

# Study of Non-Standard Neutrino Interactions with Atmospheric Neutrino Data in Super-Kamiokande I and II

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In this paper we study non-standard neutrino interactions as an example of physics beyond the standard model using atmospheric neutrino data collected during the Super-Kamiokande I(1996-2001) and II(2003-2005) periods. We focus on flavor-changing-neutral-currents (FCNC), which allow neutrino flavor transitions via neutral current interactions, and effects which violate lepton non-universality (NU) and give rise to different neutral-current interaction-amplitudes for different neutrino flavors. We obtain a limit on the FCNC coupling parameter,  $\varepsilon_{\mu\tau}$ ,  $|\varepsilon_{\mu\tau}| < 1.1 \times 10^{-2}$  at 90% C.L. and various constraints on other FCNC parameters as a function of the NU coupling,  $\varepsilon_{ee}$ . We find no evidence of non-standard neutrino interactions in the Super-Kamiokande atmospheric data.

## I. INTRODUCTION

The experimental understanding of neutrino oscillations has improved dramatically over the last ten years. In 1998 Super-Kamiokande (“Super-K”) reported an up-down asymmetry in the zenith angle distribution of muon-like ( $\mu$ -like) events and concluded the distortion was evidence for neutrino oscillations [1]. Super-K also observed an oscillation signature consistent with the  $L/E$  (pathlength over energy) dependence predicted by  $\nu_\mu$  to  $\nu_\tau$  oscillations at maximal  $\theta_{23}$  mixing [2]. Results consistent with the  $\nu_\mu \rightarrow \nu_\tau$  oscillation hypothesis have also been obtained by the tau appearance analysis in Super-K [3] and the long-baseline accelerator experiments K2K [4] and MINOS [5].

However, many alternatives to neutrino oscillations have been proposed to explain the asymmetry of the atmospheric neutrino  $\mu$ -like event sample [6]. Neutrino decoherence [7, 8], neutrino decay [9, 10], mass-varying neutrinos [11], and CPT violation effects are among the most prominent. Although most of these approaches have been ruled out by the Super-K [2, 12] and MINOS [13] data, non-standard neutrino interactions (NSI) with matter, that is interactions which are not predicted by the Standard Model, remain viable.

Among the many types of NSI models, we focus in this paper on two generic types of neutrino interactions. Flavor-changing-neutral-current (FCNC) effects represent neutrino interactions with fermions,  $f$ , in matter that induce neutrino flavor change:  $\nu_\alpha + f \rightarrow \nu_\beta + f$ , where  $\alpha$  and  $\beta$  denote neutrino flavors. Lepton non-universal (NU) interactions, on the other hand, are defined by a non-universal neutral current scattering amplitude among the three flavored neutrinos. In the Standard Model, this amplitude is identical among the neutrinos. Other theories of neutrino mass, however, predict these kinds of interactions. For instance, NSI interactions are often seen in models where the neutrino mass arises from admixtures of isosinglet neutral heavy leptons [14] or from  $R$ -parity violating supersymmetry [15]. A summary of various NSI models is presented in Ref. [16]. Although NSI are predicted by various theories, the expected phenomena do not generally depend on the particular phenomenological model and are typically characterized by dependence on the neutrino energy and the surrounding matter density. For this reason, NSI can be explored in a general context using atmospheric neutrinos. While the current constraints on NSI come from beam-based experiments [31, 32], atmospheric neutrinos can provide additional sensitivity to these interactions due to their

ample flux and the large amount of matter they traverse before detection.

This paper discusses atmospheric neutrino oscillations in the context of NSI at Super-K and is organized as follows. The data set and oscillation framework used in this paper are presented in Sec. II and Sec. III, respectively. In Sec. IV we show the results of an analysis assuming two-flavor  $\nu_\mu \leftrightarrow \nu_\tau$  neutrino oscillations amidst NSI. An analysis using an extended three-flavor framework is performed in Sec. V and finally, the results are summarized in Sec. VII.

## II. ATMOSPHERIC NEUTRINO DATA

Super-K is a 50 kiloton water Cherenkov detector located in a zinc mine in Kamioka, Japan. It is optically separated into an Inner Detector (ID) which is instrumented with 11,146 inward facing photomultiplier tubes (PMTs) at full capacity, and an outer detector (OD) used to veto cosmic ray muons. A more detailed description of the detector is presented in Ref.[17]. The run period of Super-K has been classified into four phases: The first phase corresponds to physics data taken between April 1996 and July 2001 (SK-I). After an accident at the end of 2001, Super-K resumed data taking with half the number of ID PMTs between October 2002 and October 2005 (SK-II). The remaining two run periods are divided into SK-III (2006-2008), after rebuilding the ID with its full complement of PMTs, and SK-IV (2008-present), after the data acquisition system was upgraded. In this paper we use data from the SK-I and SK-II run periods.

### 1. Classification of atmospheric neutrino data

Atmospheric neutrino events in Super-K are divided into the following four categories: fully contained (FC), partially contained (PC), upward stopping muons (UPMU stopping) and upward through-going muons (UPMU through-going). For FC and PC events, event vertices are required to be within the  $\sim 22.5$  kton fiducial volume defined by the volume inset from the ID walls by 2 m. An event whose particles are completely contained within the ID is classified as FC, while an event with particles exiting the ID and depositing energy into the outer detector (OD) is classified as PC. The PC sample is further classified into two sub-categories, PC stopping and PC through-going. The former corresponds to events with a particle that stops in the OD, while in the case of the latter, the particle exits the OD. UPMU events are produced by the charged current interactions of atmospheric muon neutrinos in the rock surrounding the detector. Muons traveling in the upward direction are selected to avoid contamination from cosmic ray muons.

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The UPMU stopping sample is defined by events which enter from outside the detector and stop inside the ID, while the UPMU through-going sample is composed of those that enter and subsequently exit the ID. The expected mean energy for each of the event class is,  $\sim 1$  GeV for FC,  $\sim 10$  GeV for PC,  $\sim 10$  GeV for UPMU stopping, and  $\sim 100$  GeV for UPMU through-going. Descriptions of the event reduction and reconstruction can be found in [17].

## 2. Monte Carlo simulation

In this paper, independent 500 year Monte Carlo (MC) samples are used for SK-I and SK-II. The analyses use

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$$H_{\alpha\beta} = \frac{1}{2E} U_{\alpha j} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} (U^\dagger)_{k\beta} + V_{\text{MSW}} + \sqrt{2} G_F N_f \begin{pmatrix} \varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu} & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{e\tau} & \varepsilon_{\mu\tau} & \varepsilon_{\tau\tau} \end{pmatrix}. \quad (1)$$


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In this equation  $U$  is the unitary PMNS matrix [20], which describes standard neutrino mixing as rotations among pairs of mass eigenstates parameterized by unique mixing angles,  $\theta_{ij}$ . Here the squared difference of the neutrino masses is denoted by  $\Delta m_{ij}^2$ ,  $V_{\text{MSW}}$  is the MSW potential in the flavor basis [21],  $G_F$  is the Fermi coupling constant,  $N_f$  is the fermion number density in matter along the path of the neutrino, and the  $\varepsilon_{\alpha\beta}$  represent the NSI coupling parameters. The non-universal couplings are represented by the flavor diagonal  $\varepsilon$  and the FCNC interactions by the off-diagonal elements. Standard neutrino oscillations are recovered when all of the  $\varepsilon_{\alpha\beta} = 0$ . Note that while the first term of the Hamiltonian carries an explicit energy dependence, the second and third terms do not and instead are functions of the local matter density. For our calculations below we employ the PREM model [24] of the Earth's density profile and chemical composition, where the proton to nucleon ratio in the mantle and core are set to be  $Y_p = 0.497$  and  $0.468$ , respectively [25]. In many of our calculations we use the average matter density along the path of the neutrino.

