply and demand.

The diagonal matrix $\boldsymbol{\sigma}$ in Eq. (13), the *industry resilience deviation matrix*, also measures the uncertainties of the dynamic process and the deviations of the *industry resilience coefficient matrix* **K**. Each nonzero diagonal element in the matrix, defined as the *industry resilience deviation coefficient,* implies the uncertainties of an economic sector during its dynamic process in the DIIM. The higher σ value indicates more volatility in a sector; therefore, it is harder to control and predict the dynamic path of its behavior. In the context of a recovery from an attack or an incident, the *industry resilience deviation coefficient,* σ, is a manifestation of a sector's characterizations (such as its structure, vulnerabilities, etc.) and the nature of the attack or incident, as well as the functions of risk management policies and other external factors during the recovery period.

All the other variables in Eq. (11) have the same definitions as in Eqs. (1)–(8). In summary, in Eq. (11) the term $\mathbf{Ax}(t) + \mathbf{c}(t) - \mathbf{x}(t)$ reflects the interdependency among the sectors of the economy; the term $\mathbf{K}dt$ represents the exponential resilience of the economy for its demand and supply to be at the same level in the long term; and the term $\boldsymbol{\sigma}dz$ captures the short-term randomness of the dynamic process. Comparing the Leontief dynamic I–O model in Eq. (7) and the DIIM formulation in Eq. (9), it is obvious that the DIIM is equivalent to the Leontief dynamic model by setting the capital investment coefficient matrix $\mathbf{B} = -\mathbf{K}^{-1}$. However, in the DIIM, **B** is not interpreted as an expression of the long-term growth of the economy. Rather, it is extended as a measure of the short-term resiliencies of industry sectors following disruptive events such as natural hazards or terrorist attacks. Since the resiliency of a sector is affected by risk management and public policies, the matrix **B** can be viewed as a risk management investment coefficient matrix, representing the willingness to invest in risk management for economic sector disasters. To characterize the randomness of the recovery, the Wiener process is supplemented to the dynamic model. It has the capability of modeling the short-term random process, and the *industry resilience deviation coefficient* (σ) provides a measure of volatility for the resilience coefficient *k*. Thus, the Wiener process in the DIIM formulation simulates the short-term uncertainties during the overall recovery trend. The effects of uncertainty from the Wiener process also depend on the amount of imbalance between supply and demand in Eq. (9). Therefore, at the beginning of the recovery, significant uncertainties are presented, and as the economic system recovers, the uncertainties decrease until the new equilibrium is reached.

Applying Eq. (1) to the as-planned production scenario and Eq. (11) to the degraded production scenario, Eqs. (14) and (15) can be obtained:

$$\mathbf{A}\hat{\mathbf{x}}(t) + \hat{\mathbf{c}}(t) - \hat{\mathbf{x}}(t) = 0, \tag{14}$$

$$\frac{d\tilde{\mathbf{x}}(t)}{\mathbf{A}\tilde{\mathbf{x}}(t) + \tilde{\mathbf{c}}(t) - \tilde{\mathbf{x}}(t)} = \mathbf{K}dt + \boldsymbol{\sigma}dz. \tag{15}$$

It follows that

$$\frac{d(\hat{\mathbf{x}} - \tilde{\mathbf{x}}(t))}{\mathbf{A}(\hat{\mathbf{x}} - \tilde{\mathbf{x}}(t)) + (\hat{\mathbf{c}} - \tilde{\mathbf{c}}(t)) - (\hat{\mathbf{x}} - \tilde{\mathbf{x}}(t))} = \mathbf{K}dt + \boldsymbol{\sigma}dz. \tag{16}$$

From Eq. (6), Eq. (16) becomes the following form, in which the normalized interdependency square matrix $\mathbf{A}^*$ follows the definition in Eq. (6):

$$\frac{d(\hat{\mathbf{x}} - \tilde{\mathbf{x}}(t))}{(diag(\hat{\mathbf{x}}))\mathbf{A}^*(diag(\hat{\mathbf{x}}))^{-1}(\hat{\mathbf{x}} - \tilde{\mathbf{x}}(t)) + (\hat{\mathbf{c}} - \tilde{\mathbf{c}}(t)) - (\hat{\mathbf{x}} - \tilde{\mathbf{x}}(t))} = \mathbf{K}dt + \boldsymbol{\sigma}dz. \tag{17}$$

Equation (17) can be transformed to

$$\frac{d[(diag(\hat{\mathbf{x}}))^{-1}(\hat{\mathbf{x}} - \tilde{\mathbf{x}}(t))]}{\mathbf{A}^*(diag(\hat{\mathbf{x}}))^{-1}(\hat{\mathbf{x}} - \tilde{\mathbf{x}}(t)) + (diag(\hat{\mathbf{x}}))^{-1}(\hat{\mathbf{c}} - \tilde{\mathbf{c}}(t)) - (diag(\hat{\mathbf{x}}))^{-1}(\hat{\mathbf{x}} - \tilde{\mathbf{x}}(t))} = \mathbf{K}dt + \boldsymbol{\sigma}dz. \tag{18}$$

By definitions of $\mathbf{q}(t)$ and $\mathbf{c}^*(t)$, the inoperability form of the DIIM is given in

$$\frac{d\mathbf{q}(t)}{\mathbf{A}^*\mathbf{q}(t) + \mathbf{c}^*(t) - \mathbf{q}(t)} = \mathbf{K}dt + \boldsymbol{\sigma}dz. \tag{19}$$

When the final equilibrium is reached, the DIIM has the same form as the static IIM in Eq. (4). Therefore, in this formulation, the DIIM encompasses the theoretical basis for interdependency, the exponential resilience of the economy, the randomness of the dynamic process, and the static IIM.

The stochastic nature of the DIIM represents the multifaceted character of the dynamic process that involves many other aspects of our society that extend beyond the concept of the standard deviation $\boldsymbol{\sigma}$. This paper emphasizes the dynamic aspect of the IIM. However, the stochastic portion is visited briefly in Section

6.4, leaving extensive discussions on the stochastic aspects of the model for a future paper. When the stochastic part of the DIIM is ignored ($\boldsymbol{\sigma} = 0$), Eq. (11) is reduced to the form resulting in

$$\dot{\mathbf{x}}(t) = \mathbf{K}[\mathbf{A}\mathbf{x}(t) + \mathbf{c}(t) - \mathbf{x}(t)], \quad (20)$$

or it takes the inoperability form in Eq. (21), where $\mathbf{q}(t)$ is related to $\mathbf{x}(t)$ as in Eq. (2):

$$\dot{\mathbf{q}}(t) = \mathbf{K}[\mathbf{A}^*\mathbf{q}(t) + \mathbf{c}^*(t) - \mathbf{q}(t)] \quad (21)$$

Equation (21) is a standard form of linear first-order differential equations [Edwards and Penney, 2000], and the solution to it is as follows, given the initial condition $\mathbf{q}(0)$:

$$\mathbf{q}(t) = e^{-\mathbf{K}(\mathbf{I}-\mathbf{A}^*)t}\,\mathbf{q}(0) + \int_0^t \mathbf{K}e^{-\mathbf{K}(\mathbf{I}-\mathbf{A}^*)(t-z)}\,\mathbf{c}^*(z)\,dz. \quad (22)$$

