Efficient ARQ Protocol for Hybrid Relay Schemes with Limited Feedback

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Abstract—When channel state information (CSI) is not available to the transmitter, outage events might happen and Automatic Repeat re-Quest (ARQ) is implemented to ensure reliable transmission in such case. In this paper, we consider a three nodes relay system with hybrid relay scheme, where the relay, based on its decoding status, could switch between decode-and-forward (DF) and compress-and-forward (CF) adaptively. We notice that CSI is required when CF is deployed and consider practical implementation issues by enhancing the ability of feedback channel from the destination to the relay to convey a few extra bits (only 2 bits in this paper) in addition to the ACK/NACK bit and propose a new ARQ scheme. The modified scheme allows the relay to utilize various relay schemes more flexibly according to its coding status and the extra feedback bits. ARQ strategies with hybrid relay schemes exhibit superior performance over direct transmission and pure DF, especially when the relay is close to the destination.

Index Terms—decode-and-forward (DF), compress-and-forward (CF), Automatic Repeat re-Quest (ARQ)

I. INTRODUCTION

The increasing worldwide demand for high volume communication of information motivates the usage of error control method based on Automatic Repeat re-Quest (ARQ). In this paper, we investigate the performance of ARQ under the framework of block fading relay system. The relay model was first introduced by Meulen [1] and substantially developed by Cover and El Gamal [2]. Three schemes are proposed including Amplify-and-forward (AF), decoded-and-forward (DF) and compress-and-forward (CF). DF has shown improvement both in achievable rate and outage behavior when the relay is close to the source [3]. However, when the relay is close to the destination, CF outperforms DF [9].

In block fading relay system, if the source has full CSI information, it is able to carefully choose optimal transmission rate. However, this assumption is unrealistic in practical implementation where we often assume only the receiver has the information. In such case, for selected transmission rate, outage events might happen eventually. The hybrid relay scheme shows improved outage performance [7]. It is natural to expect some improvement when the hybrid scheme is used with ARQ mechanism. When hybrid scheme applied, it is important to point out that although the source dose not, the relay requires full CSI knowledge for ideal Wyner-Ziv coding [6] when working in CF mode. This assumption could be valid if the relay to destination link is strong enough to support feedback of full CSI knowledge. Analysis under this assumption has been explored in [10]. We extend the analysis in some environments where this link is not that good and the feedback channel is extremely restricted. We propose more flexible hybrid scheme and modified ARQ strategy accordingly with small amount of extra feedback bits (only 2 bits from the destination in this work) in addition to the acknowledge bit. The rest of the paper is organized as follows. New ARQ strategy with extra feedback at the relay is proposed and the performance, in terms of outage probability, average reward, average inter-renewal time and throughput, is also studied in section II and III respectively. Section IV gives simulation results and the final section concludes.

Some words about notation. The mutual information of two vectors $X$ and $Y$ with i.i.d. elements is given by,

$$I(X; Y) = an, I(X'; Y')$$ (1)

Therefore, we use single-letter expression instead. We assume circularly symmetric complex Gaussian distribution (CSCGD) for the elements of the code throughout the paper to maximize the mutual information. All the logarithm is based in this paper is based on $e$ and nats is used as unit of information.

II. ARQ IN HYBRID RELAY SYSTEM

We consider the half-duplex relay model, where $S$, $R$ and $D$ denote source, relay and destination respectively and $c_0$, $c_1$ and $c_2$ are channel gains between $S$ and $D$, $S$ and $R$ and $R$ and $D$ respectively. Message $w$, is encoded into $n_c$ symbols and transmitted to the destination with the help of the relay. A complete frame with duration $T_F$ consists of two phases: relay-receive phase (phase 1 with duration $T_F=\alpha T_F$) and relay-transmit phase (phase 2 with duration $T_F=\alpha T_F$), where $\alpha$ is the half-duplex ratio representing the percentage of phase 1. We define codebooks used first as follows.

1) $X_{n_c}^w = \left[ X_{n_c}^w(0), ..., X_{n_c}^w(a) \right]$ has $an_c$ i.i.d. elements with unit power, where $n$ is frame index, the subscript denotes the node and the superscript indicates the phase where the codeword is transmitted.