## IV. ANALYSIS WITH A TWO-FLAVOR HYBRID MODEL

We consider first a model in which NSI effects in the  $\nu_\mu - \nu_\tau$  sector coexist with standard two-flavor  $\nu_\mu \leftrightarrow \nu_\tau$  neutrino oscillations. In this scenario all NSI that couple to  $\nu_e$  in Eq.(1),  $\varepsilon_{e\beta}$ , are set to zero. Allowing the remaining parameters which couple to  $\nu_\mu$  and  $\nu_\tau$  to be non-zero introduces a matter-dependent effect on the oscillations

the Honda2006 neutrino flux [18] and neutrino interactions are simulated using the NEUT interaction generator [19].

## III. OSCILLATION FRAMEWORK

In the following sections we will consider two separate NSI models derived from a more general formalism. We introduce first the more general framework, restricting its scope later to the particular NSI effects we aim to study. The full three-flavor Hamiltonian,  $H_{\alpha\beta}$ , governing the propagation of neutrinos in the presence of effects from NSI is

of  $\nu_\mu \leftrightarrow \nu_\tau$ . Since the standard two-flavor scenario ( $\theta_{12}$ ,  $\theta_{13}$ , and  $\Delta m_{21}^2 = 0$ ) does not incorporate oscillations into  $\nu_e$ , there is no separate effect from the standard matter potential,  $V_{\text{MSW}}$ . Labeling this the *2-flavor hybrid model*, we can explore NSI couplings by searching for matter-induced distortions of standard  $\nu_\mu \leftrightarrow \nu_\tau$  oscillations.

### A. Formalism

The 2-flavor hybrid model can be extracted from Eq.(1) by setting  $\Delta m_{21}^2 = 0$ . Following the formalism of M.C. Gonzalez-Garcia and M. Maltoni [22], assuming that neutrinos possess non-standard interactions with only  $d$ -quarks [16, 23], and defining  $\varepsilon_{\mu\tau} \equiv \varepsilon$  and  $\varepsilon_{\tau\tau} - \varepsilon_{\mu\mu} \equiv \varepsilon'$  the  $\nu_\mu$  survival probability in constant density matter is given by

$$P_{\nu_\mu \rightarrow \nu_\mu} = 1 - P_{\nu_\mu \rightarrow \nu_\tau} = 1 - \sin^2 2\Theta \sin^2 \left( \frac{\Delta m_{23}^2 L}{4E} R \right). \quad (2)$$

The effective mixing angle,  $\Theta$ , and the correction factor to the oscillation wavelength,  $R$ , are given by

$$\begin{aligned} \sin^2 2\Theta &= \frac{1}{R^2} (\sin^2 2\theta + R_0^2 \sin^2 2\xi + 2R_0 \sin 2\theta \sin 2\xi), \\ R &= \sqrt{1 + R_0^2 + 2R_0 (\cos 2\theta \cos 2\xi + \sin 2\theta \sin 2\xi)}, \\ R_0 &= \sqrt{2} G_F N_f \frac{4E}{\Delta m^2} \sqrt{|\varepsilon|^2 + \frac{\varepsilon'^2}{4}}, \\ \xi &= \frac{1}{2} \tan^{-1} \left( \frac{2\varepsilon}{\varepsilon'} \right), \end{aligned} \quad (3)$$

where  $\theta$  is the standard two-flavor mixing angle, and  $\xi$  is the NSI-induced effective rotation angle in matter. Both the effective mixing angle and the correction factor depend on the neutrino energy as well as the standard oscillation and NSI parameters. These parameters are shown as a function of energy for several values of  $\varepsilon$  and  $\varepsilon'$  in Fig. 1.

### B. Expected Phenomena

Because of the energy dependence of the standard oscillation term in Eq.(1), the relative dominance of NSI in the hybrid model is expected to depend on the neutrino energy. The effects of NSI on neutrino oscillation can be divided into three energy ranges: (1)  $E_\nu < 1\text{GeV}$ , (2)  $1 < E_\nu < 30\text{GeV}$ , (3)  $E_\nu > 30\text{GeV}$ .

#### (1) $E_\nu < 1\text{GeV}$

At these low energies, the eigenvalue of the vacuum term  $\Delta m_{23}^2/2E_\nu (\gtrsim 1 \times 10^{-12}\text{eV})$  is larger than that of the NSI matter term  $\sqrt{2}G_F N_f \varepsilon (\sim 1 \times 10^{-13}\text{eV})$ , assuming  $\Delta m_{23}^2 = 2.1 \times 10^{-3}\text{eV}^2$ ,  $N_f \equiv N_d \sim 3N_e$  and  $\varepsilon \sim \mathcal{O}(1)$ . Thus the  $\nu_\mu \rightarrow \nu_\tau$  transition is mostly governed by the standard two-flavor oscillation and there is no significant contribution from NSI. Note that  $\varepsilon \sim \mathcal{O}(1)$  is a conservative assumption according to the NuTeV limit  $|\varepsilon_{\mu\tau}| < 0.05$  [32].

#### (2) $1 < E_\nu < 30\text{GeV}$

This energy range corresponds to the FC Multi-GeV (visible energy greater than 1330 MeV), PC, and UPMU stopping samples. In this region the matter term competes with the vacuum term and thus the  $\nu_\mu \rightarrow \nu_\tau$  transition is no longer dominated by standard oscillations but is modified by the matter term. The left panel of Fig. 1 shows that for nonzero NU the effective mixing angle decreases with increasing neutrino energy, thereby suppressing  $\nu_\mu \rightarrow \nu_\tau$  transitions. Similarly, the right panel shows that nonzero FCNC interactions affect the frequency of oscillations. Since the effective mass splitting,  $\Delta m_{eff}^2 \equiv R\Delta m_{23}^2$ , is larger than  $\Delta m_{23}^2$ , the first oscillation maximum is expected to occur at higher neutrino energies than for standard oscillations. Focusing on the zenith angle distributions presented in the left panel of Fig. 2, the magnitude of the  $\mu$ -like deficit in the upward-

going direction is expected to become smaller due to  $\varepsilon'$ . Further, the shape of the zenith angle distribution near and above the horizon is modified in the presence of  $\varepsilon$  for the higher energy  $\nu_\mu$  samples.

#### (3) $E_\nu > 30\text{GeV}$

Above 30GeV, most atmospheric neutrinos are in the UPMU through-going sample, which ranges in energy from tens of GeV to  $\sim 100$  TeV. At these energies vacuum oscillations have less of an effect on the  $\nu_\mu \rightarrow \nu_\tau$  transition, while the transitions induced by the matter term become significant when the neutrino path length in the Earth is sufficiently long. As shown in the right panel of Fig. 2, FCNC interactions play a leading role in this energy range because the modified oscillation frequency is comparable in size to the oscillation frequency of these neutrinos in the Earth. Thus,  $\nu_\mu \rightarrow \nu_\tau$  transitions driven by FCNC are expected to occur. In contrast, at these energies NU interactions suppress the effective mixing angle and therefore are expected to produce a sub-leading effect on the data. Note that at the current values of the atmospheric mixing parameters, the standard  $\nu_\mu \rightarrow \nu_\tau$  transition is already increasingly suppressed at these energies. In summary, FCNC interactions induce a faster oscillation frequency and lead to  $\nu_\mu \rightarrow \nu_\tau$  transitions at shorter path lengths in matter, while NU suppresses the transition even at these energies.

