

Classical Electrodynamics

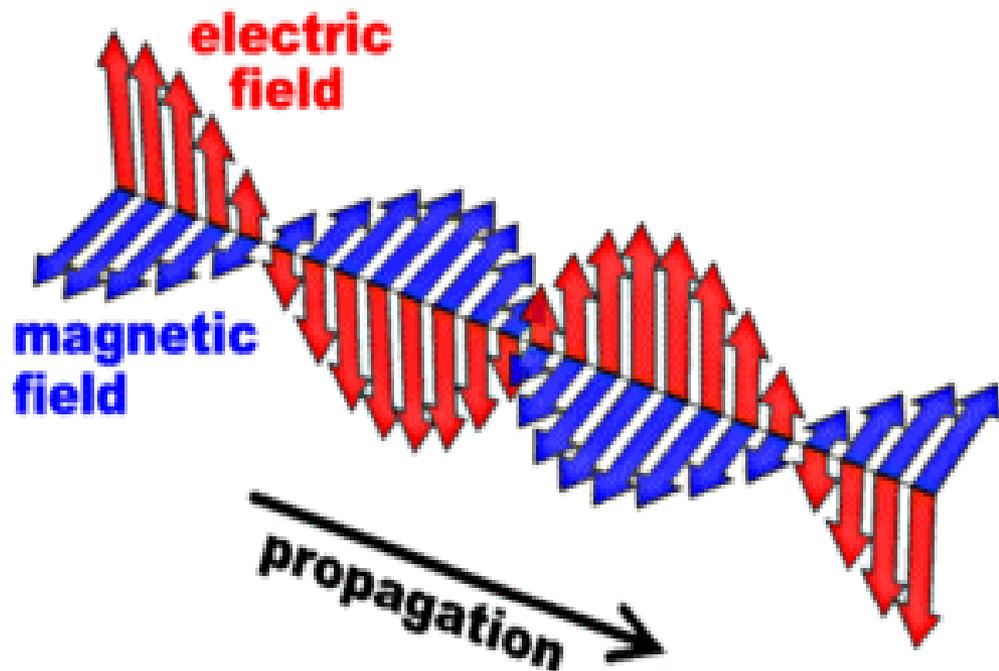
Electromagnetic waves

Agenda

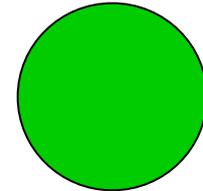
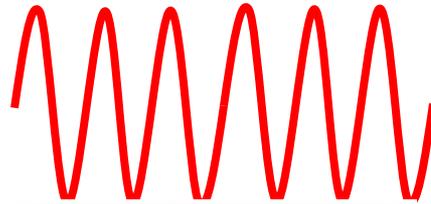
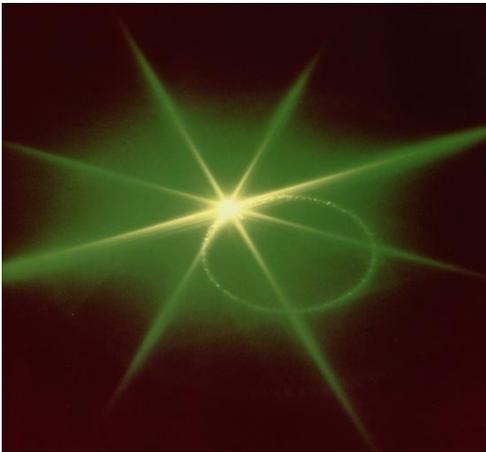
- How to describe electric and magnetic fields?
- Maxwell equations,
- How speed of light is related to the constants of electricity and magnetism?
- Electromagnetic waves. Spectrum of electromagnetic waves (different kinds of EM waves and their properties),
- How can we see? – our natural detector

What are Electromagnetic Waves?

- Electromagnetic waves are waves which do not require matter to transfer energy.
- Electromagnetic waves need no medium like sound and water waves.
- Electromagnetic wave can travel in a vacuum
- Electromagnetic waves all travel 300,000,000 meters per second in a vacuum



Light – EM wave or particle ?



- **particle (photon)**
- **EM wave**
- **dualism** -

- Isaac Newton (1642-1727)

- Huygens (1629-1695)

Fresnel (1788-1827)

wave-particle duality, De Broglie (1924)

Wave model

- electromagnetic theory

Photon model

- quantum electrodynamics

What are Electromagnetic Waves?

- The electromagnetic wave can be described by Maxwell equations:

„The time varying magnetic field acts as a source of electric field, as well as time varying electric field acts as a source of magnetic field”

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{j}$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0$$

Examples of electromagnetic generators



Forms of electromagnetic waves

- Visible light 400 to 700 nm – This is the only form of electromagnetic wave visible to the human eye.
- Ultraviolet – these cause our skin to darken (tan) and can even damage our skin. The ozone layer protects us from most UV radiation from the sun.
- X-rays – these EM waves pass through much matter. It doesn't pass through dense bone, so it is useful in determining whether a bone is broken or not.

Forms of electromagnetic waves

- Gamma rays – these EM waves are very penetrating and can severely damage cells.
- Infrared – these waves are responsible for the heat we feel. Heat seeking missiles detect infrared sources such as tanks or aircraft.

Forms of electromagnetic waves

- Microwaves – these waves are used obviously to heat food. As these waves pass through food, it causes the particles to vibrate resulting in the heating of the food. Microwaves are also used in communication.
- Radio waves – these waves include both TV and radio waves. Signals are transmitted and devices with antennas can receive the signals.

(540-1650 KHz) (88-108 MHz) Microwave

AM Radio FM

1 GHz 100 GHz

Infrared far near

100 microns 1

Visible

Ultraviolet near far

"Soft" X rays "Hard"

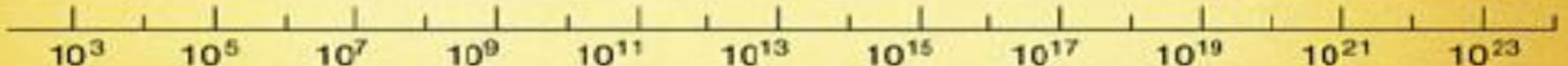
Gamma rays

700 600 500 400

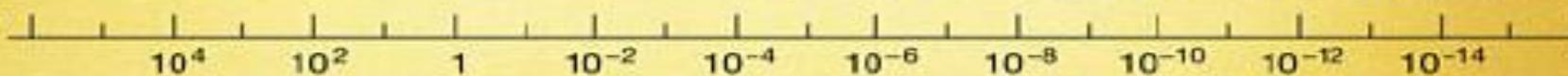
Nanometers



Frequency (Hertz)



Wavelength (meters)



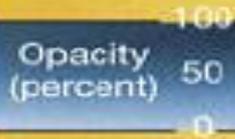
Scale



Radio window

Optical window

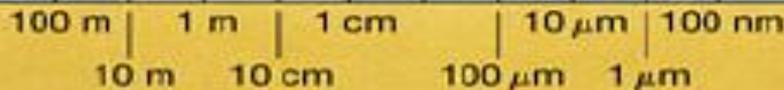
Opacity (percent)



