LETTER

Guard Zone Protected Capacity in Multi-Cell MISO Networks under Fading and Shadowing*

Qi XI(a), Nonmember, Chen HE(b), Member, and Lingge JIANG(c), Nonmember

SUMMARY  The exact power distribution of the inter-cell interference is obtained explicitly for cell edge users who are surrounded by circular guard zones. Compared with recent works, the underlying channel model is generalized from Rayleigh fading to a combination of Nakagami fading and Gamma shadowing. In addition, asymptotic analysis shows that the mean power of intercell interference changes from infinite to finite with a guard zone. Based on this interference distribution, the average capacity at the cell edge is further obtained. Special case approximation indicates that the capacity scales proportionally to the exponential of the guard zone size. Analytical capacities are validated by Monte Carlo simulations.

key words: capacity analysis, guard zone, multi-cell, shadowing

1. Introduction

Due to full spectrum reuse and the dense spatial reuse, intercell interference (ICI) becomes the dominant factor, which limits the capacity of the current and the next generation cellular networks. By leveraging stochastic modeling of ICI, Ge et al. [1] recently proposed new results in capacity analysis for multi-cell MIMO network under Nakagami fading and Gamma shadowing. However, the interference model adopted in [1] assumes that the interfering base stations are located in the entire plane. This assumption is not true in a cellular network. In fact, user equipments (UEs) are surrounded by a region without interfering base stations, referred to as guard zone. With guard zone, inter-cell interference models will be significantly different.

Although there is a substantial literature which discusses ICI distribution with the guard zone, the closed form of the distribution in a randomly distributed network under both fading and shadowing is yet to be obtained. Current modeling methods fall into three categories. The first category assumes deterministic locations of base stations [2], [3]. As cells shrinking in size, base station position becomes more and more random, making the deterministic assumption of base station locations difficult to apply. Another category assumes random distribution of interfering sources, and approximates the cumulated interference as a certain known distribution. The approximated distributions include Middleton Class A distribution [4], symmetric truncated stable distribution [5] and log-normal distribution [6], [7]. The distributions given by these methods, however, are still not in the exact form. The third category tries to obtain the closed form statistics of ICI for a network following the Poisson Point Process. Due to the high complexity of derivations, available results are still restricted to interfering channels with Rayleigh fading only [8], [9], while the effect of shadowing is not included.

This letter first derives the power distribution of ICI for cell edge users who are surrounded by the circular guard zone. Interfering base stations are assumed to follow the Poisson Point Process. Compared with channel models used in [8], [9], in this letter, fading is generalized from Rayleigh to Nakagami, the effect of shadowing is included, and the antenna setting is extended from the single antenna configuration to MISO configuration. Under this channel model, the closed form moment generating function (MGF) of the interference power distribution is derived. Asymptotic analysis to the MGF shows that the mean power of the ICI changes from infinite to finite with the guard zone. Based on the MGF of ICI, the average capacity at cell edge is further obtained. Compared with the capacity analysis in [1], the effect of noise, which is ignored in [1], is also included in this letter. Special case approximation indicates that the capacity scales proportionally to the exponential of the guard zone size. Monte Carlo simulations verify that the cell edge capacity will be significantly underestimated if the interfering base stations are assumed to be scattered in the entire plane. In addition, the shadowing effect also greatly impacts the capacity.

2. System Model

This letter focuses on down-link MISO channels for cell edge users. Without loss of generality, one single antenna UE at the cell edge is selected for analysis. For simplicity, the distance between the UE and the associated base station is fixed as $R_d$ (Fig. 1) [1]. Interfering base stations are supposed to follow the Poisson Point Process [10]. They are separated from the UE by at least the minimum cell radius ($R_i$ in Fig. 1), forming a region without interfering base stations. As the UE moves along the cell edge, the region without interfering base stations is irregular in shape but it is inner bounded by a circle of radius $R_i$, referred to as the...
guard zone. For adjacent base stations, their interference can be reduced by using the Coordinated Multi-Point (CoMP) transmission, resulting in an expanded guard zone. However, cooperative schemes cannot be successfully applied to distant base stations due to the constraint of the maximum tolerable delay spread. This constraint translates to a maximum cooperative range.