### C. Analysis Method

We evaluate the agreement between the data and Monte Carlo oscillated according to the hybrid NSI model using a  $\chi^2$  test. The SK-I and SK-II data are divided according to their reconstructed event types, momenta, and zenith angles into 400 and 350 bins, respectively. In SK-I (SK-II) there are 310 (280) bins for the FC samples, 60 (40) for the PC samples, and 30 (30) for the upward-going muon samples. Data from the two run periods are treated separately due to differences in the detector response and in the effects on the atmospheric neutrino flux from solar modulations during the runs. In order to accurately treat bins with small statistics in this binning scheme, a likelihood based on Poisson probabilities [26] is used. The complete  $\chi^2$  with 750 data bins and 90 systematic uncertainties is defined as

$$\chi^2 = 2 \sum_{i=1}^{750} \left( N_i^{exp} \left( 1 + \sum_{j=1}^{90} f_j^i \varepsilon_j \right) - N_i^{obs} + N_i^{obs} \ln \frac{N_i^{obs}}{N_i^{exp} \left( 1 + \sum_{j=1}^{90} f_j^i \varepsilon_j \right)} \right) + \sum_{j=1}^{90} \left( \frac{\varepsilon_j}{\sigma_j^{sys}} \right)^2, \quad (4)$$

where  $\varepsilon_j$  is a fitting parameter for the  $j$ -th systematic error and  $f_j^i$  is the fractional change of the event rate in

the  $i$ -th bin due to a  $1\sigma$  change in  $j$ -th systematic error.

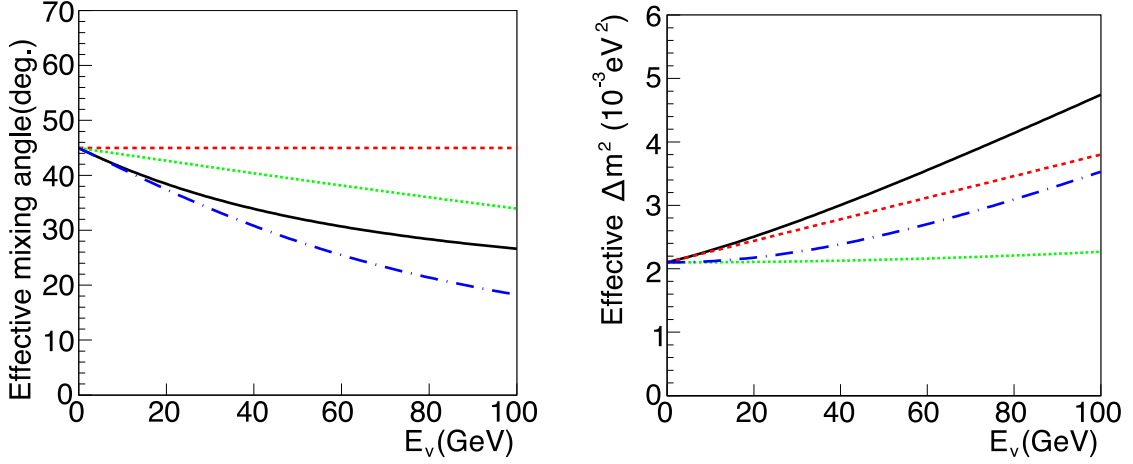


FIG. 1: (Left) Effective mixing angle in matter. (Right) Effective mass squared difference in matter. In both panels, solid black curves indicate the case with  $(\varepsilon$  (FCNC),  $\varepsilon'$  (NU)) = (0.015, 0.05), dashed red curves with (0.015, 0.0), dotted green curves with (0.0, 0.015), dashed-dotted blue curves with (0.0, 0.05). As for the vacuum parameters,  $\theta_{23} = 45^\circ$  and  $\Delta m_{23}^2 = 2.1 \times 10^{-3} \text{eV}^2$  are assumed. Matter density is defined as constant  $\rho = 5.0 \text{g/cm}^3$ .

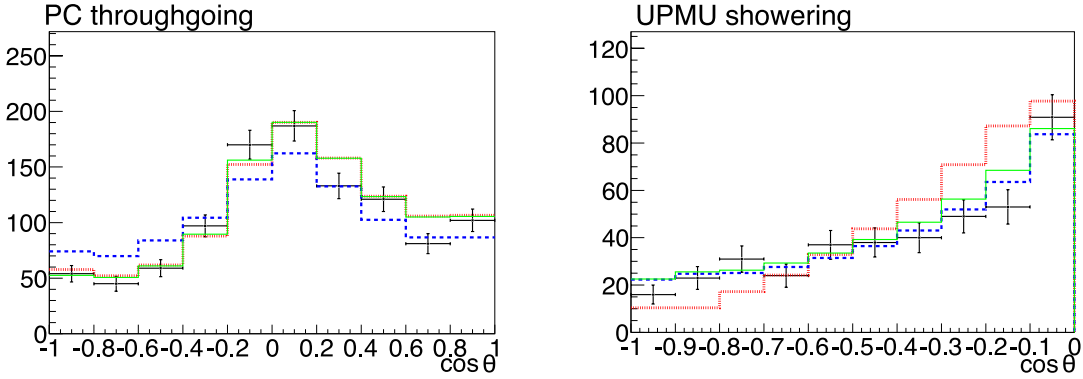


FIG. 2: Zenith angle distribution for PC through-going (left) and UPMU showering (right, included in UPMU through-going where energy loss of muon is caused by pair production, Bremsstrahlung, and photonuclear interactions) sub-samples. In the solid green line  $(\varepsilon, \varepsilon') = (1.0 \times 10^{-3}, -2.4 \times 10^{-2})$ , in the dashed blue line  $(1.0 \times 10^{-3}, -0.38)$ , and in the dotted red line  $(3.2 \times 10^{-3}, -2.4 \times 10^{-2})$ . In all lines the standard oscillation parameters are  $\theta_{23} = 45^\circ$  and  $\Delta m_{23}^2 = 2.1 \times 10^{-3} \text{eV}^2$ .

Using this equation, a value of  $\chi^2$  is evaluated at each point in a four-dimensional parameter space defined by  $\sin^2 2\theta_{23}$ ,  $\Delta m_{23}^2$ ,  $\log_{10}\varepsilon$ , and  $\log_{10}\varepsilon'$ , where  $\varepsilon$  and  $\varepsilon'$  range from  $1.0 \times 10^{-3}$  to  $3.2 \times 10^{-2}$  and from  $1.0 \times 10^{-3}$  to 0.42, respectively. The fit is performed on a  $51 \times 51 \times 51 \times 51$  grid in this space. In Eq.(4), the 90  $\varepsilon_j$  parameters are varied to minimize the value of  $\chi^2$  for each choice of oscillation parameters. The point in parameter space returning the smallest value of  $\chi^2$  is defined as the best fit. Since the effects of  $\varepsilon$  are symmetric between negative and positive values, we only consider positive  $\varepsilon$ .