If the final demand $\mathbf{c}^*(t)$ is stationary, then Eq. (22) can be further simplified as follows:

$$\mathbf{q}(t) = (\mathbf{I} - \mathbf{A}^*)^{-1}\,\mathbf{c}^* + e^{-\mathbf{K}(\mathbf{I}-\mathbf{A}^*)t}\,[\mathbf{q}(0) - (\mathbf{I} - \mathbf{A}^*)^{-1}\mathbf{c}^*]. \quad (23)$$

4. TWO BASIC DIIM APPLICATIONS

To reach equilibrium, the DIIM can be applied to the two basic types of the economy's dynamic process. These are the *demand-reduction dynamic* and the *dynamic recovery associated with the production outputs of interdependent sectors.* The demand-reduction dynamic is the extension of the demand-reduction IIM that calculates the new equilibrium output of sectors, given demand-reduction perturbations caused by psychological effect. The DIIM is also applicable to model the recovery of the economy after disruptive events that degrade the production capacity of directly affected sectors, under fixed or variable final demands. As demonstrated in Section 3.2, a key contribution of the DIIM is extending the Leontief I–O from demand-based to supply-and-demand-equilibrium-based. When the model is applied to disruptive events such as terrorist attacks or natural disasters, the sector outputs (supplies) are reduced due to impacts of the events on production capacity. Thus, as modeled through the DIIM, during the recovery period the sector outputs (supplies) will increase to meet the final demands until the equilibrium is reached. The concept of recovery rate in the DIIM therefore is introduced to measure differing sector recovery rates, given their imbalances in supply and demand.

4.1. Demand-Reduction Dynamics

The static demand-reduction IIM can be used to calculate the reduced output of economic sectors due to the initial final demand reduction from a set of sectors. The final demand reduction can result from various causes, one of which can be the psychological impact on consumers from a terrorist attack. The DIIM can be applied to the case of final demand reduction to show how the outputs change for every time period until equilibrium, after the initial perturbation to the final demand of the economy. The dynamic demand-reduction scenario can be described mathematically as the following: The term $\mathbf{A}\mathbf{x}(t)$ in Eq. (11) represents the intermediate demand of each sector at time t; therefore, the term $\mathbf{A}\mathbf{x}(t) + \mathbf{c}(t)$ represents the total demand of the sectors including the intermediate and final demands. At equilibrium, the total demand $\mathbf{A}\mathbf{x}(t) + \mathbf{c}(t)$ meets the total supply $\mathbf{x}(t)$. However, when there is a final demand reduction, $\mathbf{c}(t)$ is reduced to a smaller $\tilde{\mathbf{c}}(t)$, which makes the term $\mathbf{A}\mathbf{x}(t) + \tilde{\mathbf{c}}(t) - \mathbf{x}(t)$ negative. Therefore, according to Eq. (11), the output supply $\mathbf{x}(t)$ starts to decline until it reaches a new level, $\tilde{\mathbf{x}}(t)$, when the new equilibrium is achieved ($\mathbf{A}\tilde{\mathbf{x}}(t) + \tilde{\mathbf{c}}(t) - \tilde{\mathbf{x}}(t)$).

The solution for the demand-reduction dynamics is as follows. For calculating convenience, we adopt the inoperability form of the DIIM. Formally, for the case of demand reduction in the DIIM, it is a given that at time zero, the economy is at a business-as-usual state (completely operable, $\mathbf{q}(0) = 0$; i.e., inoperability is zero); but there are perturbations to the normalized final demand ($\mathbf{c}^* > 0$). It follows that Eq. (23) becomes

$$\mathbf{q}(t) = [\mathbf{I} - e^{-\mathbf{K}(\mathbf{I}-\mathbf{A}^*)t}](\mathbf{I} - \mathbf{A}^*)^{-1}\mathbf{c}^*. \quad (24)$$

Equation (24) fully describes the dynamic behavior of the sector that adjusts itself in the event of final demand reduction. In equilibrium, when $t \to \infty$, it can be observed from Eq. (24) that the inoperability is given by Eq. (25) if the demand reduction is held constant:

$$\mathbf{q}(\infty) = (\mathbf{I} - \mathbf{A}^*)^{-1}\mathbf{c}^*. \quad (25)$$

Equation (25) is of the same form as the static demand-reduction IIM. It is also verified that when $t = 0$, according to Eq. (24), $\mathbf{q}(0) = 0$, which is consistent with the demand-reduction assumption that the initial inoperability is zero.

As discussed in the DIIM formulation, the *industry resilience coefficient matrix* $\mathbf{K}(t)$ in the demand-reduction DIIM measures the speed of each sector's adjustment to an imbalance in supply and demand. For each sector, a greater $k(t)$ in the $\mathbf{K}$ matrix implies a more prompt adjustment of its output in response to a change in final demand. Equation (24) confirms that the *industry resilience coefficient* can affect the dynamic process exponentially. The *industry resilience coefficient* is a characteristic of a particular economic sector; therefore, to further understand and estimate accurately the coefficient for each sector requires investigating an economic sector from a holistic perspective. One of the notable approaches to that end is to use Hierarchical Holographic Modeling (HHM) [Haimes, 1981, 2004], a method that has been used in many risk analysis applications for understanding complex systems.

4.2. Dynamic Recovery of the Sectors

The DIIM can also be applied to model the dynamic recovery of industry sectors after their production is interrupted by either natural disasters or terrorist attacks. In such a case, the output of the economy $\mathbf{x}(t)$ is reduced to a smaller $\tilde{\mathbf{x}}(t)$. When the final demands are assumed to be the same, the total demand of the sectors $\mathbf{A}\tilde{\mathbf{x}}(t) + \mathbf{c}(t)$ exceeds the output supply of the sectors $\tilde{\mathbf{x}}(t)$. As the affected sectors recover from their degraded output, the mismatch between the demand and supply $\mathbf{A}\tilde{\mathbf{x}}(t) + \mathbf{c}(t) - \tilde{\mathbf{x}}(t))$ decreases as well, until the point when the sectors fully recover and have the capability to provide a level of output to meet total demand.

Formally, in the more realistic case of dynamic recovery, after an accident or attack, the initial output levels of the economic sectors are reduced, causing inoperability to these sectors ($\mathbf{q}(0) > 0$). It is assumed that the final demand of each sector stays constant; therefore, $\mathbf{c}^* = 0$. In this case, Eq. (23) can be reduced to

$$\mathbf{q}(t) = e^{-\mathbf{K}(\mathbf{I}-\mathbf{A}^*)t}\mathbf{q}(0). \qquad (26)$$

Equation (26) describes the dynamic recovery process of the economy following an interruption of its production. As $t \to \infty$, it can be observed from Eq. (26) that $\mathbf{q}(t) \to 0$. In other words, as time progresses, the economic sectors recover from their initial inoperability back to their business-as-usual status, where all the sectors are totally operable as indicated in

$$\mathbf{q}(\infty) = 0 \qquad (27)$$

Similar to the demand-reduction case, in the dynamic recovery of the sectors, the *industry resilience coefficient matrix* $\mathbf{K}$ determines the speed of *exponential recovery* for each economic sector according to the DIIM. (In the study assessing the impact of a high-altitude electromagnetic pulse (HEMP) attack on interdependent infrastructure sectors [Haimes et al., 2005a, 2005b], exponential recovery was chosen by the HEMP Commission, which was established by Congress to provide an independent assessment of the HEMP threat against the United States [CRS, 2004].) The ith diagonal element in the $\mathbf{K}$ matrix is a measure of the recovery time for the ith sector in the economy. Therefore, k_i is defined as the *interdependency recovery rate*, which describes the recovery rate of Sector i.