The UE is surrounded by a circular guard zone with radius \( R \), \( R = R_o \) for the non-cooperative case, and \( R = R_c \) for the cooperative case.

which represents the fading-shadowing effect from the UE to the interfering base station \( b \). Path loss is proportional to the \( \sigma_r \), (usually 3–6) order of the distance \( r_b \) between this base station and the UE. The Heaviside step function \( H(r_b - R) \) is used as a location constraint for interfering base stations.

The moment generating function (MGF) of ICI is defined as \( M_{I_{ei}}(s, R) = e^{\{\exp(-sI_{ei})\}} \), where \( E[\cdot] \) is the expectation operator. Substitute \( I_{ei} \) by equation (1), the MGF of ICI equals to \( E[e^{(-\sum_{b=1}^{\infty} I_{eb} e^{-s_{I_{eb}} H(r_b - R)})}] \). Using the Campbell theory [11], this MGF can be further derived as \( \exp\left(-2\pi\lambda_{BS} \int_{R}^{\infty} \left[1 - E_l\left(e^{-s_{I_{eb}} r} \right)\right] \right) \), where \( \lambda_{BS} \) is the base station density. Let \( t = s_{I_{eb}} \) and then the MGF can be obtained as [11]:

\[
M_{I_{ei}}(s, R) = \exp\left(-\pi \lambda_{BS} \alpha s t \int_0^{2\pi R - \alpha} 1 - E_l\left(e^{-s_{I_{eb}} r} \right) \right) dt \tag{2}
\]

where \( \alpha = 2/\sigma_r \). Equation (2) is not in explicit form due to the improper integral part which can be denoted as:

\[
Z(s, R) = \int_0^{2\pi R - \alpha} 1 - E_l\left(e^{-s_{I_{eb}} r} \right) dt \tag{3}
\]

Next, \( Z(s, R) \) is evaluated explicitly under the Generalized K channel. Utilizing the moment generating function of the Generalized K distribution given in [12], the expectation term in Eq. (3) can be explicitly derived as:

\[
E_l\left(e^{-s_{I_{eb}} r} \right) = \int_0^\infty e^{-s t} f(t) dt = \left(\frac{m}{\Omega}\right)^{\frac{m+1}{2}} \frac{\Gamma(m+\frac{1}{2})}{\Gamma(m)} \exp\left(-\frac{m^2}{\Omega^2}\right) \tag{4}
\]

where \( \Gamma(a, b, z) \) is the confluent hyper-geometric function of the second type. Using the change of variable \( m \lambda(\Omega t)^{-1} = z \), and substituting Eq. (4) into Eq. (3), yield:

\[
Z(s, R) = \left(\frac{m \lambda}{\Omega}\right)^{-\alpha} \int_{\alpha + m \Omega}^{\infty} \exp\left[-\alpha \Gamma(-\alpha)(\alpha + m \Omega)^{-\alpha} \right] dz \tag{5}
\]

Denote the lower limit \( m \lambda R^{-\alpha} (\Omega t)^{-1} \) in Eq. (5) as \( M \). Since variable \( R \) appears only in \( M \), the function \( Z(s, R) \) can be written as \( Z(s, M) \). To obtain the closed form of \( Z(s, M) \), the integral at \( [M, \infty) \) is evaluated by subtracting the integral at \([0, M]\) from the integral at \([0, \infty)\). The integral from zero to infinity can be evaluated as:

\[
Z(s, M = 0) = -\left(\frac{m \lambda}{\Omega}\right)^{-\alpha} \frac{\Gamma(-\alpha)(\alpha + m \Omega)^{-\alpha}}{\Gamma(\alpha)(m \Omega)} \tag{6}
\]