Atmosphere is opaque

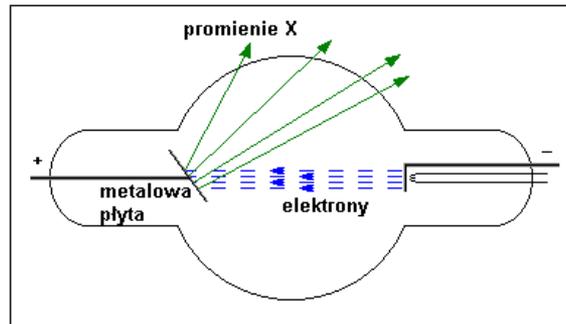
Transparent

Atmosphere is opaque



8 November 1895

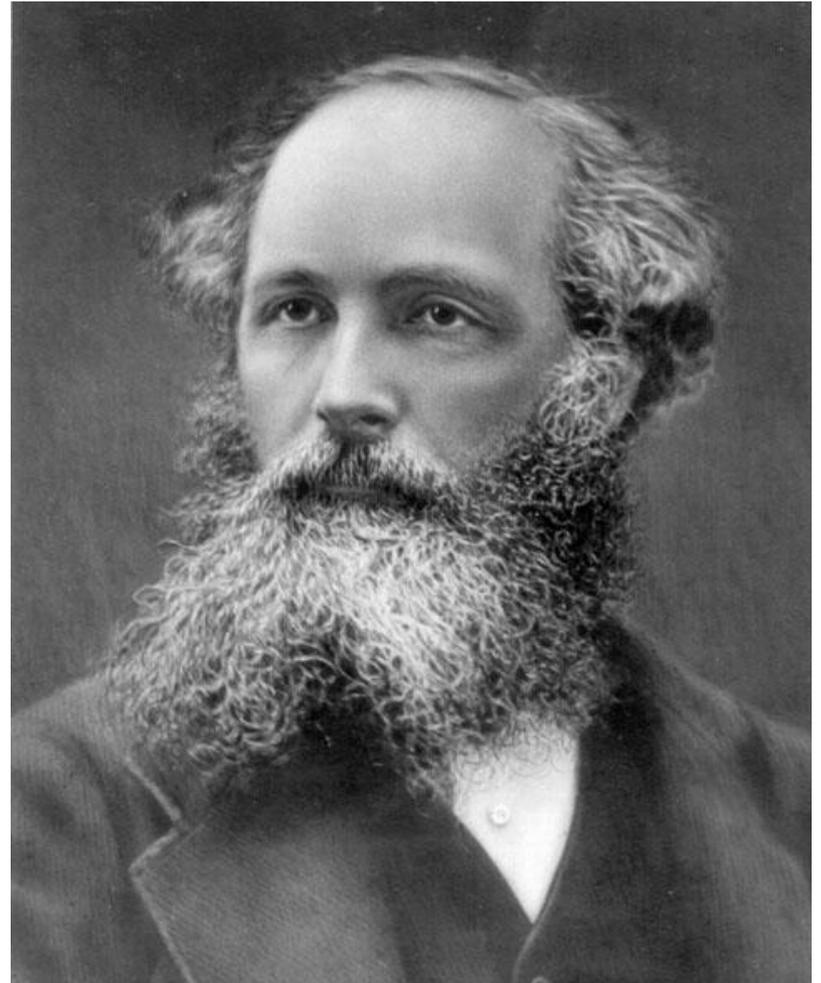
- This day a German scientist Wilhelm Röntgen started his research on cathode rays during experiments with vacuum lamps.



Otrzymywanie promieni X

Maxwell and his EM theory

- In 1865 James Clerk Maxwell created a mathematical theory that joined electricity and magnetism,
- It explained existing experiments with electromagnetism,
- ...and made new predictions.



Maxwell and his EM theory

- There are 4 equations, which can be written in integral form or differential form.

Gauss's Law (E) $\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}, \quad \nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$

Gauss's Law (B) $\oint_S \mathbf{B} \cdot d\mathbf{A} = 0 \quad \nabla \cdot \mathbf{B} = 0$

Ampere's law $\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{A} + \frac{\partial}{\partial t} \int_S \mathbf{D} \cdot d\mathbf{A} \quad \nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial}{\partial t} \mathbf{D} .$

Faraday's law $\oint_{\partial\Sigma} \mathbf{E} \cdot d\mathbf{\ell} = - \iint_{\Sigma} \frac{\partial}{\partial t} \mathbf{B} \cdot d\mathbf{A} \quad \nabla \times \mathbf{E}(\mathbf{r}, t) = - \frac{\partial \mathbf{B}(\mathbf{r}, t)}{\partial t}$

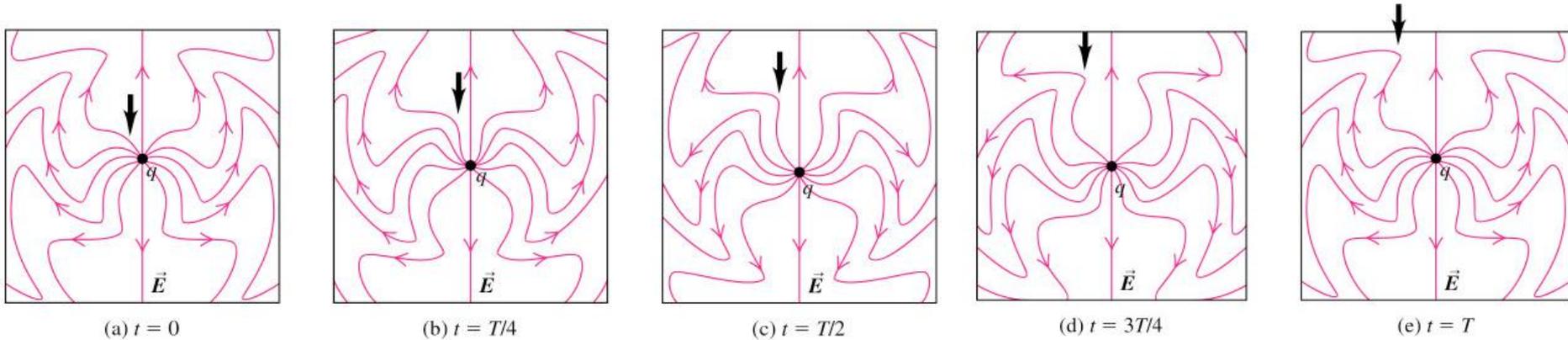
Maxwell and his EM theory

- Maxwell's equations can be used to derive the wave equation for electromagnetic waves.
- The wave speed is the speed of light c .

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}.$$

- This means that if a charged particle is *accelerating*, then EM waves are radiating outward from it.

$$\begin{aligned} \frac{d^2 y(x,t)}{dx^2} - \frac{1}{v^2} \frac{d^2 y(x,t)}{dt^2} &= 0 \\ \frac{d^2 E(x,t)}{dx^2} - \mu_0 \epsilon_0 \frac{d^2 E(x,t)}{dt^2} &= 0 \\ \frac{d^2 B(x,t)}{dx^2} - \mu_0 \epsilon_0 \frac{d^2 B(x,t)}{dt^2} &= 0 \\ \nabla^2 \vec{\mathbf{E}} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2} &= 0 \\ \nabla^2 \vec{\mathbf{B}} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{\mathbf{B}}}{\partial t^2} &= 0 \end{aligned}$$



- Electric field lines of a point charge oscillating in simple harmonic motion (during one period T)
- The arrow shows one kink in the lines of electric field as it propagates outwards from the point charge.
- Their magnetic field (**not shown in figure**) comprises circles that lie in planes perpendicular to these figures and concentric with the axis of oscillation,

Maxwell's equations

Maxwell's equations - Gauss's law

- Also known as Gauss's flux theorem,
- Formulated by Carl Friedrich Gauss in 1835,
- Relates the distribution of electric charge to the resulting electric field,

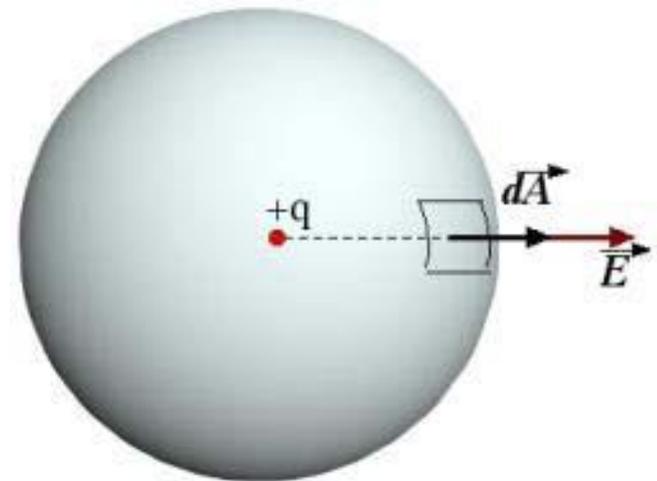
The electric flux through any closed surface is proportional to the enclosed electric charge.

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0},$$

a surface integral denoting the electric flux through a closed surface S

total charge enclosed by S divided by the electric constant.

$$\epsilon_0 = 8.854 \dots \times 10^{-12} \text{ F/m}$$



Maxwell's equations - Gauss's law

in differential form:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

Divergence of electric field

ρ – charge density

$\epsilon_0 = 8.854 \times 10^{-12}$ F/m

These two forms are correct for electric charges in vacuum

Maxwell's equations - Gauss's law

Inside the material media, the electric field causes a displacement of electric charges, which results in generation of induced charges.

$\vec{D} = \epsilon \vec{E}$  The contribution of medium can be expressed using the electric induction.

$$\oint_S \mathbf{D} \cdot d\mathbf{A} = \Phi_{D,S} = Q_{free}$$

where $\Phi_{D,S}$ is the flux of the electric displacement field \mathbf{D} through S , and Q_{free} is the free charge contained in V .

$$\nabla \cdot \mathbf{D} = \rho_{free}$$

The divergence of the electric displacement field is equal to the free electric charge density ρ_{free}

Maxwell's equations - Gauss's law for magnetism

- The magnetic field \mathbf{B} has divergence equal to zero.