To assess the *interdependency recovery rate* k_i for Sector i, it is assumed that Sector i is attacked ($q_i(0) > 0$) and other sectors are not initially perturbed ($q_j(0) = 0,\ j \neq i$). Applying Eq. (21), the recovery trajectory for Sector i becomes

$$\dot{q}_i(t) = -k_i(1 - a_{ii}^*)q_i(0) \qquad (28)$$

The solution to Eq. (28) is

$$q_i(t) = e^{-k_i(1 - a_{ii}^*)}q_i(0) \qquad (29)$$

If it is assessed by experts that it takes T_i for Sector i to recover from its initial inoperability $q_i(0) > 0$ (e.g., 100%) to a $q_i(T_i)$ (e.g., 1%) inoperability, the *interdependency recovery rate* can be assessed as Eq. (30), derived from Eq. (29):

$$k_i = \frac{\ln[q_i(0)/q_i(T_i)]}{T_i}\left(\frac{1}{1 - a_{ii}^*}\right) = \frac{\omega_i}{T_i} = \left(\frac{\lambda}{\tau}\right)_i\left(\frac{1}{1 - a_{ii}^*}\right). \qquad (30)$$

Let the symbol ω_i in Eq. (30) denote

$$\frac{\ln[q_i(0)/q_i(T_i)]}{1 - a_{ii}^*},$$

τ is the time when inoperability reduces to some value q_τ, the recovery constant is λ, and collectively the ratio $(\lambda/\tau)_i$, representing the term $\ln[q_i(0)/q_i(T_i)]/T_i$, is the recovery-rate parameter. The notation a_{ii}^* represents the ith diagonal element in the interdependency matrix $\mathbf{A}^*$. The smaller a_{ii}^* value, thus greater $1 - a_{ii}^*$, indicates the higher degree of dependence for the sector and hence faster recovery, taking into account interdependency with other sectors. Equation (30) implies that the greater a_{ii}^* leads to the greater *interdependency recovery rate* k_i.

Therefore, the actual recovery rate k_i of the ith sector is comprised of two components: its own recovery rate and its interdependency with other sectors. Due to its importance, the term $1 - a_{ii}^*$ is formally defined as the *interdependency index* for Industry i, denoted as θ_i, in

$$\theta_i = 1 - a_{ii}^*. \qquad (31)$$

To compare the degree of interdependency between the two arbitrary sectors of the economy, Sectors i and j, the ratio of their interdependency indices can be taken and defined as the *interdependency ratio* of Sectors i and j. It is denoted as ρ_{ij} in

$$\rho_{ij} = \frac{\theta_i}{\theta_j} = \frac{1 - a_{ii}^*}{1 - a_{jj}^*}. \tag{32}$$

For the ith sector, we observe that its *interdependency index* θ_i, like the recovery rate λ/τ, also affects its recovery exponentially. Therefore, from the risk management perspective, according to Eq. (30), if the *interdependency index* θ_i (or $1 - a_{ii}^*$) of the ith sector can be decreased in the scenario of an terrorist attack or natural disaster, it will increase the interdependency recovery rate k_i. This results in a quicker recovery, less economic loss, and fewer other losses. The reduction of a sector's interdependency index can be achieved through various risk management options, including prevention efforts, adding redundancies, and many others.

5. MULTIOBJECTIVE FORMULATIONS

To manage risk, options must be adopted to affect either the recovery rate λ/τ or the interdependency index θ_i, or both, so that the *interdependency recovery rate* k_i can be increased. This will speed up the recovery process and reduce potential losses during the recovery of the economy from an attack. In this context, the *industry resilience coefficient matrix* **K** can be viewed as the *risk management investment coefficient matrix*, which represents the decision-maker's willingness to invest in risk management. Therefore, this definition of **K** is comparable to the traditional definition of the *capital investment coefficient matrix* **B** in the economics literature previously discussed and illustrated in Eq. (7).

If a finite set of n risk management options is defined as r_i, i = 1, 2, …, n, and the cost function for risk management options is denoted as $C = C((r_1, r_2, \ldots, r_i, \ldots, r_n))$, a two-objective optimization problem can be formulated as in Eq. (33), and Pareto-optimal solutions [Haimes, 2004] can be generated for the risk-cost-benefit analyses using the DIIM. One or more risk management options can be chosen among the total n candidates.

$$\underset{r_1, r_2, \ldots, r_i, \ldots r_n}{\textbf{Minimize}}\ Q = \sum_{j=1}^{N} \left(\hat{x}_j \int_0^{\infty} q_j(t, \mathbf{K}(r_1, r_2, \ldots, r_i, \ldots, r_n))\, dt \right),$$

$$\underset{r_1, r_2, \ldots, r_i, \ldots r_n}{\textbf{Minimize}}\ C = C(r_1, r_2, \ldots, r_i, \ldots, r_n). \tag{34}$$

In Eq. (33), the notation Q represents the total economic loss during the entire recovery period. For each sector of the economy, q_j represents the inoperability and $\hat{x}_j$ is its nominal output. Although the cost of risk management and the total economic loss are both measured in dollar units, they are treated as two noncommensurate objectives for a number of reasons. First of all, the sources of expense are different. The majority of the risk management cost most likely falls to the government, but the total loss will probably be spread among all industry sectors of the economy. In addition, risk management is a deterministic investment in order to mitigate the effects of a disaster or attack; however, the economic loss incurred during recovery has more uncertainties and usually involves stochastic distributions. Another important aspect is that in both risk management and loss from an attack, the economic metric is only one perspective. It is a surrogate of many other noncommensurate impacts, including political, social, and environmental considerations.