2) $X_{n_c}^w = \left[ X_{n_c}(0), ..., X_{n_c}((1-\alpha)n_c) \right]$ has $(1-\alpha)n_c$ i.i.d. elements with unit power.

3) $X_{n_c}^w = \left[ X_{n_c}(0), ..., X_{n_c}((1-\alpha)n_c) \right]$ has $(1-\alpha)n_c$ i.i.d. elements with unit power and forms a MISO code with $X_{n_c}^w$.

In phase 1 of the $n$-th frame, the source broadcasts $X_{n_c}$ while the relay and the destination decode. The channels are assumed to be block Rayleigh fading channels. During the whole frame, the channel gains keep constant. The received signal at the relay and the destination can be written as

$$\text{Received Signal at Relay: } R = h_{SR}X + Z$$

$$\text{Received Signal at Destination: } D = h_{DR}R + Z$$
\[ Y_i = c_i \sqrt{\mu_i} X_i + n_i, \]
\[ Y_i = c_i \sqrt{\mu_i} X_i + n_i, \]
where \( \mu_i \), for \( i=0,1,2 \), are average SNR, \( n_{i,a} \) and \( n_{i,d} \) are the noise at the relay and destination respectively, both modeled by i.i.d. CCSGD with normalized variances \( \sigma_1^2 \) and \( \sigma_2^2 \). At the end of phase 1, if the decoding at the destination is successful, a positive acknowledgement (ACK) is sent back to the source and the transmission of the current message is ended. On the contrary, if the destination is not able to decode, the decoding and the transmission of the current message is ended. On the one hand, if the destination detects error but relay decodes successfully, DF is applied. Otherwise, we will perform more flexible relay according to the feedback bits. If \( \gamma_{0,n} \) is in the region \([\mu_0a_2, \mu_0a_3], [\mu_0a_3, \mu_0a_4] \) and \([\mu_0a_4, +\infty] \), we set \( a_3 \) as \( (\alpha - 1) / \mu_i \) and \( a_4 \) as \( (\alpha - 1) / \mu_i \).

Fig. 1 Flow chart of modified hybrid relay scheme

Fig. 1 is a flow chart of the operation at the relay. We give some explanation for the reason of this setting. When \( \gamma_{0,n} \) is above \( \mu_0a_3 \), the destination is able to decode. When the destination detects error but relay decodes successfully, DF is applied. Otherwise, we will perform more flexible relay according to the feedback bits. If \( \gamma_{0,n} \) is in the region \([\mu_0a_2, \mu_0a_3], [\mu_0a_3, \mu_0a_4] \), with our selection of \( a_3 \), the quality of the source to destination link is good enough for the successful decoding even with direct transmission only. When \( \gamma_{0,n} \) is below \( \mu_0a_3 \), there is no chance of successful decoding for direct transmission and we try to deploy CF to boost mutual information conveyed. However, if the link between the relay and the destination has low quality \( (\gamma_{0,n} < \mu_0a_3) \), when deploy CF, the compression noise is high and the amount of mutual information conveyed to the destination is almost same as direct transmission [9]. The chance of successful decoding for CF could be very small as well. Keeping working in CF mode may not be an efficient way because we are likely wasting the time of phase 2. So we let the system switch back to broadcasting phase to start a new frame to save time. If the relay to destination link is good \( (\gamma_{2,n} > \mu_0a_1) \), we look at \( \gamma_{0,n} \) to decide if we should do CF. When \( \gamma_{0,n} < \mu_0a_2 \), the amount of side information is too small to do efficient CF [7]. If \( \gamma_{0,n} \) lies in the region \([\mu_0a_2, \mu_0a_3], \) we can carefully choose the compression rate and transmission rate of the relay to avoid breaking two constraints of Wyner-Ziv coding [6].