#### D. Two-Flavor Non-Standard Interaction Analysis Result

The result of a scan on this parameter space gives a best fit at

$$\begin{aligned} \sin^2 2\theta_{23} &= 1.00, & \Delta m_{23}^2 &= 2.2 \times 10^{-3} \text{eV}^2, \\ \varepsilon &= 1.0 \times 10^{-3}, & \varepsilon' &= -2.7 \times 10^{-2} \\ \chi_{min}^2 &= 838.9 / 746.0 \text{ d.o.f.} \end{aligned} \quad (5)$$

The best fit value assuming only standard 2-flavor oscillations with the same samples and binning is  $\chi_{min}^2 = 834.3/748$  d.o.f. at  $\sin^2 2\theta = 1.00$  and  $\Delta m^2 = 2.1 \times 10^{-3} \text{eV}^2$ . The slightly larger  $\chi_{min}^2$  value from the two-flavor hybrid model compared to the standard analysis is caused by the different numerical method used in the NSI analysis: the chosen ranges of  $\varepsilon$  and  $\varepsilon'$  do not allow the two-flavor hybrid model to reduce completely to

standard oscillations.

Fig. 3 shows the allowed neutrino oscillation parameter regions,  $\sin^2 2\theta_{23}$  and  $\Delta m_{23}^2$ , from this analysis and from the standard oscillation analysis. The three contours correspond to the 68%, 90% and 99% C.L., defined by  $\chi^2 = \chi_{min}^2 + 2.30, 4.61,$  and  $9.21,$  respectively. There is no inconsistency between the two sets of allowed regions. Since the difference in the minimum  $\chi^2$  values is not large, no significant contribution to standard two-flavor oscillations from NSI effects is found in this analysis

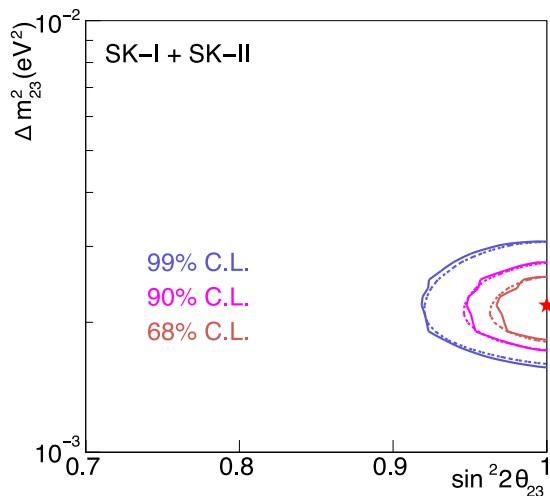


FIG. 3: Allowed oscillation parameter regions derived by the 2-flavor hybrid model analysis (solid curves), where undisplayed parameters  $\varepsilon$  and  $\varepsilon'$  are integrated out. For reference, the result of standard 2-flavor oscillation is added (dashed curves).

The allowed regions of the NSI parameters are shown in Fig. 4, where there undisplayed parameters  $\sin^2 2\theta_{23}$  and  $\Delta m_{23}^2$  have been minimized over. At 90% C.L. the obtained limits on the NSI parameters in the  $\nu_\mu - \nu_\tau$  sector are

$$|\varepsilon| < 1.1 \times 10^{-2} \text{ and } -4.9 \times 10^{-2} < \varepsilon' < 4.9 \times 10^{-2}. \quad (6)$$

These limits can be compared with a phenomenological study using the SK-I (79 kton yr) and MACRO atmospheric neutrino data [16], in which the limits on FCNC and NU at 90% C.L. are  $2.0 \times 10^{-2} < \varepsilon < 1.3 \times 10^{-2}$  and  $-4.7 \times 10^{-2} < \varepsilon' < 4.2 \times 10^{-2}$ .

### E. Discussion

As described in Sec. IV B, NSI are expected to affect multi-GeV muon neutrinos, and are consequently constrained by the Super-K  $\mu$ -like samples. Fig. 5 shows the allowed region spanned by the NSI parameters for three sub-samples of the data. Since the minimum  $\chi^2$  is located in negative  $\varepsilon'$  space, the negative  $\varepsilon'$  plane is

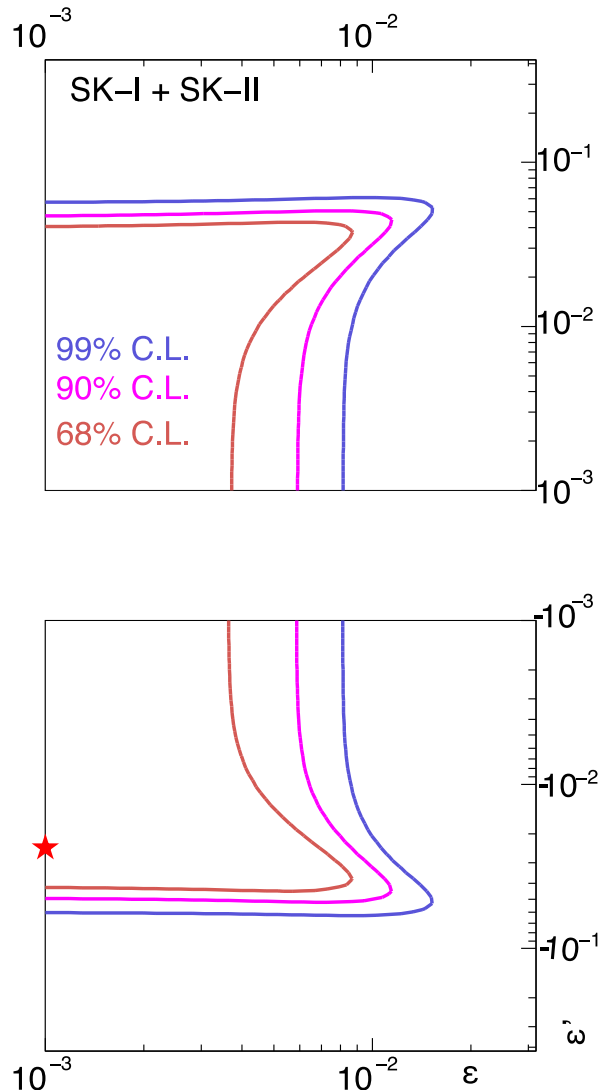


FIG. 4: Allowed NSI parameter regions in the 2-flavor hybrid model. The horizontal axis shows  $\varepsilon$  (FCNC) and the vertical axis shows  $\varepsilon'$  (NU). Undisplayed parameters  $\sin^2 2\theta_{23}$  and  $\Delta m_{23}^2$  are integrated out. The star represents the best-fit point for the NSI parameters.

presented. The strongest constraint on  $\varepsilon$  comes from the UPMU through-going sample (solid curve), while  $\varepsilon'$  is most tightly constrained by the PC and UPMU stopping (dashed curve) samples. The constraint from the sub-GeV samples is too weak to be visible in this figure.