Equation (33) indicates that the overall economic loss is a function of the *risk management investment coefficient matrix* **K.** This functional relationship can be further specified by inserting Eq. (26) (the formula of the inoperability) into Eq. (33). A more explicit expression for the first objective in Eq. (33) can be derived in Eq. (34), in which the symbol $\mathbf{I}_{1\times n}$ represents a summation vector, which is an identity row-vector with the value 1 in each of the n columns. In Eq. (34), the *risk management investment coefficient matrix* **K** becomes the surrogate of the decision variable, which is manifested through the risk management options:

$$\underset{r_1, r_2, \ldots, r_i, \ldots, r_n}{\textbf{Minimize}}\ Q =$$

$$\mathbf{I}_{1\times n}[diag(\hat{x})(\mathbf{I} - \mathbf{A}^*)^{-1}\mathbf{K}(r_1, r_2, \ldots, r_i, \ldots, r_n)^{-1}\mathbf{q}(0)]. \tag{34}$$

It is evident from Eq. (34) that the total loss borne by the economy is a linear function of the reciprocals of the *risk management investment coefficients*. Moreover, a greater k_i, i = 1, 2, …, n (the diagonal element of **K**) renders a smaller economic loss Q, with fixed nominal sector output, interdependency coefficients, and initial perturbations, Therefore, risk management options that result in higher values of the *risk management investment coefficients* will reduce more of the overall economic loss following an attack. Equation (34) can be readily transformed into Eq. (35) by virtue of Eq. (30). Elements in the diagonal matrices $diag(\mathbf{T})$ and $diag(\boldsymbol{\omega})$ follow the same definitions as in Eq. (30):

$$\underset{r_1, r_2, \ldots, r_i, \ldots, r_n}{\textbf{Minimize}}\ Q =$$

$$\mathbf{I}_{1\times n}[diag(\hat{\mathbf{x}})(\mathbf{I}-\mathbf{A}^*)^{-1}diag(\mathbf{T}(r_1, r_2, \ldots, r_i, \ldots, r_n))diag^{-1}(\boldsymbol{\omega})\mathbf{q}(0)] \tag{35}$$

Equation (35) shows that in fact the overall economic loss incurred during the recovery is a linear function of the recovery times. Therefore, by shortening the recovery time of the attacked or damaged sectors, the overall economic loss can be reduced. In the following discussion, the matrix and vector notations are equivalently represented by their elements, which facilitate the interpretations of the multiobjective formulation at the sector level. To this end, the matrix notations in Eqs. (34) and (35) are further simplified into more compact forms in Eqs. (36) and (37) respectively, as follows:

$$\underset{r_1,r_2,\ldots,r_i,\ldots,r_n}{\textbf{Minimize}}\ Q = \sum_{j=1}^{n}\left[\left(\sum_{i=1}^{n}\hat{x}_i d_{ij}\right)\frac{q_j(0)}{k_j(r_1, r_2, \ldots, r_i, \ldots, r_n)}\right], \tag{36}$$

$$\underset{r_1,r_2,\ldots,r_i,\ldots,r_n}{\textbf{Minimize}}\ Q = \sum_{j=1}^{n}\left[\left(\sum_{i=1}^{n}\hat{x}_i d_{ij}\right)\frac{q_j(0)}{\omega_j}\,T_j(r_1, r_2, \ldots, r_i, \ldots, r_n)\right], \ldots, \tag{37}$$

where d_{ij} denotes the element in the ith row and jth column of matrix $(\mathbf{I} - \mathbf{A}^*)^{-1}$. In Eqs. (36) and (37), if Sector j is not initially perturbed, $q_j(0)$ is zero; therefore, $q_j(0)$ disappears from the summation, leaving only those terms representing the sectors initially attacked. In Eq. (37), the overall economic loss is a linear function of the recovery times of the sectors initially affected by the attack, weighted by the term $(\sum_{i=1}^{n} \hat{x}_i d_{ij})\ q_j(0)/\omega_j$. The summation $(\sum_{i=1}^{n} \hat{x}_i d_{ij})$ represents the interdependency of the economy, and the term measures $q_j(0)/\omega_j$ the degree of initial perturbation rendered by the attack. For example, if it is assumed that there are only two sectors, r and s, recovering from attacks and the *risk management investment coefficients* are the decision-variables in risk management, a multiobjective optimization problem can be specified as in

$$\underset{r_1,r_2,\ldots,r_i,\ldots,r_n}{\textbf{Minimize}}\ Q = \left(\sum_{i=1}^{n}\hat{x}_i d_{ir}\right)\frac{q_r(0)}{k_r(r_1, r_2, \ldots, r_i, \ldots, r_n)} + \left(\sum_{i=1}^{n}\hat{x}_i d_{is}\right)\frac{q_s(0)}{k_s(r_1, r_2, \ldots, r_i, \ldots, r_n)},$$

$$\underset{r_1,r_2,\ldots,r_i,\ldots,r_n}{\textbf{Minimize}}\ C = C(r_1, r_2, \ldots, r_i, \ldots, r_n).$$

Alternatively, this can be formulated in Eq. (39), where the recovery time of each sector is adopted as the surrogate of the decision variable:

$$\underset{r_1,r_2,\ldots,r_i,\ldots,r_n}{\textbf{Minimize}}\ Q = \left(\sum_{i=1}^{n}\hat{x}_i d_{ir}\right)\frac{q_r(0)}{\omega_r}\,T_r(r_1, r_2, \ldots, r_i, \ldots, r_n) + \left(\sum_{i=1}^{n}\hat{x}_i d_{is}\right)\frac{q_s(0)}{\omega_s}\,T_s(r_1, r_2, \ldots, r_i, \ldots, r_n),$$

$$\underset{r_1,r_2,\ldots,r_i,\ldots,r_n}{\textbf{Minimize}}\ C = C(r_1, r_2, \ldots, r_i, \ldots, r_n). \tag{39}$$

Equations (38) and (39) are multiobjective optimization problems that can be solved through a number of methods, such as the Surrogate Worth Trade-Off (SWT) [Haimes, 2004]. Pareto-optimal solutions or the efficient frontiers for risk management can be derived to provide explicit tradeoffs and insights for the decision-makers. Note that, although the cost of risk management options is deterministic, the effectiveness and thus the efficacy of risk management is stochastic by its nature. In this multiobjective framework, the effectiveness of risk management is calculated through the deterministic IIM as the reduced economic loss with risk management compared with the scenario if no risk management is deployed. Therefore, the deterministic effectiveness of risk management should be interpreted as the expected value of its actual stochastic distribution. This multiobjective formulation for risk management is illustrated and discussed in more detail in the example in Section 7.

With the DIIM, a general risk management framework for an economic system is formulated in this section as multiobjective optimization problems. Decision-makers need to balance the cost of risk management and its associated benefit, e.g., economic-loss reduction during the recovery period. Essentially, in the dynamic model, the implementation of more risk management efforts results in a greater recovery rate for the economy sectors, which in turn shortens the recovery period and reduces potential damages. The system recovery is also a function of some random factors.

6. AN ILLUSTRATIVE EXAMPLE: A TWO-INDUSTRY ECONOMIC SYSTEM

A two-industry economy example illustrates the theory and methodology of the DIIM, primarily to demonstrate its basic analytical concepts and procedures. Miller and Blair [1985] introduced the original static version of this example. In Table I, the numbers in the shaded cell are the amounts of commodities in monetary units (million of dollars) flowing between the industries. The third column represents the amount of the industries' final demands and the third row of the table represents the amount of value added to the industries. The last column and last row represent the total outputs and inputs of the industries.