\[
\text{cons1: } R_{r,n} = (1-\alpha) I(X^r;Y^r) \geq R_{r,n} \\
\text{cons2: } R_{r,n} = \alpha I(Y^r,W) - I(Y^r,W) \geq R_{r,n}
\]

where \( W \) is an auxiliary random variable with variance \( \sigma_{n,d}^2 \) (compression noise), \( R_{0,n} \) is relay supported rate, \( R_{r,n} \) is relay transmission rate, \( R_{r,n} \) is ideal transmission rate of the compressed signal and \( R_{r,n} \) is selected transmission rate of the compressed signal respectively. With Gaussian input, we can choose \( R_{r,n} \) and \( R_{r,n} \) as

\[
\text{Let } f(\alpha, \gamma_{0,n}, \gamma_{1,n}) = (1-\alpha) \log \left( 1 + \frac{\gamma_{0,n}}{1 + \gamma_{1,n}} \right)
\]

\[
R_{r,n} = f(\alpha, \mu_0a_3, \mu_0a_4) \leq (1-\alpha) I(X^r;Y^r) = f(\alpha, \gamma_{0,n}, \gamma_{1,n})
\]

\[
R_{r,n} = \alpha \log \left( 1 + \frac{\gamma_{0,n}}{1 + \mu_{0a_1}} \right) \geq \alpha I(Y^r,W) - I(Y^r,W) = \log \left( 1 + \frac{\gamma_{0,n}}{1 + \gamma_{1,n}} \right)
\]

Let \( R_{r,n} \geq R_{r,n} \), we can deploy CF and recover the ‘observation’ with small distortion. When equalities are satisfied, the mutual information is maximized.

The stage diagram is shown in fig. 2. State \( A_n \) stands for the state where the source is broadcasting in phase 1 of the \( n \)-th frame. State \( B_{m,n} \) is the state where the relay has decoded \( w \) in phase 1 of the \( n \)-th frame, the system operates in DF mode and phase 2 is repeated for the \( m \)-th time. State \( C_n \) defines the stage where the relay quantizes, compresses the message and forwards the compressed version during phase 2. State \( G \) is a state representing the situation where the relay keeps silent and the source transmits during phase 2. If the
system is in this state, the destination is definitely able to decode the packet with our setting. State D denotes the state where the destination has decoded the message successfully. State E is the state where the transmission of current message fails after maximum retransmission limit N is reached and w is discarded. Define
\[ E_{\gamma_i} = \{ \gamma_i < e^{-T_i} \} \]
and
\[ E_{\gamma_i} = \{ \gamma_i \geq e^{-T_i} \} \]

where \( \gamma_i \) is instantaneous SNR and follows exponential distribution with mean \( \mu_i \) for \( i = 0, 1, 2 \). The transition probabilities are updated in Appendix A as,

\[
P(A_i \rightarrow D) = \Pr(\{ \gamma_i \text{ satisfies } R \})
\]

\[
P(A_i \rightarrow B_i) = \Pr(\{ \gamma_i < R, \gamma_i \text{ satisfies } R \})
\]

\[
P(A_i \rightarrow C_i) = \Pr(\{ \gamma_i \geq \mu_i, \gamma_i \leq \mu_i \})
\]

\[
P(A_i \rightarrow A_{i+1}) = 1 - P(A_i \rightarrow D) - P(A_i \rightarrow B_i) - P(A_i \rightarrow C_i) - P(A_i \rightarrow G)
\]

\[
P(B_i \rightarrow B_i) = \Pr(\{ \gamma_i > R, \gamma_i \text{ satisfies } R \})
\]

\[
P(B_i \rightarrow B_{i+1}) = \Pr(\{ \gamma_i < R, \gamma_i \text{ satisfies } R \})
\]

\[
P(B_i \rightarrow D) = 1 - P(B_i \rightarrow B_i) - P(B_i \rightarrow B_{i+1}) - P(B_i \rightarrow G)
\]

\[
P(C_i \rightarrow A_i) = \Pr(\{ \gamma_i > R, \gamma_i \geq \mu_i, \gamma_i \leq \mu_i \})
\]

\[
P(C_i \rightarrow D) = 1 - P(C_i \rightarrow A_i)
\]

\[
P(C_i \rightarrow E) = P(C_i \rightarrow A_i)
\]

\[
P(C_i \rightarrow D) = 1 - P(C_i \rightarrow E)
\]

with
\[
I_{\gamma_i}(\gamma_i) = I(X_{\gamma_i}; Y_{\gamma_i}) = \alpha \log(1 + \gamma_i)
\]

\[
I_{\gamma_i}^+ = \alpha \log(1 + \gamma_i) + (1 - \alpha) \log(1 + \gamma_i + \gamma_i)
\]

\[
I_{\gamma_i}^- = \alpha \log(1 + \gamma_i) + (1 - \alpha) \log(1 + \gamma_i + \gamma_i)
\]

\[
I_{\gamma_i} = \log \left( \frac{1 + \gamma_i}{1 + \gamma_i} \right)
\]

\[
\gamma_i = g(\alpha, \gamma_i, \gamma_i, f(\alpha, \mu_i, \mu_i), g(\alpha, \mu_i, \mu_i)) = \frac{1}{1 + \gamma_i} \left( 1 - \frac{\gamma_i}{1 + \gamma_i} \right)
\]