These constraints on  $\varepsilon$  can be understood as follows. As shown in the right panel of Fig. 1, FCNC interactions (dashed red curve) increase the effective neutrino oscillation frequency thereby shortening the oscillation length for neutrinos at a fixed energy. Above 30 GeV, where the atmospheric sample is dominated by UPMU events, the oscillation length in the absence of NSI is already greater than the diameter of the Earth. The addition of FCNC effects on the other hand can shorten the oscillation length enough to induce oscillations in the steepest

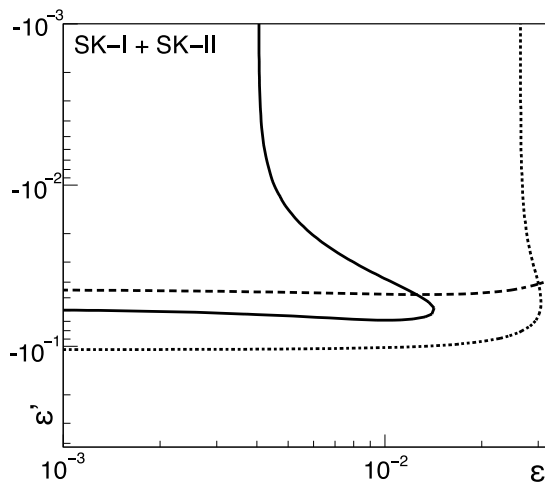


FIG. 5: Allowed NSI parameter regions for various event sub-samples. The solid curve indicates the allowed region from the UPMU through-going sample, the dashed curve is that from the PC and UPMU stopping samples, and the dotted curve is from the FC Single-ring Multi-GeV and Multi-ring samples. Each contour corresponds to  $\chi^2 = \chi^2 + 2.30$  (68% C.L.).

upward-going bins of the UPMU sample. This is an apparent contradiction with the data and results in a tight constraint on  $\varepsilon$ . Below  $\sim 30$  GeV, the oscillation length is sufficiently short in both standard and NSI cases to cause oscillations in the FC and PC samples. For this reason it is more difficult for these samples to discriminate between standard- and FCNC-induced oscillations.

As for the most stringent limit on NU derived from the PC and UPMU stopping samples, since a large  $\varepsilon'$  gives a small magnitude of the  $\mu$ -like deficit in the upward-going direction and a distortion of the shape of the zenith angle distribution near and above the horizon (see dashed blue line in the left panel of Fig. 2), a tight constraint on NU can occur. Further limits on NU by other sub-samples can be understood by the discussion in Sec. IV B.

## V. ANALYSIS WITH A THREE-FLAVOR HYBRID MODEL

We now consider a *3-flavor hybrid model* in which NSI in the  $\nu_e - \nu_\tau$  sector coexist with standard 2-flavor  $\nu_\mu \leftrightarrow \nu_\tau$  oscillations and all other NSI are zero. By introducing couplings between  $\nu_e$  and  $\nu_\tau$ , this model allows flavor transitions of all types. That is, an overall  $\nu_\mu \rightarrow \nu_e$  transition becomes possible due to the  $\varepsilon_{e\tau}$  induced  $\nu_\tau \rightarrow \nu_e$  conversion working in conjunction with the standard  $\nu_\mu \leftrightarrow \nu_\tau$  oscillation:

$$\nu_\mu \xrightarrow{\theta_{23}} \nu_\tau \xrightarrow{\varepsilon_{e\tau}} \nu_e. \quad (7)$$

Note that since the other FCNC epsilons have been set to zero, there is no direct transition between  $\nu_\mu$  and  $\nu_e$ . This analysis therefore aims to constrain possible NSI

effects by examining changes in the  $\nu_e$  and  $\nu_\mu$  fluxes on top of standard oscillation-like effects.

### A. Formalism

An evolution matrix from time  $t_0$  to  $t$  can be obtained by diagonalizing the Hamiltonian Eq. (1) in terms of the leptonic mixing matrix in matter  $U'$  and the effective eigenvalues  $\hat{H} = \text{diag}(E_1, E_2, E_3)$ . In the case of constant matter density, the evolution matrix in natural units is

$$S_{\beta\alpha}(t, t_0) = \sum_{i=1}^3 (U'_{\alpha i})^* U'_{\beta i} e^{-iE_i(t-t_0)}, \quad \alpha, \beta = e, \mu, \tau. \quad (8)$$

Thus the neutrino oscillation probability under the effect of NSI can be expressed as

$$P_{\alpha\beta} = |S_{\beta\alpha}(t, t_0)|^2. \quad (9)$$

However, in order to account for the varying matter density in the Earth (from 2.5 to 13 g/cm<sup>3</sup>), we divide the neutrino path into several constant density steps and calculate the evolution matrix in each. The oscillation probability is obtained from the eigenvalues of the product of these matrices.

### B. Expected Phenomena

As in the 2-flavor hybrid model, the effects of NSI are expected to vary with the neutrino energy. We consider three energy ranges: (1)  $E_\nu < 1\text{GeV}$ , (2)  $1 < E_\nu < 15\text{GeV}$ , and (3)  $E_\nu > 15\text{GeV}$ .

#### (1) $E_\nu < 1\text{GeV}$

Since the standard oscillation eigenvalue  $\Delta m^2/2E_\nu$  is much greater than the matter potential  $\sqrt{2}G_F N_d$ ,  $\nu_\mu \rightarrow \nu_\tau$  transitions induced by these oscillations are expected to be dominant and the effects from NSI can be ignored.

#### (2) $1 < E_\nu < 15\text{GeV}$

In this energy range, the matter term has a sizable effect on the  $\nu_\mu \rightarrow \nu_\tau$  transition as it did in the 2-flavor hybrid model. Moreover, since the current limits on the NSI parameters governing  $\nu_e \rightarrow \nu_\tau$  transitions are poor:  $\varepsilon_{e\tau} \sim \mathcal{O}(10^{-1})$  [30], large effects from NSI are possible.

The modification of the  $\nu_e$  flux by NSI can be parameterized by two oscillation probabilities:  $P(\nu_e \rightarrow \nu_e)$  and  $P(\nu_\mu \rightarrow \nu_e)$  in Eq. (7). Normalizing by the  $\nu_e$  flux ( $\Phi_e$ ), and disregarding any  $\nu_\mu$  contamination, the resulting  $e$ -like distributions can be approximately expressed as  $P(\nu_e \rightarrow \nu_e) + rP(\nu_\mu \rightarrow \nu_e)$ , where  $r \equiv \Phi_\mu/\Phi_e$  is the neutrino flavor ratio [27]. This ratio grows with increasing neutrino energy and though it is highly dependent on the neutrino zenith angle, near the horizon it is  $\sim 2$  up to around 10 GeV. Thus, if  $P(\nu_e \rightarrow \nu_e) + rP(\nu_\mu \rightarrow \nu_e) \lesssim 1$ ,

as is the case when  $\varepsilon_{e\tau}$  and  $\varepsilon_{\tau\tau}$  have comparable values, then since  $P(\nu_e \rightarrow \nu_e)$  is suppressed by  $\varepsilon_{e\tau}$  and  $P(\nu_\mu \rightarrow \nu_e)$  by  $\varepsilon_{\tau\tau}$ , the number of  $e$ -like events at the horizon can be expected to decrease. In contrast, the number of  $e$ -like events in the upward-going bins would effectively increase because large values of  $\varepsilon_{e\tau}$  produce additional  $\nu_e$  from  $\nu_\tau$  created during standard oscillations. This effect is enhanced by the larger flavor ratio in these regions and enables the multi-GeV  $e$ -like samples to help constrain  $\varepsilon_{e\tau}$ . These effects are shown in the left panel of Fig. 6.

Effects driven by  $\varepsilon_{\tau\tau}$  on the other hand may be described in terms of the limiting case where the other NSI are set to zero. In this case the problem reduces to the 2-flavor hybrid model with  $\varepsilon = 0$  and  $\varepsilon' = \varepsilon_{\tau\tau}$ . Therefore a constraint on  $\varepsilon'$  similar to that from the 2-flavor case can be expected.