It can be seen in the first column that Industry *i* uses a $150 million input from its own production as well as $200 million from the output of Industry *j*, and $650 million is the value added, which adds up to a total input of $1000 million for Industry *i*. The value added is measured as the sum of compensation of employees, taxes on production and imports less subsidies, and gross operating surplus [U.S. Department of Commerce, BEA, 2005]. The first row shows that Industry *i* produces $150 million for its own internal intermediate consumption, $500 million as input to Industry *j*, and $350 million for its final demand. Therefore, the total output from Industry *i* equals its total input. The same is true for Industry *j,* and thus the entire economy is balanced (i.e., total input equals total output).

The commodity flow between the two industries in the shaded area of Table I can be denoted as matrix **Z** in

$$\mathbf{Z} = \begin{bmatrix} 150 & 500 \\ 200 & 100 \end{bmatrix}. \tag{40}$$

The Leontief technical coefficient matrix for the demand-side I–O model (**A**) is obtained by dividing each element of the commodity flow matrix (**Z**) by the respective column sum. For the two-industry example, this is constructed as Eq. (41). The symbols x_m, x_n are used to denote the total output of the industries and x_{mn} represents the *m*th-row, *n*th-column element in matrix **Z**:

Table I. Commodity Flow of a Two-Industry Economy (Millions of Dollars)

Industries	*i*	*j*	Final Demand (c)	Total Output (x)
i	150	500	350	1000
j	200	100	1700	2000
Value Added	650	1400		
Total Input ($\mathbf{x}^T$)	1000	2000		

$$\mathbf{A} = \{x_{mn}/x_n\}\ \forall m,n = \begin{bmatrix} 150/1000 & 500/2000 \\ 200/1000 & 100/2000 \end{bmatrix}$$
$$= \begin{bmatrix} 0.15 & 0.25 \\ 0.20 & 0.05 \end{bmatrix}. \tag{41}$$

The normalized technical coefficient matrix $\mathbf{A}^*$ used in the IIM is derived in Eq. (42) by applying Eq. (6):

$$\mathbf{A}^* = \begin{bmatrix} 0.15 & 0.25(\frac{2000}{1000}) \\ 0.20(\frac{1000}{2000}) & 0.05 \end{bmatrix} = \begin{bmatrix} 0.15 & 0.5 \\ 0.1 & 0.05 \end{bmatrix}. \tag{42}$$

The interdependency indices for the two industries can be calculated in Eq. (43) according to Eq. (31):

$$\theta_i = 1 - a^*_{ii} = 0.15,$$
$$\theta_j = 1 - a^*_{jj} = 0.05. \tag{43}$$

Therefore, it can be concluded from Eq. (43) that Industry *i* is three times more interdependent than Industry *j*. By its definition in Eq. (32), the *interdependency ratio* is 3 to 1.

This discussion can also be verified from the commodity flows in Table I, in which half (500/1000) of Industry *i*'s total output is consumed by Industry *j*. However, only one-tenth (200/2000) of Industry *j*'s output is consumed by Industry *i.* Therefore, Industry *i*'s production is much more dependent on Industry *j* than vice versa. The *interdependency index* and *interdependency ratio* measure the degrees of interdependency for the two industry sectors in this example.

6.1. Deterministic Demand-Reduction Dynamics

It is assumed in this case that the two industries are totally operable at time zero ($q_i(0) = q_j(0) = 0$), but there is a 10% demand reduction in Industry *j* ($c_j^* = 10\%$), which is causing the outputs of both industries to change. The system equations in (44) for this two-industry economy can be derived according to Eq. (21) in its inoperability form:

$$\begin{cases} q_i(k+1) - q_i(k) = k_i[0.15q_i(k) + 0.5q_j(k) + c_i^* - q_i(k)], \\ q_j(k+1) - q_j(k) = k_j[0.1q_i(k) + 0.05q_j(k) + c_j^* - q_j(k)]. \end{cases} \tag{44}$$

It is assumed for simplicity that the *industry resilience coefficients* for both industries are equal to 0.2 ($k_i = k_j = 0.2$). Their inoperability trajectories without risk management can be simulated in Figure 3 according to Eq. (44).

In Figure 1, Industry *j*'s inoperability increases exponentially and stabilizes at around 11.2% at equilib-

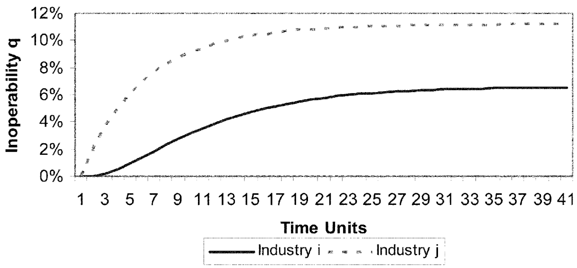


Figure 1. Inoperability dynamics for demand-reduction scenario.

rium. Affected by the demand reduction in Industry *j,* Industry *i* also becomes inoperable and stabilizes with 6.6% inoperability. It can be verified that the inoperabilities of the two industries at the equilibrium state are consistent with the results from the static demand-reduction IIM model.

6.2. Deterministic Dynamic Recovery with Demand Reduction after Attacks

It is assumed that there is an attack on Industry *j,* which results in a 10% inoperability ($q_j(0) = 10\%$), but Industry *i* is not directly attacked ($q_i(0) = 0$). It is assumed that the final demand reduction to Industry *i* is 0 ($c_i^* = 0$) and the final demand reduction to Industry *j* is 3% ($c_j^* = 3\%$). If it is further assumed that $k_i = k_j = 0.2$, the dynamic recovery process with the final demand-reduction scenario can be described in Figure 2 according to the simulation results from Eq. (44).

In Figure 2, Industry *j* recovers from its 10% inoperability rendered by the attack and Industry *i* has become inoperable due to its interdependency with Industry *j.* However, the two industries cannot completely recover to the totally operable state because of the demand reduction in Industry *i.* Their final inoperabilities are determined by their demand reductions the same as they are in Section 6.1. It can again be verified that the final inoperabilities of the two industries at the equilibrium state are consistent with the results from the static IIM.

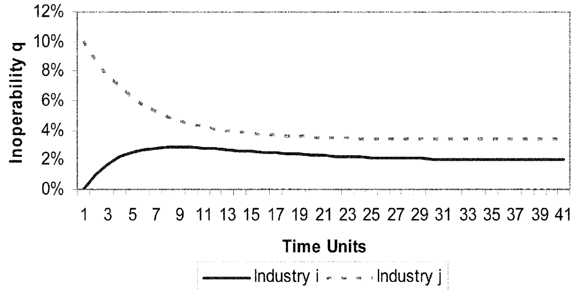


Figure 2. Sector recoveries with final demand reduction.

6.3. Stochastic Recovery

In the DIIM, the *industry resilience deviation coefficients* are introduced to model the uncertainties of the *industry resilience coefficients*. The stochastic component in these equations is modeled through the *Brownian motion*, which reflects the short-term uncertainty of the industry recovery in contrast to the long-term exponential recovery process. Similar to the *industry resilience coefficient*, the *industry resilience deviation coefficient* also characterizes an economic sector during its recovery. To illustrate the stochastic aspect of the DIIM, the *industry resilience deviation coefficients* of the two industries are incorporated into this scenario. Furthermore, the *industry resilience coefficients* are derived from the recovery-time estimation instead of being directly given to the model as assumptions. The following discussion describes the process.