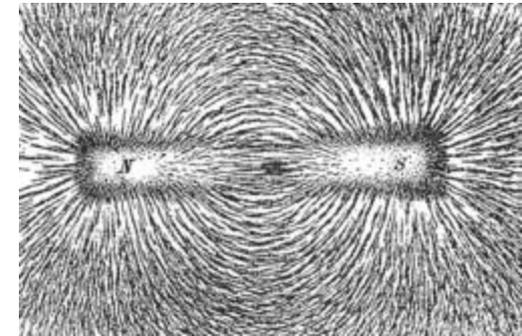
$$\nabla \cdot \mathbf{B} = 0$$

(magnetic monopoles does not exist – only magnetic dipoles)

the net flux of the magnetic field out of the surface

$$\oint_S \mathbf{B} \cdot d\mathbf{A} = 0$$

- For each volume element in space, there is exactly the same number of "magnetic field lines" entering and exiting the volume. No total "magnetic charge" can build up in any point in space.*



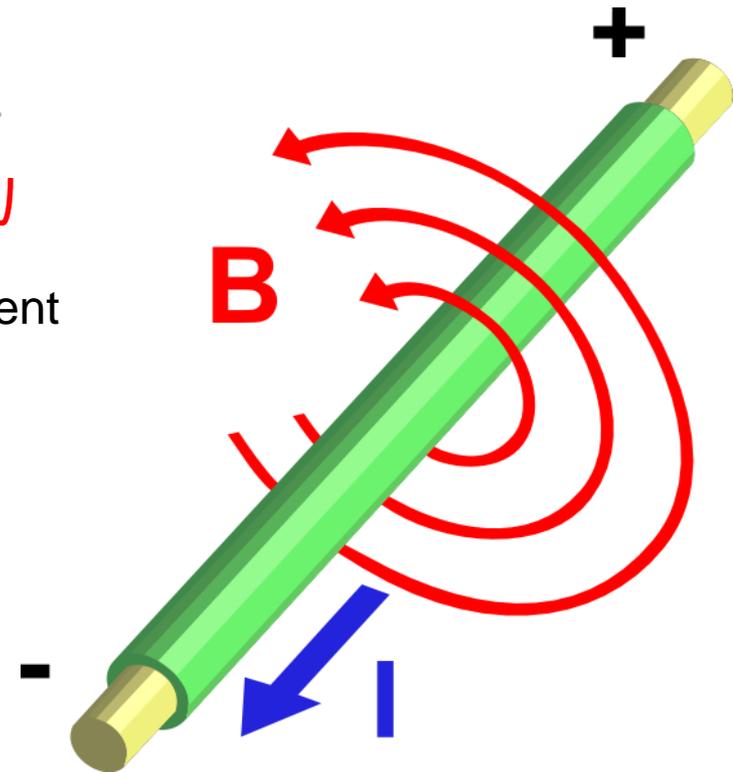
Maxwell's equations – Ampère's law

- Relates the integrated magnetic field around a closed loop to the electric current passing through the loop,
- Corrected by Maxwell (by including the displacement current)

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{A} + \underbrace{\frac{\partial}{\partial t} \int_S \mathbf{D} \cdot d\mathbf{A}}_{\text{Displacement current}}$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial}{\partial t} \mathbf{D} .$$

The magnetic field is generated by current and electric field changes



Maxwell's equations – Faraday's law of induction

A changing magnetic field creates an electric field

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\frac{\partial \mathbf{B}(\mathbf{r}, t)}{\partial t}$$

$$\oint_{\partial \Sigma} \mathbf{E} \cdot d\boldsymbol{\ell} = - \iint_{\Sigma} \frac{\partial}{\partial t} \mathbf{B} \cdot d\mathbf{A} = - \frac{\partial}{\partial t} \iint_{\Sigma} \mathbf{B} \cdot d\mathbf{A}$$

One restriction:

Due to existence of only partial time derivatives, this equations can be used when the test charge is stationary in a time varying magnetic field.

When a charged particle moves in a magnetic field, it does not account for electromagnetic induction.

Maxwell's equations – Faraday's law of induction

- It is possible to show that electric field variation causes generation of electric current flow, perpendicular to the movement direction,
- In a copper pipe we drop a magnet and register the time of the fall,
- When we repeat the experiment, but in the copper pipe with thin cuts (0,5 mm) made along the pipe, the magnet will fall 5 times slower.



Maxwell's equations – continuity equations

$$\mathbf{D}(\mathbf{r}, t) = \epsilon_0 \mathbf{E}(\mathbf{r}, t) + \mathbf{P}(\mathbf{r}, t)$$

Electric induction \mathbf{D} can be expressed as sum of electric constant (ϵ_0) \times \mathbf{E} and polarization density (\mathbf{P}),

$$\mathbf{H}(\mathbf{r}, t) = \frac{1}{\mu_0} \mathbf{B}(\mathbf{r}, t) - \mathbf{M}(\mathbf{r}, t)$$

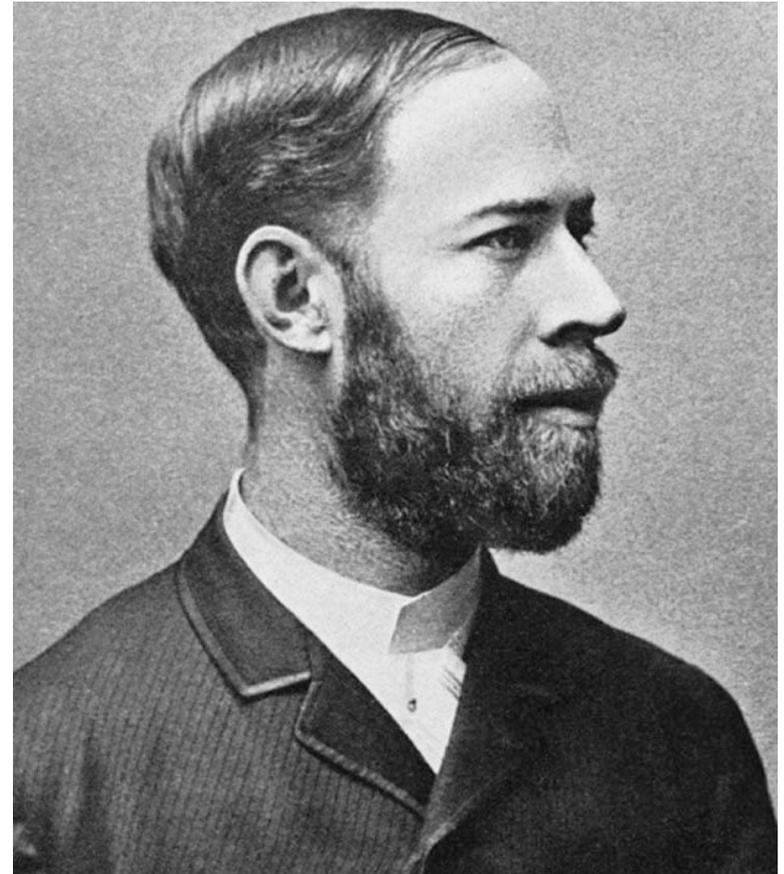
Magnetic field intensity \mathbf{H} can be expressed as difference of magnetic field density \mathbf{B} per magnetic constant and magnetization \mathbf{M}

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

Two fields (\mathbf{E} and \mathbf{H}) are orthogonal to each other and moves with the speed c

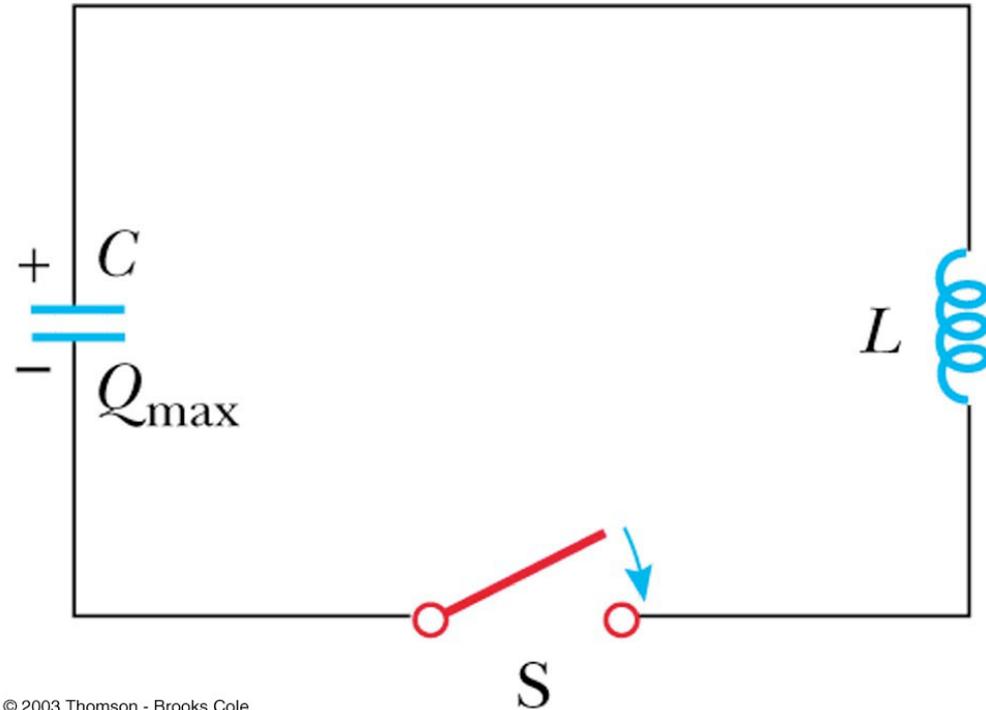
The EM waves

- The EM waves predicted by Maxwell were discovered by Heinrich Hertz in 1887.
- He used an LC circuit with an alternating source.
- This discovery was exploited by Marconi (Radio)