### (3) $E_\nu > 15\text{GeV}$

Above a few tens of GeV the  $\nu_e$  flux decreases as their parent muons increasingly reach the ground before decaying, making it possible to neglect the  $\nu_e$  contribution to the  $\nu_\mu$  flux from oscillations induced by  $\varepsilon_{e\tau}$ . Conversely, changes in that flux can be clearly recognized as NSI-driven  $\nu_\tau \rightarrow \nu_e$  conversion at these energies. As in the 2-flavor hybrid case,  $\varepsilon_{\tau\tau}$  can be expected to suppress the effective mixing angle and increase the effective mass splitting in the  $\nu_\mu \rightarrow \nu_\tau$  sector. Both of these effects can be seen in the right panel of Fig. 6.

As pointed out in Ref. [29] the atmospheric neutrino sample cannot effectively constrain  $\varepsilon_{ee}$ . This is due to the fact that when  $\varepsilon_{e\tau}$  is zero, the eigenstates of the Hamiltonian in matter are identical to the vacuum eigenstates. In this case, the matter eigenvalues are no longer dependent upon  $\varepsilon_{ee}$  and the problem reduces to two-flavor NSI mixing with  $\nu_\mu \leftrightarrow \nu_\tau$  transitions in matter modified by only  $\varepsilon_{\tau\tau}$ . Despite the lack of sensitivity to  $\varepsilon_{ee}$ , at fixed non-zero values it dictates a parabolic relationship among the NSI parameters, which in the limit where one of the matter eigenvalues is small takes the form

$$\varepsilon_{\tau\tau} \sim \frac{3|\varepsilon_{e\tau}|^2}{1 + 3\varepsilon_{ee}}. \quad (10)$$

This is a general feature which will present itself in the shape of our allowed regions below.

## C. Analysis Method

The analysis procedure for the 3-flavor hybrid model follows that used for the 2-flavor hybrid model. A value of  $\chi^2$  is evaluated at each grid point in the three-dimensional parameter space of  $\varepsilon_{ee}$ ,  $\varepsilon_{e\tau}$  and  $\varepsilon_{\tau\tau}$ , where 51 points are chosen for each. Since  $\varepsilon_{e\tau}$  enters the oscillation equations as  $|\varepsilon_{e\tau}|$  [29] when  $\theta_{13}$  and solar oscillations ( $\Delta m_{12}^2$  and  $\theta_{12}$ ) are not considered, we only test positive values of the parameter.

$\varepsilon_{ee}$	Best-fit $\varepsilon_{e\tau}$	Best-fit $\varepsilon_{\tau\tau}$	Minimum $\chi^2$
-0.50	0.016	-0.016	831.1
-0.25	0.016	0.024	829.9
0.00	0.024	0.016	830.9
0.25	0.000	-0.016	831.4
0.50	0.000	-0.016	831.4

TABLE I: The allowed NSI parameters at the 90% C.L. as a function of  $\varepsilon_{ee}$ .

We first set the standard 2-flavor parameters to  $(\sin^2 \theta_{23}, \Delta m^2) = (0.5, 2.1 \times 10^{-3} \text{eV}^2)$  as motivated by the results from the 2-flavor hybrid model and other Super-K analyses [28]. Here the standard oscillation parameters are taken as fixed values. This assumption will be verified later by comparing the allowed NSI parameter regions derived using slightly different standard oscillation parameters. Modifications to the fitting results when  $\theta_{13}$  is non-zero are presented in Sec. VI

## D. Results of the 3-Flavor NSI analysis

The allowed regions for the 3-flavor NSI parameters are shown in Fig. 7. Allowed regions for five fixed values of  $\varepsilon_{ee}$  (-0.50, -0.25, 0.00, 0.25, and 0.50) are presented. The three contours correspond to the 68%, 90% and 99% C.L. regions as defined by  $\chi^2 = \chi_{min}^2 + 2.30, 4.61,$  and  $9.21,$  respectively. The best-fit values for each  $\varepsilon_{ee}$  are summarized in Tbl. I.

In these figures, except for the projection for  $\varepsilon_{ee} = -0.25$ , parabola-like regions in the  $\varepsilon_{e\tau} - \varepsilon_{\tau\tau}$  are shown. In particular, the allowed region extends to negative  $\varepsilon_{\tau\tau}$  values when  $\varepsilon_{ee} = -0.5$ , while they extend to the positive values above  $\varepsilon_{ee} = 0$ . This feature is consistent with a transition of the matter eigenvalue hierarchy discussed in Ref. [29].

Next, we evaluate the influence of using fixed standard oscillation parameters. Fig. 8 shows the allowed NSI parameter regions using four slightly different sets of standard oscillation parameters. The oscillation parameters are taken from the 90% C.L. allowed region from the standard SK 2-flavor analysis. Although slight changes appear when the NSI fits are repeated using these parameters, their minimum  $\chi^2$  values are larger than that of the original fit by less than 1.5 units. Therefore no significant change in the allowed parameter regions is expected even if the standard oscillation parameters are allowed to vary during the NSI fit. Finally note that the current allowed regions of  $\sin^2 \theta_{23}$  and  $\Delta m^2$  are constrained more tightly by analyses using the full SK data set, which is larger than the sample used here.



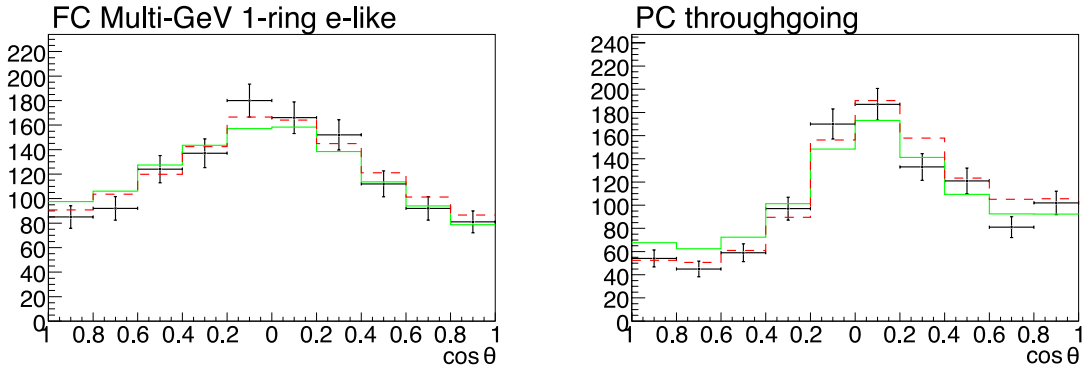


FIG. 6: Zenith angle distribution for typical  $e$ -like (left, FC Multi-GeV 1-ring  $e$ -like) and  $\mu$ -like (right, PC throughgoing) samples. The solid green line is the MC prediction at  $\varepsilon_{ee} = 0.0$ ,  $\varepsilon_{e\tau} = 0.2$ , and  $\varepsilon_{\tau\tau} = 0.2$ . The dashed red line is that for standard neutrino oscillations. In all lines the standard oscillation parameters are  $\theta_{23} = 45^\circ$  and  $\Delta m_{23}^2 = 2.1 \times 10^{-3} \text{eV}^2$ .