Consider a terrorist attack scenario where Industry *j* is rendered 90% inoperable (10% operable) and Industry *i* is assumed to be not directly affected. Suppose that, due to some risk management actions, both industries can recover in 60 days from 100% inoperability to only 1%. From Eq. (30), the *industry resilience coefficients* for Industries *i* and *j* are calculated in

$$k_i = \left(\frac{\lambda}{\tau}\right)_i \left(\frac{1}{1-a_{ii}^*}\right) = 0.0904,$$

$$k_j = \left(\frac{\lambda}{\tau}\right)_j \left(\frac{1}{1-a_{jj}^*}\right) = 0.0808, \ldots, \qquad (45)$$

where $1 - a_{ii}^* = 0.15$, $(\lambda/\tau)_i = 0.0136$, $1 - a_{jj}^* = 0.05$, and $(\lambda/\tau)_j = 0.004$. For both industries, the *industry resilience deviation coefficients* are assumed for simplicity to be half of their *industry resilience coefficients* in value ($\sigma_i = 0.5k_i$, $\sigma_j = 0.5k_j$). The system equations for this scenario can be formulated in Eq. (46) if the DIIM model in Eq. (9) is applied:

$$\begin{cases} x_i(t+\Delta t) - x_i(t) = [a_{ii}x_i(t) + a_{ij}x_j(t) + c_i(t) - x_i(t)][k_i\Delta t + \sigma_i\,\varepsilon_i(t)\sqrt{\Delta t}], \\ x_j(t+\Delta t) - x_j(t) = [a_{ji}x_i(t) + a_{jj}x_j(t) + c_j(t) - x_j(t)][k_j\Delta t + \sigma_j\,\varepsilon_j(t)\sqrt{\Delta t}]. \end{cases} \qquad (46)$$

In Eq. (46), $\varepsilon_i(t)$ and $\varepsilon_j(t)$ are two independent random variables with standard normal distributions $N(0,1)$, and the time increment Δt is assumed to be 0.25 day. Through simulation, the recovery process of the two industries is depicted in Figure 3. For this scenario, the final demand of the economy is assumed to be constant. The outputs are measured in monetary units, although

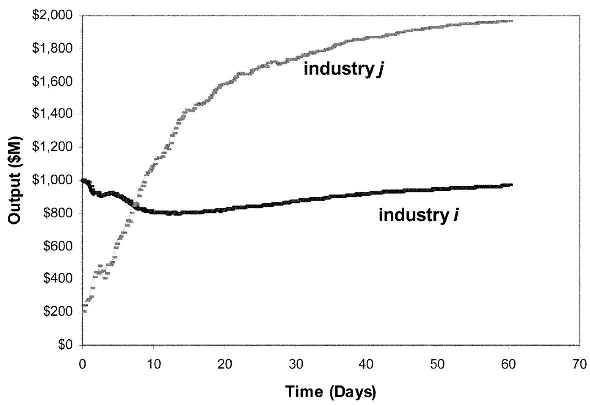


Figure 3. Recovery of the industry sectors, with uncertainty.

they can be transformed into the inoperability form through Eq. (2).

In Figure 3, Industry *j* begins to recover right after the attack. Industry *i* is affected due to its interdependency with Industry *j*, even though it is not directly attacked. When equilibrium is reached, Industries *i* and *j* are totally operable with their output levels at $1000 million and $2000 million, respectively. Unlike in the previous scenarios, the trajectories in Figure 3 are not smooth. This is caused by the short-term uncertainty during the recovery, modeled in the DIIM. The stochastic model therefore depicts the recovery of the economic sectors as an exponential form in the long run, combined with the stochastic characteristics in the short run. Figure 3 also reflects that, for both sectors, the degrees of uncertainty are higher at the beginning of the recovery period than they are in the later periods towards the end. In the final analysis, the stochastic component, as a supplementary parameter to the *industry resilience coefficient* in the DIIM, not only facilitates measuring recovery uncertainty for the economy sectors, it also enhances the detailed modeling capability of the DIIM.

7. ASSESSING AND MANAGING THE RISK OF A TERRORIST ATTACK TO INFRASTRUCTURE SYSTEMS USING THE DIIM AND THE BEA/RIMS II DATABASES

One advantage of using the IIM and the DIIM for interdependency analysis of economic sectors is that the underlying transaction data are readily available and supported by major ongoing data-collection efforts. The Bureau of Economic Analysis (BEA) of the U.S. Department of Commerce provides national I–O accounts, which can be used to generate the Leontief technical coefficient matrix for nearly 500 sectors of the U.S. economy. The Regional Input-Output Multiplier System (RIMS II) supplements and complements the national I–O account. It can be used to derive the Leontief technical coefficient matrix for economic sectors within a specific region. In the following example, the DIIM is applied to assess and manage the risks of a terrorist attack scenario using the BEA/RIMS II databases for the Commonwealth of Virginia.

7.1. Risk Assessment of Infrastructure Systems Using the DIIM

From the BEA/RIMS II data, the technical coefficient matrix $\mathbf{A}$ or $\mathbf{A}^*$ can be generated for Virginia using the methods in Haimes et al. [2005a, 2005b]. From $\mathbf{A}^*$, the *interdependency index* of each sector can be calculated by Eq. (21). Table II lists the Interdependency Index for some sectors and their ratios with the truck transportation sector.

The results in Table II show that the *air transportation* and *hospital and nursing* sectors are among the most interdependent sectors in Virginia, while the *insurance* and *broadcasting* sectors are comparatively less interdependent with other sectors of the economy. The *Interdependency Ratio over Truck Transportation Sector* column shows the relative degree of sector interdependency compared with truck transportation. According to Table II, the *insurance* and *broadcasting*

Table II. Sector Interdependency Index Examples and Ratios with Truck Transportation (Virginia)

Sector Name	Interdependency Index	Interdependency Ratio over Truck Transportation Sector
Insurance carriers and related activities	0.7329	0.82
Broadcasting and telecommunications	0.7754	0.87
Computer and electronic product manufacturing	0.9053	1.01
Machinery manufacturing	0.9565	1.07
Air transportation	0.9990	1.11
Hospitals and nursing and residential care facilities	0.9999	1.12
Truck transportation	0.8963	1.00