Hertz's Basic LC Circuit

- When the switch is closed, oscillations occur in the current and in the charge on the capacitor,
- When the capacitor is fully charged, the total energy of the circuit is stored in the electric field of the capacitor



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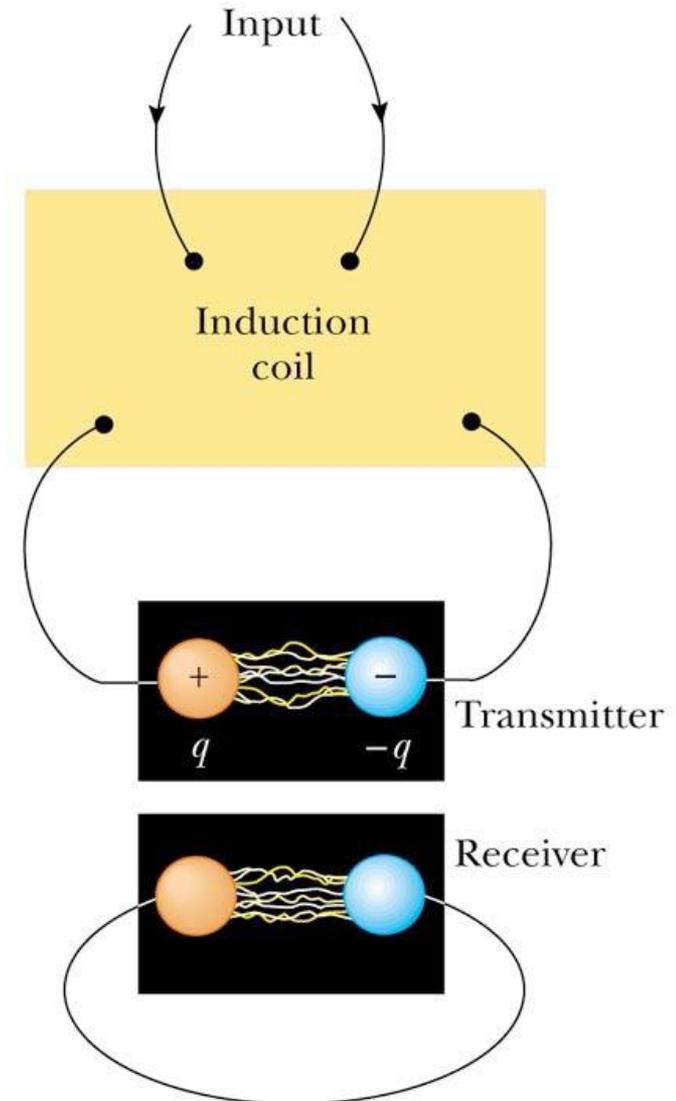
– At this time, the current is zero and no energy is stored in the inductor.

Hertz's Basic LC Circuit

- As the capacitor discharges, the energy stored in the electric field decreases ,
- At the same time, the current increases and the energy stored in the magnetic field increases,
- When the capacitor is fully discharged, there is no energy stored in its electric field,
 - The current is at a maximum and all the energy is stored in the magnetic field in the inductor,
- The process repeats in the opposite direction,
- There is a continuous transfer of energy between the inductor and the capacitor.

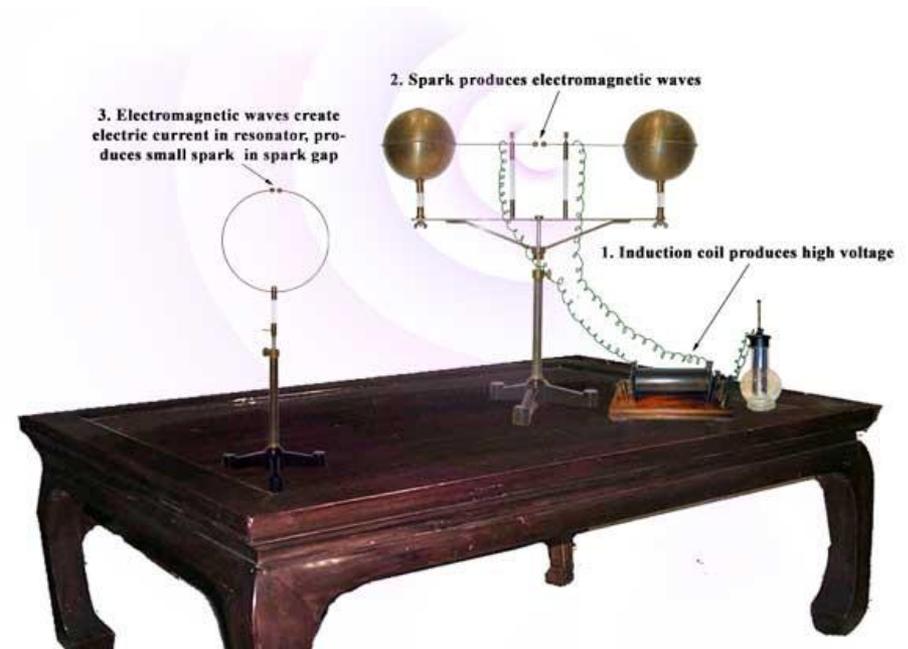
Hertz's experimental setup

- An induction coil is connected to two large spheres forming a capacitor,
- Oscillations are initiated by short voltage pulses,
- The inductor and capacitor form the transmitter,



Hertz's experiment

- Several meters away from the transmitter is the receiver
 - This consisted of a single loop of wire connected to two spheres,
 - It had its own inductance and capacitance
- When the resonance frequencies of the transmitter and receiver matched, energy transfer occurred between them,



Hertz's conclusions

- Hertz hypothesized the energy transfer was in the form of waves
 - These are now known to be electromagnetic waves
- Hertz confirmed Maxwell's theory by showing the waves existed and had all the properties of light waves
 - They had different frequencies and wavelengths

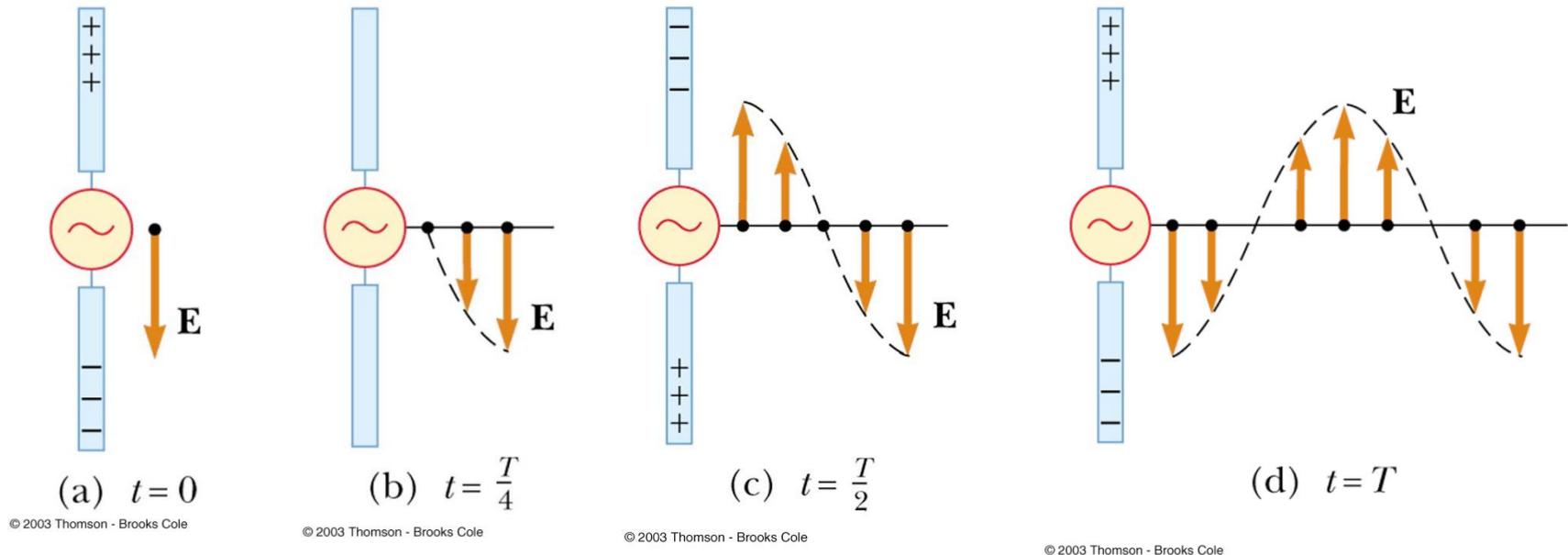
Hertz's Measure of the Speed of the Waves