which is consistent with the result in [1]. For the general
\[
\mathcal{M}_{\text{ICI}}(s, R) = \exp \left\{ \pi \lambda_{BS} / \Omega s \right\} \left( m \lambda / \Omega s \right)^{-\alpha} \Gamma(-\alpha) \Gamma(\alpha + \lambda) \Gamma(\alpha + m) / \Gamma(\lambda) \Gamma(m) + R^2 \left[ 1 / \alpha - (m \lambda R^{2/\alpha} / \Omega s)^{\lambda} \right. \\
\times \left. \Gamma(-\lambda + m) / \Gamma(\lambda) \Gamma(m) \right) + \left[ (m \lambda R^{2/\alpha} / \Omega s)^{\lambda} \right. \\
\times \left. \Gamma(-\lambda + m) / \Gamma(\lambda) \Gamma(m) \right) \}
\]

Inserting Eqs. (6), (7) into Eq. (5), and further substituting the expression of \( Z(s, R) \) into Eq. (3), gives the closed form MGF of ICI. The Meijer-G function \( G^{2,2}_{2,3}(\cdot) \) can be represented by more junior functions, namely, the generalized hyper-geometric function \([13, eq.(9.303)]\). The simplified MGF is shown on the top of this page (eq.(8)), where \( \text{F}_2(a, b; c, d; z) \) represents the generalized hyper-geometric function. Interference power distribution of ICI can be further obtained by performing the inverse Laplace transform.

Since the moments of the interference distribution are determined by the behavior of the MGF around zero, asymptotic analysis at \( s \to 0 \) is conducted to get more insights into Eq. (8). The case \( s \to 0 \) corresponds to \( M \to \infty \). Expanding the Meijer G function at infinity \([14]\), yields:

\[
\lim_{M \to \infty} \frac{M^{-\alpha}}{\Gamma(\lambda) \Gamma(m)} G^{2,2}_{2,3} \left( M, 1, \alpha + 1, \alpha + \lambda, \alpha + m, 0 \right) = \\
= \frac{1}{\alpha} \Gamma(-\alpha) \Gamma(\alpha + \lambda) \Gamma(\alpha + m) / \Gamma(\lambda) \Gamma(m) M^\alpha - \lambda m_1 / (\alpha - 1) M + \lambda(1 + \lambda) m_1 (1 + m_1) / 2(\alpha - 2) M^2 + O \left( 1 / M^3 \right)
\]

where \( O(\cdot) \) represents the big O notation. Substituting the series expansion (9) into Eq. (5) and then into Eq. (3), after the cancellation of terms, the asymptotic form of MGF when \( R > 0 \) can be derived as:

\[
\lim_{s \to 0} \mathcal{M}_{\text{ICI}}(s, R) = \exp \left\{ c_1 R^{2 - \sigma} s + c_2 R^{-\sigma + \frac{1}{2}} s^2 + O(s^3) \right\}
\]

where \( c_1 = 1 / \pi am_1 \Omega \lambda_{BS} / (\alpha - 1)m, c_2 = -1 / 2(\alpha - 2)m^2 \lambda \). Equation (10) shows that the basic property of the interference power distribution is changed due to the guard zone. Without the guard zone, the interference power follows alpha stable distribution \([11]\), whose mean is unbounded. While based on Eq. (10), even with a small guard zone (as long as \( R \neq 0 \)), the mean power of the interference is changed to a finite value given by \(-c_1\).

### 4. Capacity Analysis

Although decoding interference symbols and intended symbols simultaneously can achieve higher rate \([15]\), these schemes are difficult to implement in practice. Therefore, interference symbols are treated as noise in this letter. Furthermore, full channel state information is assumed to be available at the base stations. Under such conditions, cell edge capacity can be calculated using the MGF of ICI and the MGF of the intended link \([9]\):

\[
C(R) = \int_0^\infty \left\{ \mathcal{M}_{\text{ICI}} \left( \frac{P_I R^{2 - \sigma}}{\sigma_n^2} s \right) \left[ 1 - \mathcal{M}_{\text{BS}} \left( \frac{P_{0} R^{2 - \sigma}}{\sigma_n^2} \cdot \exp(-\alpha s) \right) \right] \right\} ds
\]

where \( C(R) \) represents cell edge capacity with a guard zone of radius \( R \). \( P_I \) represents signal power from interfering base stations, and \( P_0 \) represents the power from the associated base station. \( \mathcal{M}_{\text{ICI}} \) is given by Eq. (8), and \( \mathcal{M}_{\text{BS}} \) is the MGF of the intended link given by Eq. (4).