Table III. Top 10 Inoperable Sectors in the Attack Scenario

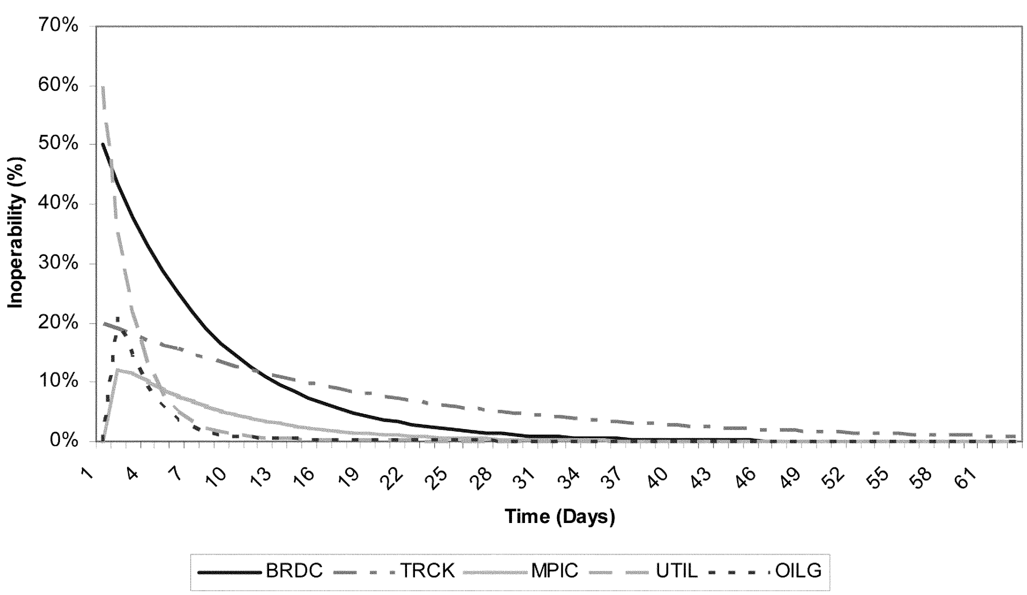
Sector Symbol	Sector Names
BRDC	Broadcasting and telecommunications
TRCK	Truck transportation
MPIC	Motion picture and sound recording industries
UTIL	Utilities
OILG	Oil and gas extraction
PETR	Petroleum and coal products manufacturing
PERF	Performing arts, museums, and related activities
PIPE	Pipeline transportation
RENT	Rental and leasing services and lessors of intangible assets
ELEC	Electrical equipment and appliance manufacturing

sectors are less interdependent on the *truck transportation* sector than are other sectors.

To illustrate the DIIM method using the Virginia regional data from BEA/RIMS II, we assume that a terrorist attack renders 20% inoperability to the *truck transportation* sector, 50% inoperability to the *broadcasting and telecommunications* sector, and 60% inoperability to the *utilities* sector. We assume that other sectors are initially unaffected in this scenario, and that no risk management was implemented. Individual sector recovery times for these three sectors, from initial inoperability after the attack to 1% inoperability, are assessed as follows: 60 days for *truck transportation*, 30 days for *broadcasting and telecommunications*, and 10 days for *utilities*. According to the given assumptions above, the *industry resilience coefficient* of the *truck transportation* sector can be calculated according to Eq. (29) as

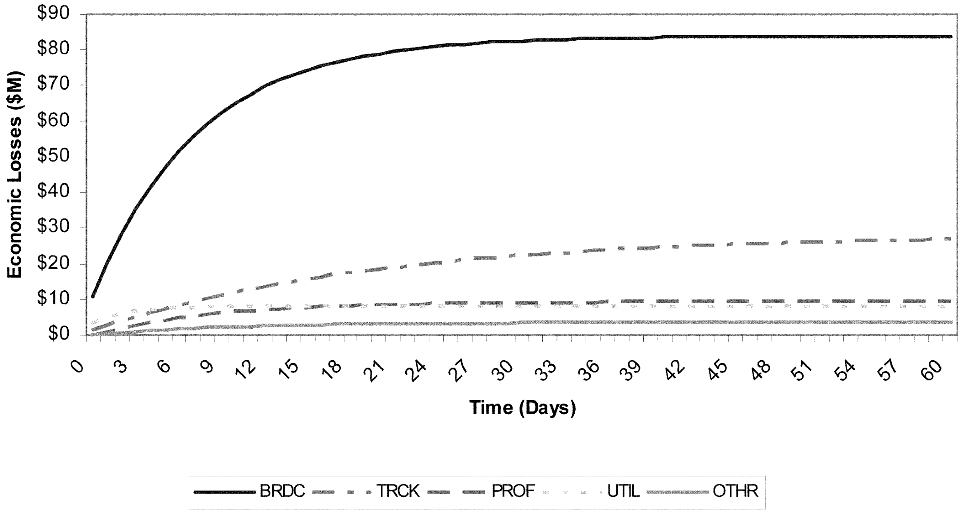
$$k_t = \frac{\ln[q_t(0)/q_t(T)]}{T}\frac{1}{1-a^*_{t,t}} = 0.0557/day, \ldots, \quad (47)$$

where $T = 60$, $q_t(0) = 0.2$, $q_t(T) = 0.01$, and $1 - a^*_{t,t} = 0.1037$. The symbol k_t represents the *resilience coefficient* of *truck transportation.* In this scenario, the subscripts t, b, and u are used to notate the *truck transportation*, *broadcasting and telecommunications*, and *utilities* sectors, respectively. The *resilience coefficients* for the *broadcasting and telecommunications* and *utilities* sectors are calculated as $k_b = 0.1682$/day and $k_u = 0.4124$/day using the approach shown in Eq. (47). Since other sectors of the economy are not directly attacked, it is assumed that they can quickly adjust their outputs during the recovery period; therefore, for simplicity their coefficient values are set to 1 in this scenario.



(See Table III for sector descriptions.)

Figure 4. Recovery of the top 5 inoperable sectors.



(See Table IV for the sector descriptions.)

Figure 5. Cumulative economic loss of the top 5 affected sectors.

Applying the DIIM in the inoperability form as in Eq. (19), the top 10 inoperable sectors can be determined and presented through simulation in Table III.

The detailed recoveries of the five most-affected sectors are depicted in Figure 4. *Truck transportation, broadcasting and telecommunications*, and *utilities* begin to recover from their initial inoperabilities after the attack. Though not directly attacked, other sectors that are interdependent with these three are also affected and inoperable during the recovery period. Through the DIIM, the recovery process of the interdependent sectors of the economy is modeled more explicitly. The inoperability of each sector can be obtained through the model at any time during the recovery.

Another supplementary metric that can be used to assess the consequences of the attack scenario shows the economic losses in dollar units during the recovery. Figure 5 depicts the cumulative economic loss for the top five most-affected sectors over a 60-day period.

Table IV lists the total economic loss for the sectors, including those in Figure 5, after 60 days.

It is important to note that comparing Tables III and IV reveals that the severity rankings based on inoperability and economic loss are not necessarily identical. For example, the *professional, scientific, and technical services* sector is the third most-affected sector in terms of economic loss, but it is not in the top 10 list of sectors based upon the inoperability measure. Therefore, both

Table IV. Top 10 Sectors Having Economic Losses in the Attack Scenario

Sector Symbol	Sector Names	Total Economic Losses after 60 Days ($M)
BRDC	Broadcasting and telecommunications	$83.9
TRCK	Truck transportation	$26.9
PROF	Professional, scientific, and technical services	$9.5
UTIL	Utilities	$8.2
OTHR	Other services	$3.6
BANK	Federal Reserve banks, credit intermediation and related services	$3.5
REAL	Real estate	$2.5
ADMI	Administrative and support services	$2.0
WTRD	Wholesale trade	$1.7
RENT	Rental and leasing services and lessors of intangible assets	$1.5

Table V. Costs and Benefits of the Three Risk Management Options

Risk Management Options	No Risk Management (Base)	r_t	r_b	$r_t + r_b$
Overall Economic Loss ($M)	$163.8	$143.9	$88.2	$68.2
Recovered Economic Loss ($M)	$0	$19.9	$75.6	$95.6
Risk Management Cost ($M)	$0	$0.0	$0.0	$0.0

inoperability and economic loss metrics are important for the development of risk management options.