With transition probabilities, we can evaluate the throughput by investigating the random reward \( \Phi \) associated with selected rate \( R \) and random inter-renewal time \( T \) [8].

### III. THROUGHPUT ANALYSIS

To evaluate the average transmission time, we investigate the time already used when the system is in state \( A_0 \). When the system departs from state \( A_1 \), it may go through different route \( rot \), which is a combination of sub-route \( A_j \rightarrow A_{j+1} \) and \( A_j \rightarrow C_j \rightarrow A_{j+1} \). The probability of \( rot \) is

\[
\Pr(rot) = \Pr(A_1 \rightarrow C_1) \Pr(C_1 \rightarrow A) \Pr(A_1 \rightarrow A) \cdots \Pr(A_{N-1} \rightarrow A)
\]

With \( i.i.d. \) \( \gamma_i \), we have

\[
P_{aw} = \Pr(\text{success}) = \Pr(C_1 \rightarrow A_{n+1}) = \Pr(A_{n+1} \rightarrow E)
\]

\[
P_{aw} = \Pr(A_{n+1} \rightarrow E) = \Pr(A_n \rightarrow E) = 1 - n
\]

Supposing \( \text{rot} \) consists of \( \ell \) for \( 0 < \ell < n-1 \), sub-routes \( A_j \rightarrow A_{j+1} \) and \( n-1-l \) sub-routes \( A_j \rightarrow C_j \rightarrow A_{j+1} \), the probability and the delay of route \( \text{rot} \) can be expressed as,

\[
\Pr(rot) = \left( P_{aw} \right) (P_{aw} P_{aw})^{n-1-l}
\]

Define

\[
P_{aw} = \left( n-1 \right) (P_{aw} P_{aw})^{n-1-l}
\]

\[ E_{\text{AA}(K)} \] stands for the case where the destination decodes in phase 1 of \( K \)-th frame. \( E_{\text{AB}(L,K)} \) defines the situations where the relay decodes in \( L \)-th frame but the destination cannot decode until \( K \)-th frame \( (K \geq L) \). \( E_{\text{AC}(M)} \) describes those situations where the system is able to decode in CF mode in the \( M \)-th frame. We also consider the events where the system is working in direct transmission mode. The average time associated with successful decoding is given by,

\[
E_{\text{aa}}(n, l) = \sum_{n=1}^{\infty} E_{\text{aa}}(n, l)
\]

\[
E_{\text{aa}}(n, l) = \sum_{n=1}^{\infty} E_{\text{aa}}(n, K, l)
\]

\[
E_{\text{ac}}(n) = P_{aw} \Pr(C_1 \rightarrow D)(l - \alpha + n - l) - (K - n)(l - \alpha)
\]

\[
E_{\text{ac}}(n) = \sum_{n=1}^{\infty} E_{\text{ac}}(n)
\]

\[
E_{\text{ac}}(n, l) = P_{aw} \Pr(A_{n+1} \rightarrow E) l - \alpha + n - l)
\]

\[
E_{\text{ac}}(n, l) = \sum_{n=1}^{\infty} E_{\text{ac}}(n, l)
\]

In above cases, \( \Phi = R \). When outage events happen, \( \Phi \) equals zero and the related time should also be considered. The average time associated with outage events in CF and DF mode is

\[
E_{\text{aw}}, P_{aw} \Pr(C_1 \rightarrow E)(l - \alpha + N - l)
\]

\[
E_{\text{aw}}(n, l) = P_{aw} \Pr(A_{n+1} \rightarrow E) (l - \alpha + n - l) + (N - n)(l - \alpha)
\]

\[
E_{\text{aw}}(n, l) = \sum_{n=1}^{\infty} E_{\text{aw}}(n, l)
\]