- Hertz measured the speed of the waves from the transmitter
- – He used the waves to form an interference pattern and calculated the wavelength,
 - From $v = f \lambda$, v was found
 - v was very close to 3×10^8 m/s, the known speed of light
- This provided evidence in support of Maxwell's theory

Electromagnetic Waves Produced by an Antenna

- When a charged particle undergoes an acceleration, it must radiate energy
 - If currents in an ac circuit change rapidly, some energy is lost in the form of EM waves
 - EM waves are radiated by any circuit carrying alternating current
- An alternating voltage applied to the wires of an antenna forces the electric charge in the antenna to oscillate

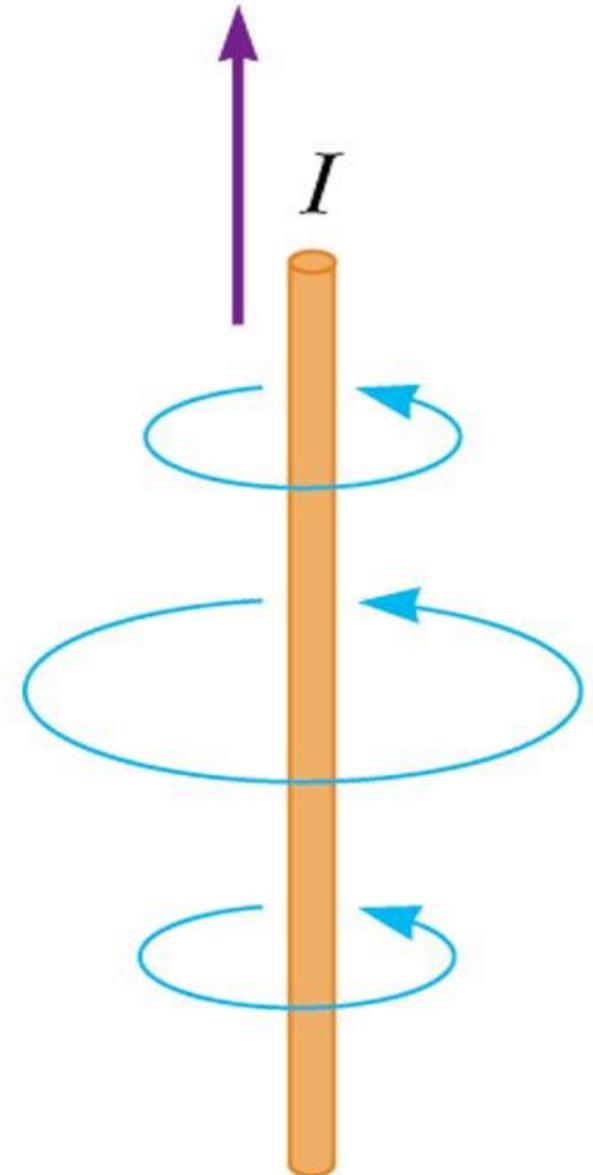
Electromagnetic Waves produced by an Antenna



- Two rods are connected to an AC source, charges oscillate between the rods (a)
- As oscillations continue, the rods become less charged, the field near the charges decreases and the field produced at $t = 0$ moves away from the rod (b)
- The charges and field reverse (c)
- The oscillations continue (d)

Electromagnetic Waves produced by an Antenna

- Because the oscillating charges in the rod produce a current, there is also a magnetic field generated
- As the current changes, the magnetic field spreads out from the antenna

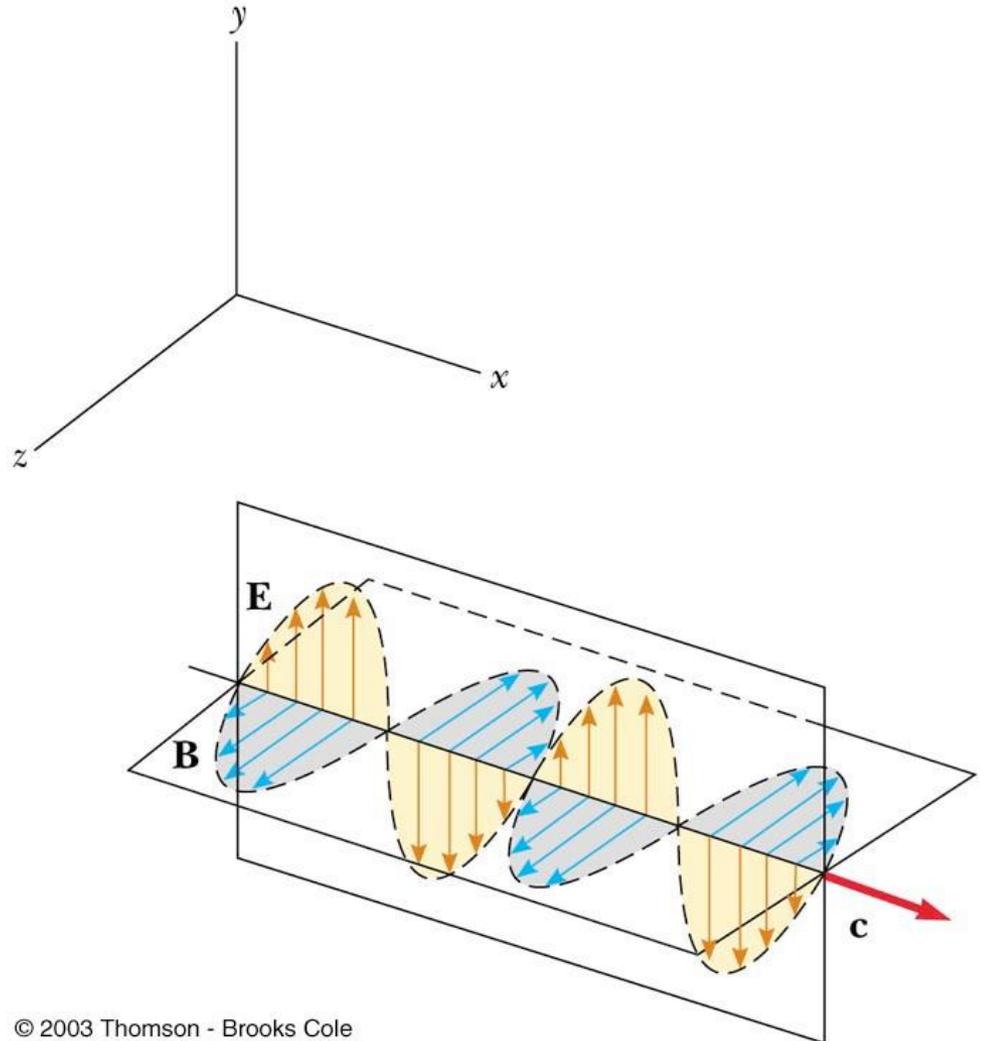


Charges and Fields, Summary

- Stationary charges produce only electric fields
- Charges in uniform motion (constant velocity) produce electric and magnetic fields,
- Charges that are accelerated produce electric and magnetic fields and electromagnetic waves.
- A changing magnetic field produces an electric field
- A changing electric field produces a magnetic field
- These fields are *in phase*
 - At any point, both fields reach their maximum value at the same time