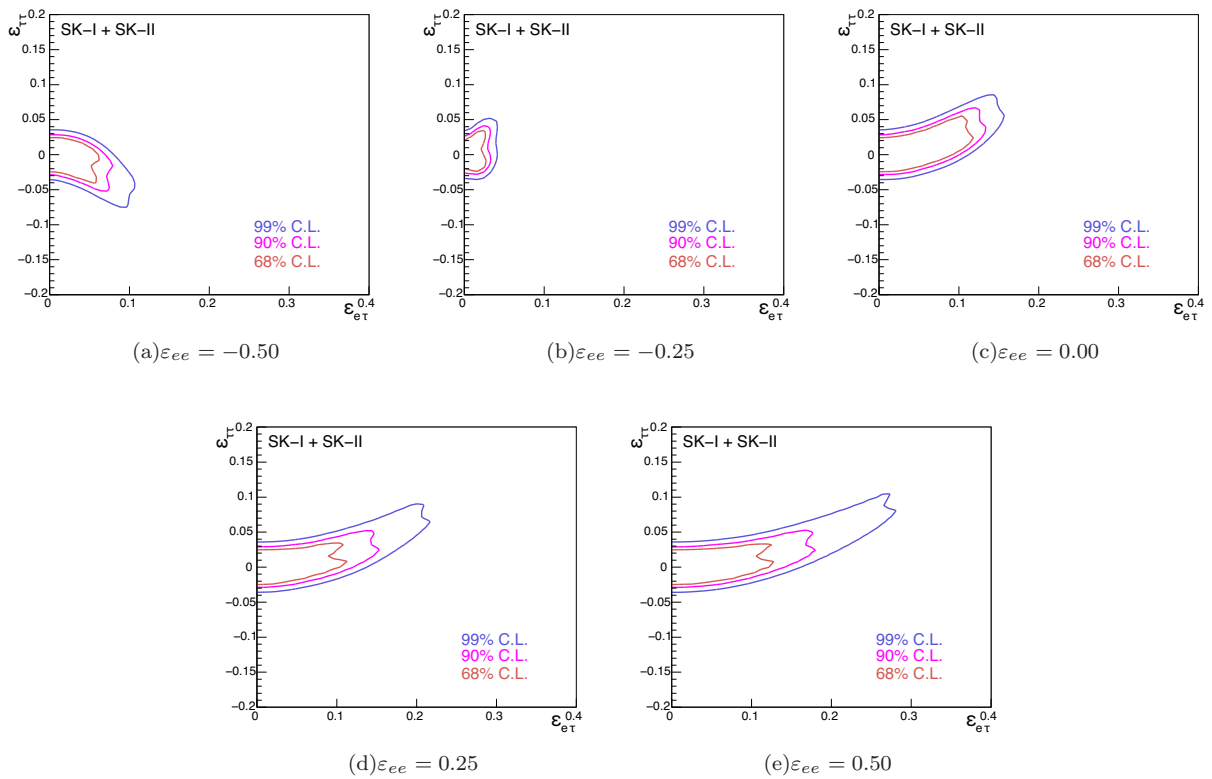


FIG. 7: Allowed NSI parameter regions at fixed  $\varepsilon_{ee}$  at 68%, 90% and 99% C.L., where contours are drawn at  $\chi^2 - \chi_{min}^2 = 2.30, 4.61, \text{ and } 9.21$ , respectively.

### E. Discussion

Fig. 9 shows the allowed parameter regions for three subsets of the atmospheric sample. The high energy  $\nu_\mu$ -rich UPMU throughgoing sample (indicated by the solid curve) and PC+UPMU stopping samples constrain  $\varepsilon_{\tau\tau}$  as expected. However, these samples provide no significant contribution to the constraint on  $\varepsilon_{e\tau}$  because they lack an  $e$ -like component. On the other hand, the FC

multi-GeV samples (shown by the dotted curve) include several  $e$ -like sub-samples which better constrain  $\varepsilon_{e\tau}$ .

### VI. ANALYSIS WITH A THREE-FLAVOR HYBRID MODEL WITH NONZERO $\theta_{13}$

There are several scenarios in which sub-dominant effects from standard neutrino oscillations may affect the

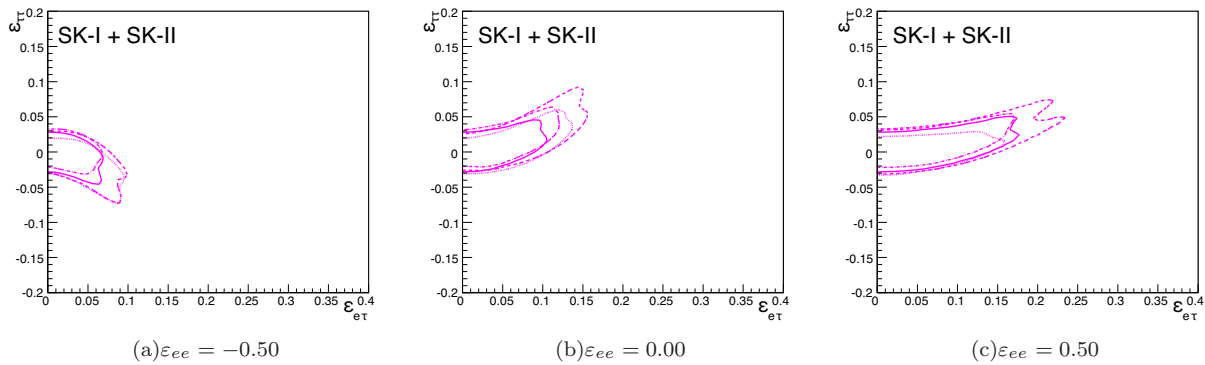


FIG. 8: Allowed NSI parameter regions in the  $\varepsilon_{e\tau}$  vs.  $\varepsilon_{\tau\tau}$  plane at the 90% C.L. for  $\varepsilon_{ee} = -0.5, 0.0,$  and  $0.5$  using four sets of standard oscillation parameters. The solid curve corresponds to  $(\sin^2 \theta_{23}, \Delta m^2) = (0.5, 1.7 \times 10^{-3} \text{eV}^2)$ , the dashed curve to  $(0.5, 2.7 \times 10^{-3} \text{eV}^2)$ , the dotted curve to  $(0.39, 2.1 \times 10^{-3} \text{eV}^2)$ , and the dashed-dotted curve to  $(0.61, 2.1 \times 10^{-3} \text{eV}^2)$ .

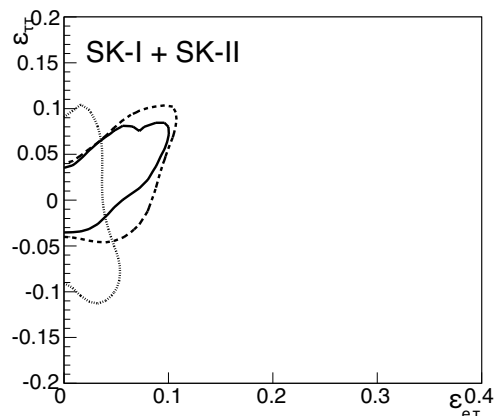


FIG. 9: Allowed NSI parameter regions at 90% C.L. derived for three subsamples at  $\varepsilon_{ee} = -0.25$ . The solid curve indicates the allowed region given by the UPMU throughgoing sample, the dashed curve shows the PC and UPMU stopping sample, and the dotted curve is the constraint from the FC multi-GeV samples.

allowed NSI parameters. In this section we consider how our limits change when effects from nonzero  $\theta_{13}$  are included in the analysis.