In state \( A_{N+1} \), there is some possibility that the system should stop if the CSI criterion is not satisfied. The related average time is

\[
E_{\text{aw}} = P_{aw} \Pr(A_{n+1} \rightarrow E) (l + 1) - \alpha + n - l)
\]
The outage probability is obtained

\[ P_{out} = P_1^w P_2^w + \sum_{n=0}^{N-1} \sum_{r=0}^{n-1} P_1^n \Pr(A_n \rightarrow B_{n, r}) P_0^{nn} \Pr(B_{N-n+1, \alpha} \rightarrow E) \]  

The average random reward and average random inter-renewal time \( T \) are expressed as

\[
E(\Phi) = R(1 - P_{out})
\]

\[
E(T) = E_{\Phi} + E_{\Phi \Phi} + E_{\Phi \Phi} + E_{\Phi \Phi} + E_{\Phi \Phi} + E_{\Phi \Phi} + E_{\Phi \Phi}
\]

The average throughput is able to be maximized,

\[ T = \max_{k=n,n+1} \frac{E(\Phi)}{E(T)} = \max_{k=n,n+1} \frac{R(1 - P_{out})}{E(T)} \]  

Note \( a_1 \) and \( a_2 \) should also be optimized through numerical computing.

IV. Simulation Results, Comparisons, and Discussion

In this section, we report the results of the maximum throughput of the various strategies proposed. It is important to compare these protocols with three strategies described as follows,

i. Direct Transmission (DT).

ii. Pure DF (PDF): In this case, the relay only performs DF. If decoding at relay is unsuccessful, a new frame starting with broadcasting phase is transmitted.

iii. Modified Hybrid scheme with full CSI at the relay \( MHS_{CSI} \): This is when the relay has full CSI knowledge and the feedback bits from the destination.

The transition probabilities are obtained from multiple-integration in this paper, which are calculated through numerical integration. Fig. 3 depicts the throughput gain (throughput / throughput of direct transmission) of proposed schemes. In both scenarios, \( MHS_{CSI} \) has the largest throughput. Explanation is addressed as below. The system may benefit from the case that \( \gamma_0 \) is below certain threshold \( \mu_0 a_2 \). In such case, the ‘amount’ of side information is so small that it is hardly able to help the Wyner-Ziv coding. Therefore, the compression noise is very large and the total mutual information conveyed to the destination is so small that outage events are very likely to happen. In other words, we are very possible to ‘waste’ the time when we still try to do CF in phase 2. Stopping current frame and starting a new frame might be a wiser choice. The inter-renewal time can be reduced by the shortening of the frame and the throughput is improved accordingly. In short, the transition from state \( A_n \) to \( A_{n+1} \) directly might enhance the throughput. Interestingly, with optimal selection of \( a_1 \), the performance of \( MHS_{Efb} \) is quite close to \( MHS_{CSI} \). Fig. 4 depicts the scenario where \( \mu_2 \) is approaching infinity while keeps \( \mu_0 \) and \( \mu_1 \) constant at 0dB. The performance of PDF saturates because of the limitation of the low quality link from source to the relay. If the link from source to relay is poor, with increased \( \mu_2 \), the possibility of successful decoding at the relay is still low and the system has less possibility to enjoy the MISO channel capacity. However, with hybrid scheme, when \( \mu_2 \) is increased, the system is able to do CF and benefits from receive diversity. In extreme case, \( \mu_2 \) is approaching infinity, the relay and the destination can be considered as co-located, share all the information and achieve almost full receive diversity.

V. CONCLUSION

In this paper, we investigate the ARQ performance in hybrid relay system. New strategy is proposed and its performance is analyzed under the assumption of very small number of extra feedback bits from the destination to the relay to indicate the region where \( \gamma_0 \) and \( \gamma_2 \) lies. The new scheme exhibits substantial improvement over PDF, especially when the relay is close to the designation. Future work includes the extension to Hybrid-ARQ scheme which is able to buffer previous
frames at both the relay and the destination combines all the available signals to decode.