7.2. Risk Management of Infrastructure Systems Using the DIIM

Based on the risk assessment, the risk management options are considered and the multiobjective optimization framework discussed in Section 4 is applied in this scenario to illustrate the cost-benefit tradeoff analysis and risk management efficacy calculation. This tradeoff analysis is enhanced through the deterministic DIIM by considering the economic impacts of an attack with and without risk management [Haimes and Chittester, 2005]. It is assumed that there are three risk management scenarios to reduce the recovery time of the *truck transportation* and *broadcasting and telecommunications* sectors in this case study: (i) Risk management option r_t, cost $19.9M, can reduce the recovery time of *truck transportation* from 60 days to 30 days. (ii) Risk management option r_b, cost $75.6M, can reduce the recovery time of *broadcasting and telecommunications* from 30 days to 10 days; and (iii) r_t and r_b can be combined. For simplicity, it is assumed that no risk management options are considered for the *utilities* sector in this scenario.

From Eq. (36), the economic losses for these three scenarios can be estimated by

$$Q = \left(\sum_{i=1}^{n} \hat{x}_i d_{it}\right) \frac{q_t(0)}{\omega_t} T_t(r_t, r_b) + \left(\sum_{i=1}^{n} \hat{x}_i d_{ib}\right) \frac{q_b(0)}{\omega_b} T_b(r_t, r_b) + \left(\sum_{i=1}^{n} \hat{x}_i d_{iu}\right) \frac{q_u(0)}{\omega_u} T_u. \tag{48}$$

In summary, the economic loss, recovered economic loss, and the cost of risk management are presented for each of the three risk management options in Table V. The risk management costs are given by the assumptions. When no risk management is taken, the overall economic loss can be derived by Eq. (48), where $T_t(r_r, r_b) = 60$ days and $T_b(r_t, r_b) = 30$ days. When risk management options r_t and r_b are considered, the recovery times for *truck transportation* and *broadcasting and telecommunications* sectors are reduced to 30 days and 10 days, respectively, in Eq. (48), and thus reduce the overall economic losses. The combination of r_t and r_b is also considered. The recovered economic loss in Table V is defined as the overall economic loss without

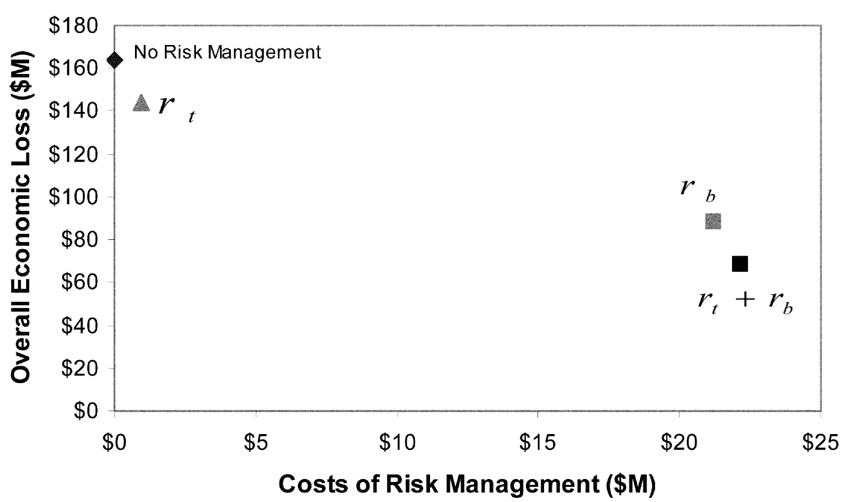


Figure 6. Costs and overall economic losses of the risk management options.

risk management, minus its value when various risk management options are in place.

Figure 6 further illustrates the tradeoffs between the costs of risk management and its benefits (recovered economic losses). For example, the three options presented in this case study are all Pareto-optimal solutions. Among them, $(r_t + r_b)$ can recover most losses with the "no risk management" scenario, but it requires the highest risk management investment.

8. CONCLUSIONS

This paper discusses the theory, methodology, and applications supporting the development of the DIIM. As an extension of the IIM, the DIIM captures the dynamics of the economic sector when there is a demand-reduction perturbation or an interruption caused by an attack or a natural disaster. Two basic categories of dynamic scenarios using the DIIM are addressed: (i) demand reduction and (ii) the recovery of sectors from attacks. The DIIM uses two measures—inoperability and economic loss—to quantify the consequences of risks. Thus, it provides important metrics for quantifying the efficacy of risk management. An interdependency index is also defined, as a measurement of one sector's interdependency with other sectors. Industry resilience coefficients are introduced into the DIIM as key parameters which characterize both the sectors' recovery speeds and the degrees of interdependency among them. These coefficients can be estimated through specific industry historical databases available or through expert opinions. One approach in this paper used surrogates for estimates of recovery times to calculate the industry resilience coefficients. In the event of disasters or terrorist attacks, the recovery time and the resilience coefficients are also affected by the deployment of risk management and public policies. Therefore, the industry resilience coefficients can represent the efficacy of risk management efforts that are planned for each of the sectors. Cost-benefit-risk tradeoff analyses can be conducted based upon the model and the framework proposed in this paper. One application of the DIIM is to assess the consequences to the economic system of a terrorist attack or natural disaster, using the BEA/RIMS II commodity-flow databases. As the example showed, the DIIM can calculate the inoperabilities and economic losses for each of the sectors throughout a given recovery period.

The IIM and its dynamic model retain several assumptions and limitations inherited from the original Leontief I-O structure. These include linearity, lack of behavioral content, lack of substitution possibilities, and others [Rose, 2004]. Therefore, in order to generate meaningful results in practice, the assumptions need to be verified to ensure the applicability of the model. Another limitation of the IIM and DIIM is that the models primarily deal with economic losses and associated risks; however, there are many other aspects of risk that are not directly reflected. These include life losses, personal freedom loss due to political changes, losses due to insurance, injuries, and changes in the international political landscape. In future research, the dynamic model can be expanded to include the transient period during the attack, which generates inputs to the recovery period that is addressed by the DIIM in this paper. In the current model, the capacity loss due to a major disruption is simplified as decreasing output. Therefore, another improvement of the model is to include capacity and resource constraint in major disruptions. The paper introduces the concept of a not-unlikely event in the context of a single terrorist attack. Although this is a reasonable assumption, we recognize that it is difficult to predict under various assumptions all terrorist scenarios as well as their implications. To better understand the nature of a terrorist threat, it is imperative to identify the possible consequences of terrorist attacks to specific infrastructure systems. In future research, optimization of terrorist scenarios will also be incorporated in the proposed model to obtain more accurate results.

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