To get more information about the impact of the guard zone, special case analysis is conducted for the high SNR regime, when the intended link is subjected to path loss only. Although this case is not practical, it is helpful for understanding the general trend of the guard zone protected capacity. In this special case, \( \mathcal{M}_{\text{BS}} \) decays to zero faster than a negative exponential function of \( s^\sigma \) (eq. (2)), the capacity \( C(R) \) is dominated by the integral over the neighborhood of \( (P_I / \sigma_n^2) s = 0 \). In the high SNR regime, the scaling factor \( (P_I / \sigma_n^2) \) is large thus \( s \) also approaches to zero. As \( s \to 0 \), \( \exp(-\alpha s) \approx 1 - \alpha s \approx 1 - \exp \left( - \frac{P_{0} R^{2 - \sigma}}{\sigma_n^2} s \right) \approx \frac{P_{0} R^{2 - \sigma}}{\sigma_n^2} s \). Substituting these two simplifications and the first order asymptotic result (eq.(10)) into the formula of \( C(R) \) above, the special case capacity can be approximated by \( C(R) \approx -\frac{P_{0} R^{2 - \sigma}}{\sigma_n^2} R^{\frac{1}{2} - \sigma} \), where \( c_1 \) is given by Eq. (10). This approximation shows that the special case capacity scales proportionally to \( R^{\frac{1}{2} - \sigma} \). Since \( \frac{1}{2} - \sigma = \sigma - 2 > 1 \), this indicates a significant impact of the guard zone.
5. Result and Discussion

The analytical results in Sect. 3 and Sect. 4 are verified by Monte Carlo (MC) simulation. Simulation area is in size of 10 km x 10 km with UE located at the center. Urban micro scenario is considered with default base station density set to be 1/π x 0.5 km². The minimum cell radius $R_c$ and the maximum cooperation range $R_{cc}$ are set as 0.3 km and 1 km respectively. Other parameters are configured as: $m = 1$, $\Omega = 1.256$, $\sigma_r = 4.0$, $R_s = 0.5$ km. In all the scenarios, theoretical results match well with simulation results.

Figure 2 makes comparisons with the capacity results of [1]. Comparisons are done in the high SNR regime, with $P_0 = P_1 = 30$ dBm and $\sigma_n^2 = -100$ dBm. Under all numbers of Tx antennas at the associated base station, and all scenarios of base station density, capacity will be significantly underestimated if interfering base stations are assumed to be scattered in the entire plane. On average, this underestimation reaches 27% for non-cooperative cases, and 69% for cooperative cases, which is consistent with the trend given by the special case analysis in Sect. 4.

Figure 3 makes comparisons with the capacity results generated by [8], [9] (No Shadowing). Two shadowing levels are simulated, with $\sigma_f = 0.1$ and 0.3 respectively. Theoretical capacities given by [8], [9] are not able to capture the impact of shadowing effect. The impact of shadowing is severer at low SNR regime. This is because both the intended link and the interference links are impacted by shadowing and their effects to the capacity are compromised at the high SNR regime.

6. Conclusion

Closed-form statistics of the guard zone protected ICI are obtained under nakagami-$m$ fading and gamma shadowing. Asymptotic analysis shows that the property of ICI distribution is fundamentally changed by the guard zone. Average cell edge capacity is further derived. Both special case analysis and numerical simulations demonstrate that assuming interfering base stations, scattering in the entire plane, will significantly underestimate the average capacity. The impact of fading environment to capacity is also studied.

References