Appendix A

If the system is in state $A_n$, the average mutual information is

\begin{align*}
I(\gamma_n) &= I(X'_n:Y'_n) = \log(1 + \gamma_n) \\
I(\gamma_n) &= I(X'_n:Y'_n) = \log(1 + \gamma_n)
\end{align*}

The destination is able to decode if $aI(\gamma_{0,n}) \geq R$ and the transition probability $P(A_n \rightarrow D)$ is

$$P(A_n \rightarrow D) = \Pr(aI(\gamma_{0,n}) \leq R) = \Pr(\gamma_n \geq e^{R - 1})$$

If the destination detects error, but the relay is able to decode when $aI(\gamma_{1,n}) \geq R$ and $aI(\gamma_{0,n}) < R$. $P(A_n \rightarrow B_{1,n})$ is given by,

$$P(A_n \rightarrow B_{1,n}) = \Pr(aI(\gamma_{1,n}) \leq R, E_{\text{out}}^{b_1}) = \Pr(\gamma_n < e^{R - 1}, \gamma_n \geq e^{R - 1})$$

Afterwards, the system begins to work in DF mode. When phase 2 is repeated for the $m$-th ($m \leq N+1-n$) time, the maximum average mutual information conveyed during this period is given by,

$$I(\gamma_{2,m+1}, \gamma_{2,m+1}) = I(X_{2,m+1}':Y_{2,m+1}', \gamma_{2,m+1}) = \log(1 + \gamma_{2,m+1} + \gamma_{2,m+1})$$

The information conveyed to the destination consists of two parts. The first part is $aI_0$ which is achieved during phase 1, another part is $(1-\alpha)I_{\text{out}}$ accumulated in phase 2. The destination processes two parts jointly and is able to decode if

$$R \leq I_{\text{out}}^{a} = aI(\gamma_{0,n}) + (1-\alpha)I_{\text{out}}(\gamma_{2,m+1}, \gamma_{2,m+1})$$

It is important to point out that $I_{\text{out}}^{a}$ and $I_{\text{out}}^{a}$ are not independent. The transition probability $P(B_{m,n} \rightarrow B_{m+1,n})$ is the probability of unsuccessful decoding in DF mode conditioned to the event where the destination is not able to decode in previous ($m-1$) phase 2 and the relay decodes the message in phase 1 of the $n$-th frame. The transition probability is

$$P(B_{m,n} \rightarrow B_{m+1,n}) = \Pr(I_{\text{out}}^{a} < R | I_{\text{out}}^{a} < R, \ldots, I_{\text{out}}^{a} < R, aI(\gamma_{0,n}) < R)$$

Note that since $I_{\text{out}}^{a}$ and $I_{\text{out}}^{a}$ are not independent, the condition part in the transition probability expression cannot be eliminated.

The transition probability $P(A_n \rightarrow G)$ is, given the new setting, expressed as

$$P(A_n \rightarrow G) = \Pr(E_{\text{out}}, \mu, \mu, \mu, \mu, \mu)$$

The transition probability $P(G \rightarrow D)$ is 1 because of the selection of $\delta$. The system works in CF mode only when the quality of link relay to destination is above some threshold and CF can be efficiently applied. The transition probability is given by,

$$P(A_n \rightarrow G) = \Pr(E_{\text{out}}, \mu, \mu, \mu, \mu, \mu)$$

In CF mode, we select transmission rate of the relay as,

$$R_n = (1-\alpha)\log\left(1 + \frac{\gamma_n}{1 + \mu_n}\right) + R_n = (1-\alpha)\log\left(1 + \frac{\gamma_n}{1 + \mu_n}\right)$$

and transmission rate of the compressed signal as,

$$I_{\text{out}}^{a} = aI(\gamma_{0,n}) + (1-\alpha)I(\gamma_{2,m+1}, \gamma_{2,m+1})$$

When equalities are satisfied, the total mutual information conveyed to the destination is maximized as,

$$I_{\text{out}}^{a} = aI(\gamma_{0,n}) + (1-\alpha)I(\gamma_{2,m+1}, \gamma_{2,m+1})$$

The transition probability $P(C_n \rightarrow A_{n+1})$ is

$$P(C_n \rightarrow A_{n+1}) = \Pr(I_{\text{out}}^{a} < R | E_{\text{out}}, \gamma_n \geq \mu, \mu, \mu, \mu, \mu)$$

The rest of the transition probabilities can be easily obtained due to the complementary counterparts.

REFERENCES