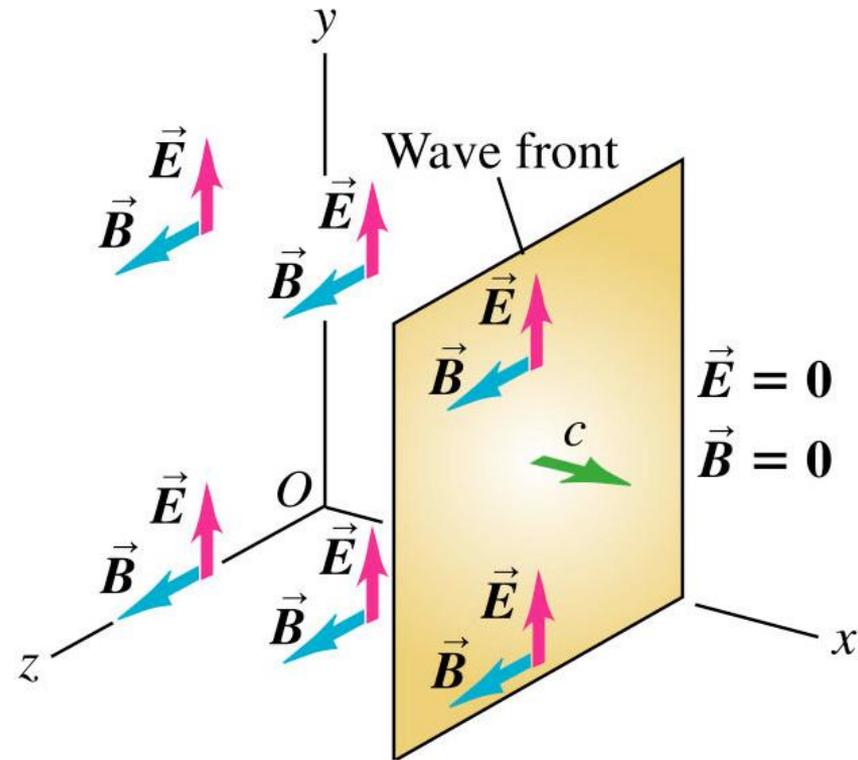
Electromagnetic Waves are Transverse Waves

- The **E** and **B** fields are perpendicular to each other
- Both fields are perpendicular to the direction of motion
 - Therefore, EM waves are transverse waves



A Plane Wave

- A *wave front* is the locus of adjacent points such that the points are all at the same phase.
- A *plane wave* is one in which the wave fronts are planes perpendicular to the direction of propagation.

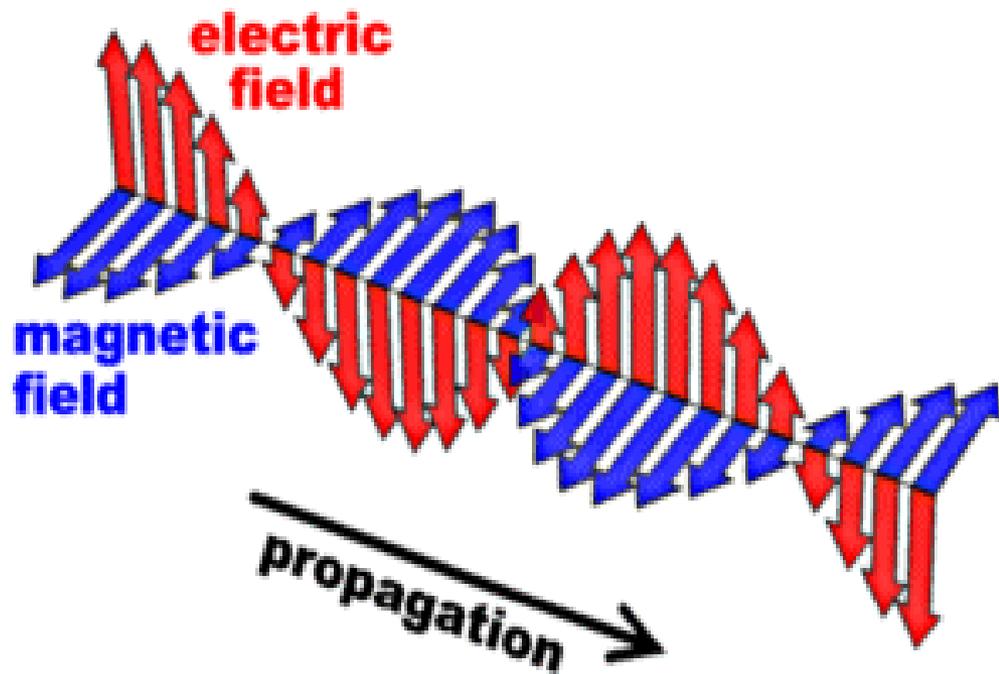


$$\vec{E}(x, t) = \hat{j} E_{\max} \cos(kx - \omega t)$$
$$\vec{B}(x, t) = \hat{k} E_{\max} \cos(kx - \omega t)$$
$$E_{\max} = c B_{\max}$$

Example 1

A sinusoidal electromagnetic wave of frequency 6.10×10^{14} Hz travels in vacuum in the $+z$ -direction. The \mathbf{B} field is parallel to the y -axis and has amplitude 5.80×10^{-4} T.

Write the vector equations for \mathbf{E} and \mathbf{B} .



$$\mathbf{32.3:} \quad \vec{\mathbf{B}}(z,t) = B_{\max} \cos(k(z-ct)) \hat{\mathbf{j}} = B_{\max} \cos(2\pi f(z-ct)) \hat{\mathbf{j}}$$

$$\Rightarrow \vec{\mathbf{B}}(z,t) = (5.80 \times 10^{-4} \text{ T}) \cos\left(2\pi(6.10 \times 10^{14} \text{ Hz})(z - (3.00 \times 10^8 \text{ m/s})t)\right) \hat{\mathbf{j}}$$

$$\Rightarrow \vec{\mathbf{B}}(z,t) = (5.80 \times 10^{-4} \text{ T}) \cos\left((1.28 \times 10^7 \text{ m}^{-1})z - (3.83 \times 10^{15} \text{ rad/s})t\right) \hat{\mathbf{j}}$$

$$\vec{\mathbf{E}}(z,t) = (B_y(z,t) \hat{\mathbf{j}}) \times (c \hat{\mathbf{k}})$$

$$\Rightarrow \vec{\mathbf{E}}(z,t) = \left(1.74 \times 10^5 \frac{\text{V}}{\text{m}}\right) \cos\left(\left(1.28 \times 10^7 \frac{\text{rad}}{\text{m}}\right)z - \left(3.83 \times 10^{15} \frac{\text{rad}}{\text{s}}\right)t\right) \hat{\mathbf{i}}$$

Example 2

The electric-field amplitude near a certain radio transmitter is $3.85 \times 10^{-3} \text{ V/m}$. What is the amplitude of **B**?

How does this compare to the Earth's magnetic field?

$$\mathbf{32.8:} \quad B_{\max} = \frac{E_{\max}}{c} = \frac{3.85 \times 10^{-3} \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 1.28 \times 10^{-11} \text{ T.}$$

$$\text{So } \frac{B_{\max}}{B_{\text{earth}}} = \frac{1.28 \times 10^{-11} \text{ T}}{5 \times 10^{-5} \text{ T}} = 2.56 \times 10^{-7}, \text{ and thus } B_{\max} \text{ is much weaker than } B_{\text{earth}}.$$

Energy in an EM wave

- Both the electric field and the magnetic field have energy.
- Waves transport energy.
- The EM wave transports energy at the speed of light!

$$u = u_E + u_B$$

$$u = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$

$$B = \frac{E}{c} = \sqrt{\mu_0 \epsilon_0} E$$

$$u = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2\mu_0} \left(\sqrt{\mu_0 \epsilon_0} E \right)^2 = \epsilon_0 E^2$$

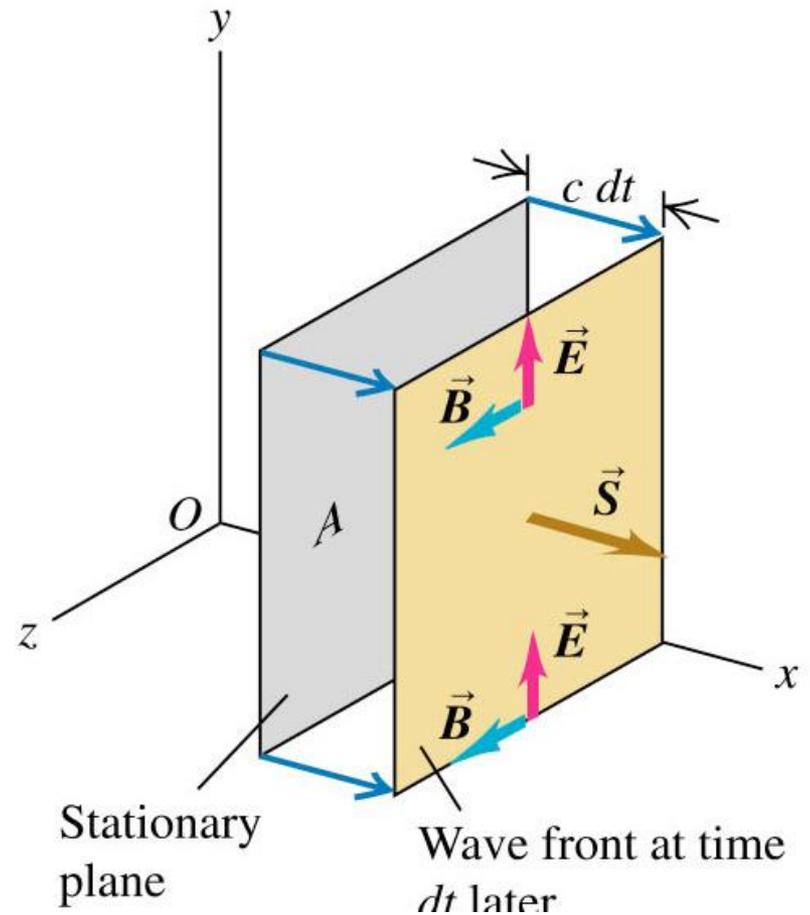
$$u_E = u_B$$

Energy Flow

- Let S be the rate of energy flow per unit area.
- Energy per unit time per unit area.
- S is called the *Poynting vector*.