Present data suggest that  $\theta_{13}$  is small relative to the other mixing angles,  $\theta_{23}$  and  $\theta_{12}$ . The Chooz experiment has placed the most stringent limit on the parameter, indicating that  $\sin^2 \theta_{13} < 0.04$  [33]. Due to its small size its effects were ignored in the main analysis. However,  $\theta_{13}$  is expected to induce  $\nu_\mu \rightarrow \nu_e$  transitions at multi-GeV energies possibly producing an excess of  $\nu_e$  that could be misinterpreted as the effect of  $\varepsilon_{e\tau}$ . Here the analysis of section V is repeated for the normal hierarchy,  $\Delta m_{23}^2 > 0$ , and inverted hierarchy,  $\Delta m_{23}^2 < 0$ , with  $\theta_{13}$  fixed at the Chooz limit. Including  $\theta_{13}$  breaks the positive-negative

symmetry of  $\varepsilon_{e\tau}$  so the fit is expanded to cover both regions. The results of the fit are again presented as a function of  $\varepsilon_{ee}$  in Fig. 10.

Focusing on the normal hierarchy case, the addition of  $\theta_{13}$  to the fit tends to improve the constraint on  $\varepsilon_{e\tau}$ . The  $\nu_\mu \rightarrow \nu_e$  oscillation probability in constant density matter is proportional to  $\sin^2 \Theta_{13}$ , where  $\Theta_{13}$  is the effective  $\theta_{13}$  mixing angle given by,

$$\Theta_{13} \sim \theta_{13} + \phi, \quad \tan 2\phi \sim \frac{a \sin 2\theta_{13}}{\Delta m_{31}^2 - a \cos 2\theta_{13}}. \quad (11)$$

In this equation  $a$  is the product of the MSW matter potential and the neutrino energy,  $\pm 2\sqrt{2}G_F N_e E_\nu$ , where the sign is positive (negative) for neutrinos (antineutrinos). The structure of the denominator can create a resonant enhancement of the oscillation probability depending upon the energy of the neutrino and the density of matter it traverses. If  $\theta_{13}$  is nonzero, the upward-going  $\nu_e$  flux is expected to increase in the 2-10 GeV range, coinciding with the region where  $\varepsilon_{e\tau}$  can enhance the  $\nu_e$  flux. Accordingly, part of the  $\nu_e$  flux induced by  $\varepsilon_{e\tau}$  is now occupied by events from  $\theta_{13}$  transitions which therefore results in a tighter constraint on the parameter. However, the resonance behavior of Eqn.(11) is contingent upon the signs of the mass hierarchy and MSW matter potential. This, coupled with unequal neutrino and antineutrino fluxes in the atmospheric data, results in the asymmetric constraint on  $\varepsilon_{e\tau}$  seen in Fig. 10.

## VII. CONCLUSIONS

We have studied non-standard neutrino interactions in the context of atmospheric neutrinos propagating in the Earth. Two analyses were presented considering possible effects from both flavor changing neutral current and lepton universality violating interactions. Analysis of the SK-I and SK-II atmospheric neutrino data shows no evidence of NSI and provides the following constraints. For

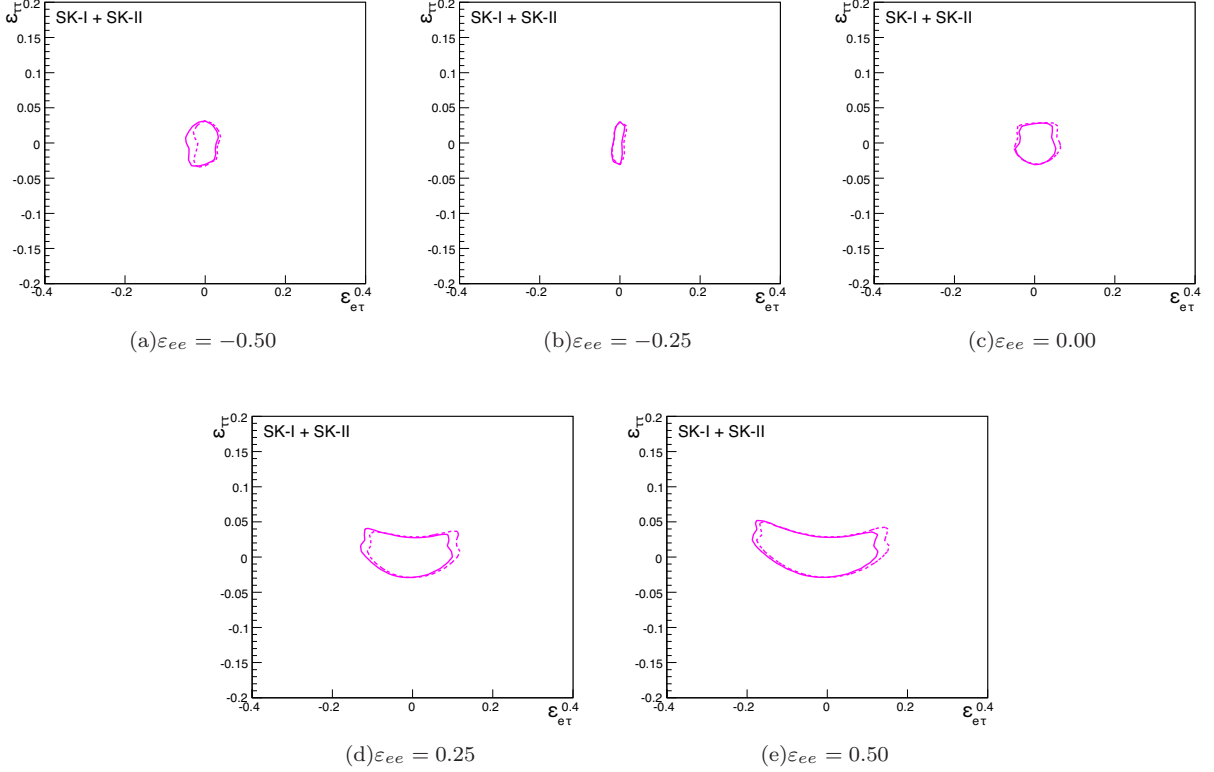


FIG. 10: Allowed NSI parameters regions in the  $\varepsilon_{e\tau}$  vs.  $\varepsilon_{\tau\tau}$  plane at the 90% C.L. when  $\sin^2\theta_{13} = 0.04$ . The solid curve and dashed curve indicate the normal inverted hierarchy fits, respectively.

NSI in the  $\nu_\mu - \nu_\tau$  sector, the 2-flavor hybrid model allows contributions from NSI in the form

$$|\varepsilon_{\mu\tau}| < 1.1 \times 10^{-2} \quad \text{and} \\ -4.9 \times 10^{-2} < \varepsilon_{\tau\tau} - \varepsilon_{\mu\mu} < 4.9 \times 10^{-2}, \quad (12)$$

at the 90% C.L., where  $\varepsilon$  and  $\varepsilon'$  are replaced with  $\varepsilon_{\mu\tau}$  and  $\varepsilon_{\tau\tau} - \varepsilon_{\mu\mu}$ , respectively. In the 3-flavor hybrid model, the allowed regions are presented for different values of  $\varepsilon_{ee}$  since the atmospheric data have no ability to constrain this parameter.

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