$$S = \epsilon_0 c E^2 = \frac{EB}{\mu_0}$$

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$



Poynting Vector

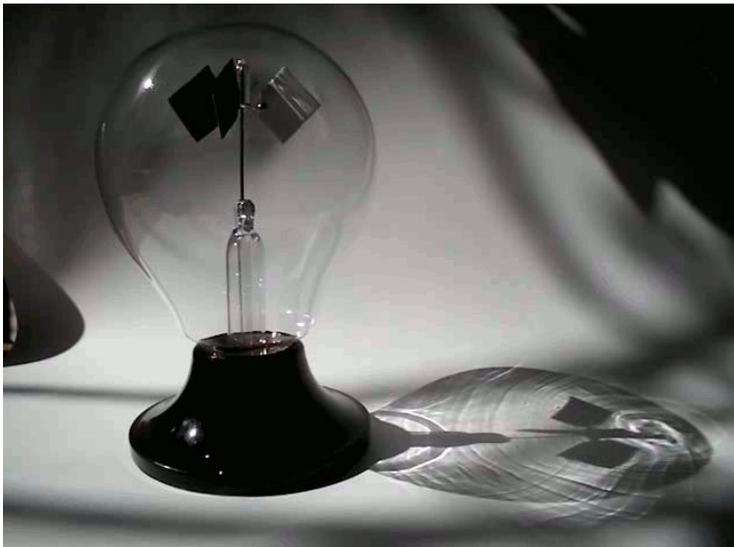
- The time-averaged value of S is *the intensity*.
- Assuming a plane wave (sinusoidal) in a vacuum.

$$\begin{aligned}\bar{\mathbf{S}}(x,t) &= \frac{1}{\mu_0} \bar{\mathbf{E}}(x,t) \times \bar{\mathbf{B}}(x,t) \\ &= \frac{1}{\mu_0} \left[\hat{\mathbf{j}} E_{\max} \cos(kx - \omega t) \right] \times \left[\hat{\mathbf{k}} B_{\max} \cos(kx - \omega t) \right] \\ &= \frac{E_{\max} B_{\max}}{\mu_0} \cos^2(kx - \omega t) (\hat{\mathbf{j}} \times \hat{\mathbf{k}}) \\ \bar{\mathbf{S}}(x,t) &= \hat{\mathbf{i}} \frac{E_{\max} B_{\max}}{\mu_0} \cos^2(kx - \omega t) \\ I = S_{ave} &= \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 \epsilon_0} = \frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} E_{\max}^2 = \frac{1}{2} \epsilon_0 c E_{\max}^2\end{aligned}$$

Momentum in an EM Wave

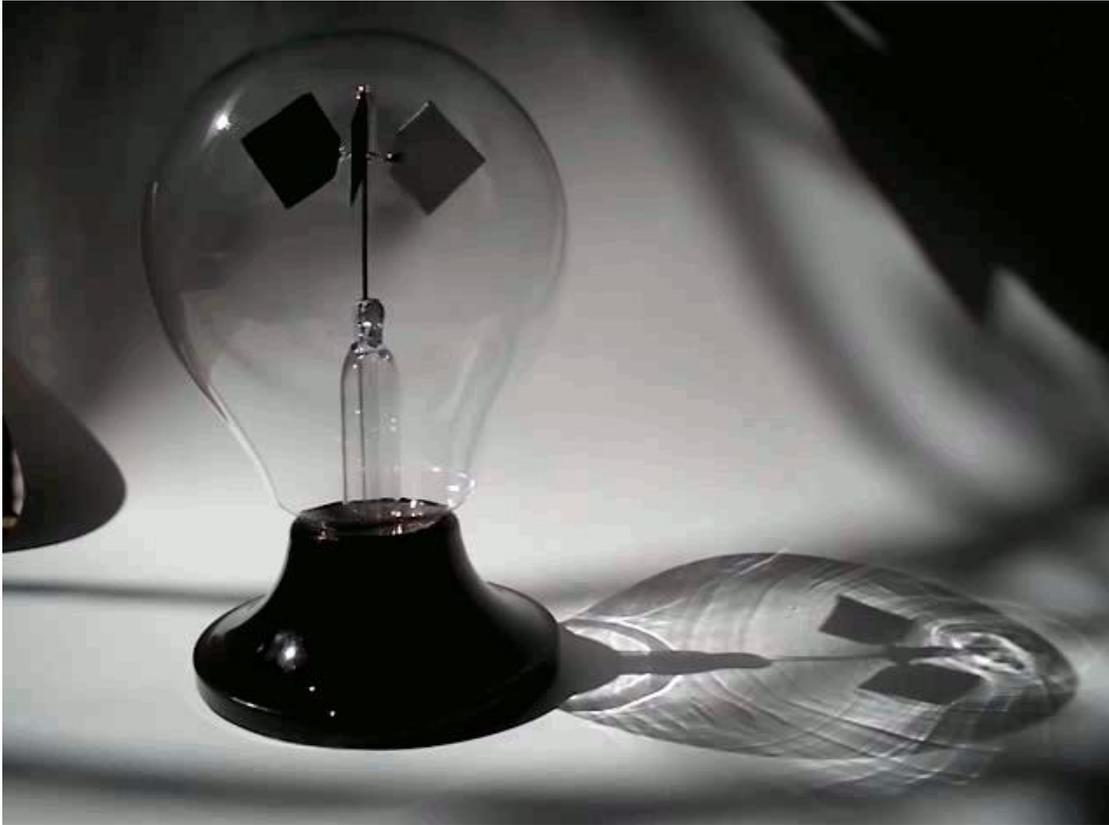
- EM waves carry momentum too,
- EM waves can exert a pressure on an object,
- The flow rate of momentum is a pressure,

$$\frac{1}{A} \frac{dp}{dt} = \frac{S}{c} = \frac{EB}{\mu_0 c}$$
$$\left(\frac{1}{A} \frac{dp}{dt} \right)_{ave} = \frac{S_{ave}}{c} = \frac{I}{c}$$



$$\frac{1}{A} \frac{dp}{dt} = \frac{F}{A} = P_{rad}$$
$$P_{rad} = \frac{S_{ave}}{c} = \frac{I}{c}$$

Crookes radiometer



- also known as the *light mill*,
- consists of an airtight glass bulb, containing a partial vacuum.
- Inside are a set of vanes (white and black) which are mounted on a spindle.
- The vanes rotate when exposed to light, with faster rotation for more intense light,
- It gives possibility to measure the electromagnetic radiation intensity.

STOP:

- The idea of work is not related to the pressure of light!
- The darker side of the vane is heated faster than the light one,
- The pressure difference causes the vane to move cold (light) side forward.

Example 3

An intense light source radiates uniformly in all directions.

At a distance of 5 m from the source the radiation pressure on a perfectly absorbing surface is 9.0×10^{-6} Pa.

What is the total average power output?

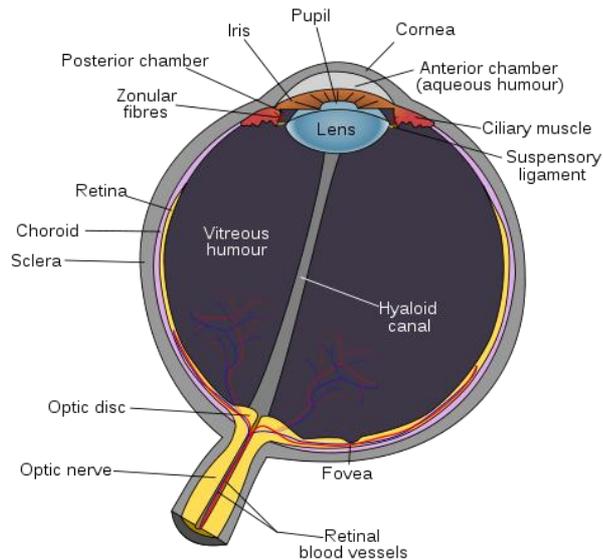
32.17: $p_{\text{rad}} = I/c$ so $I = cp_{\text{rad}} = 2.70 \times 10^3 \text{ W/m}^2$

Then $P_{\text{av}} = I(4\pi r^2) = (2.70 \times 10^3 \text{ W/m}^2) (4\pi) (5.0 \text{ m})^2 = 8.5 \times 10^5 \text{ W}$

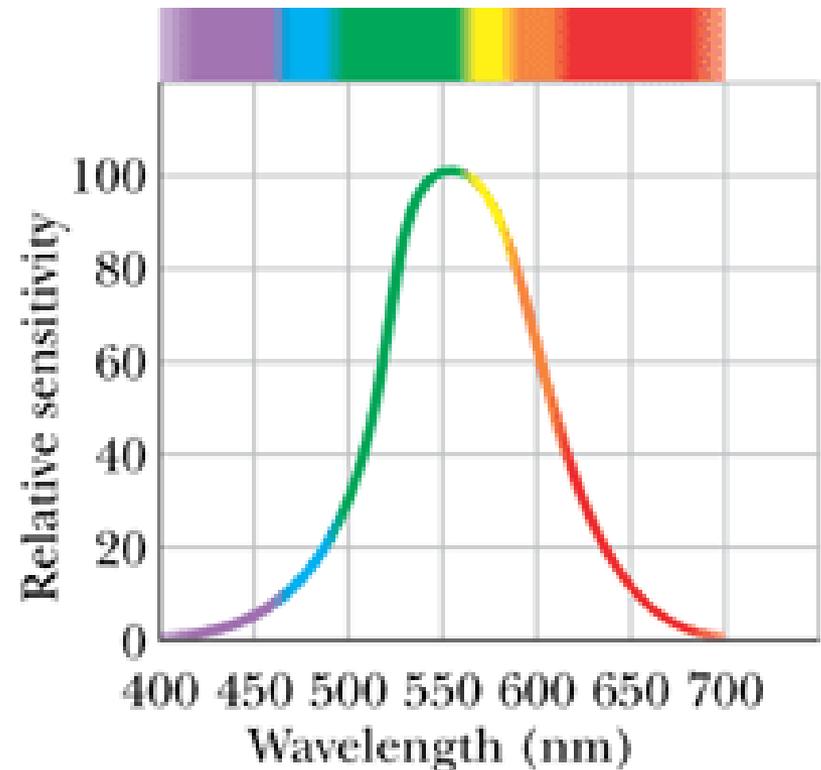
How can we see



- Our natural detectors – eyes,



- The retina contains two major types of light-sensitive photoreceptor cells used for vision: the rods and the cones.



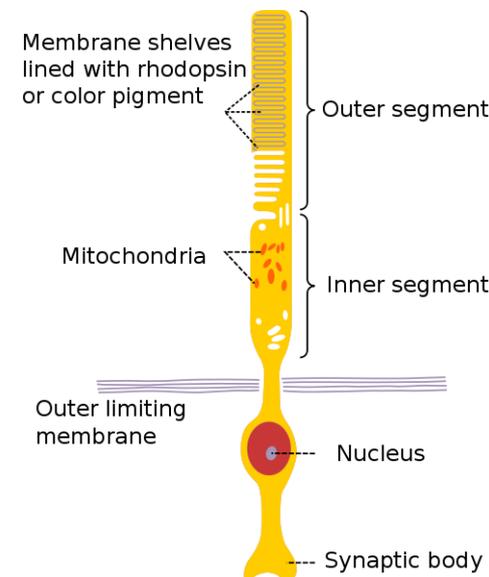
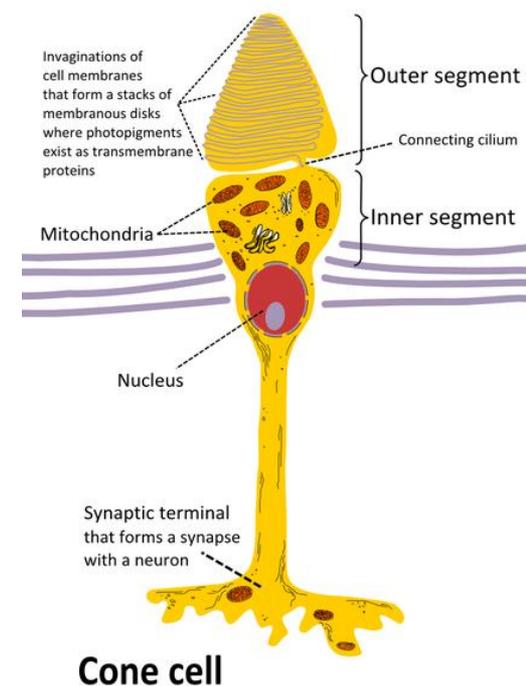
How can we see

- Rods

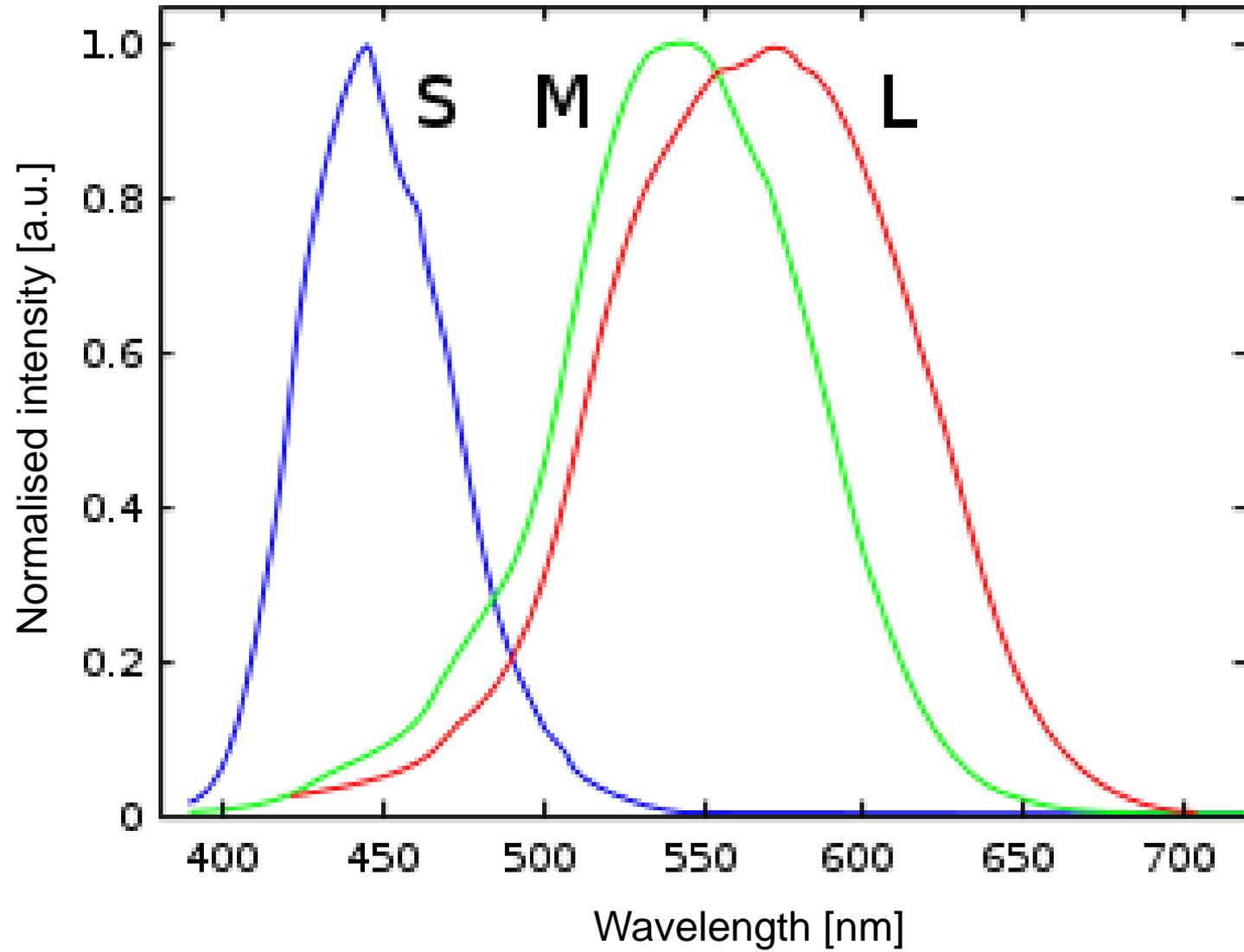
- cannot distinguish colours,
- responsible for low-light, monochrome (black&white) vision,
- they work well in dim light as they contain a pigment (but saturates at higher intensities).

- Cones

- function best in relatively bright light,
- less sensitive to light than the rod cells,
- allow the perception of color.
- Are also able to perceive finer detail and more rapid changes in images (response times to stimuli are faster than those of rods),
- We have three kinds of cones with different response curves and thus respond to variation in color in different ways (trichromatic vision).



Sensitivity of cones



- Interference
- Diffraction
- Refraction

..... Next week