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APPLICATION OF THE FUZZY DECISION TREES FOR THE TASKS OF ALTERNATIVE CHOICES

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The article deals with solving the alternative choice problem on the base of fuzzy decision trees or fuzzy positional games, which peculiarity lies in using fuzzy quality estimation of decisions consequence and nature states. The proposed approach and numerical algorithm can be used for a large number of transport services problems. Among them, there are the investment decisions in the transport sector, the choice of types of trucks, cars, buses etc, the choice of variants for transportation organization, financial services problems and others. Corresponding example of practical calculations in the specialized program Fuzicalc for fuzzy numbers has been given and described.

Keywords: alternative choice problem, fuzzy decision trees, fuzzy positional games, Fuzicalc program

1. Introduction

The problem of decision-making is one of the most common in business, production and service activities of transport enterprises. In particular, such corresponding tasks arise in making a variety of investment and financial decisions when choosing a park (types of vehicles) and various equipment, choice of variants for transport services and traffic, for variants of interaction between producers and consumers of transportation and logistics services, and so on. These decisions are made in competitive alternatives and market conditions. In fact, there is a considerable number of different options, alternatives, possibilities and situations. At the same time, the decisions must take into account the various factors, trends, developments, which are of fairly vague, fuzzy character [1, 2]. The above reasons require the application of appropriate mathematical tools, models and algorithms for recording and processing the uncertainties, in particular on the basis of the fuzzy sets theory.

2. Traditional Approach to the Use of Decision Trees

Decision trees are one of the most frequently used methods of selecting the best variants of actions on the set of available possibilities. Many tasks of alternative analysis and selection of decisions require to solve situations and environmental states in which one set of policies and states generates a different state of this or similar type. If there are two (or more) consecutive sets of solutions, and subsequent decisions ,based on the results of the previous and / or two (or more) sets of states of the environment (i.e., there is a whole chain of decisions emanating from one to another, which corresponds to the events, occurring with some probability), then such situations can be represented by formal models in the form of positional or multi-stage games. The graphical representations of these games are called decision trees.

For building a decision tree, the decision-maker must determine (in accordance with his views) a sequence of decisions and environmental states, with estimated probabilities and gains (losses) for any combination of alternatives and environmental states. Thus, we could argue that the concept of expected value is an integral part of the method of decision trees. Traditionally, the use of decision trees in the tasks of alternative choice use point estimates for probabilities of states of nature, gains or losses. Thus, we consider the estimated expected values, and they themselves explicitly in the problem solving process are not represented. In addition, for the traditional variants of using the decision trees it is not possible to use qualitative estimates of the parameters of the problem.

3. Solution to the Multi-Criteria Choice Problem, Using Fuzzy Decision Trees

Reflection of the concept of expected value can be achieved if we move from the point estimates to the estimates in the form of fuzzy numbers. In this case, the fundamental changes in the process of decision tree passing is not required. All calculations will be realized on the basis of so-called "fuzzy calculations", and the final results will be presented in the form of fuzzy numbers. Modeling the level of uncertainty in this case can be realized by extending the base set of relevant assessments and choice of type of membership function.

When in the final evaluation uses the fuzzy numbers along with the numerical results the distribution of its validity as the corresponding membership function will be obtained. And the type of this function (the degree of fuzziness) will characterize the degree of fuzziness of the solution. All this will give more information for the decision-makers.

3.1. Main Features of Fuzzy Decision Trees

If the fuzzy decision trees contain only quantitative evaluation of alternatives and states of nature in the form of the fuzzy numbers, then (if we want to find the best solution) we must use the traditional calculation methods, grounded in the basis of so-called soft calculations. The results will be represent as a set of fuzzy numbers with corresponding membership functions:

$$M = \{\mu_f(z) : f = \overline{1,F}\}, \tag{1}$$

where $\mu_f(z)$ – is the membership function of fuzzy number that represents the integral evaluation of a tree branch with the *f* – number.

In case of quantitative estimations, the integral conclusions, which are presented by fuzzy numbers (1), are located in natural order on the real numerical axis. Therefore, if under conditions of the problem we are interested in maximizing the effect, then score the best solution should be placed on the right side of the real axis. However, if we solve the inverse problem, such as to minimize the risk, the assessment of the best solution should be in the left side of the real axis.

Obtained (for each branch of fuzzy decision tree) integral estimates will be in the form of fuzzy numbers with certain membership functions (Fig. 1), where *f, h, h+1* – are the numbers of branches of a decision tree.

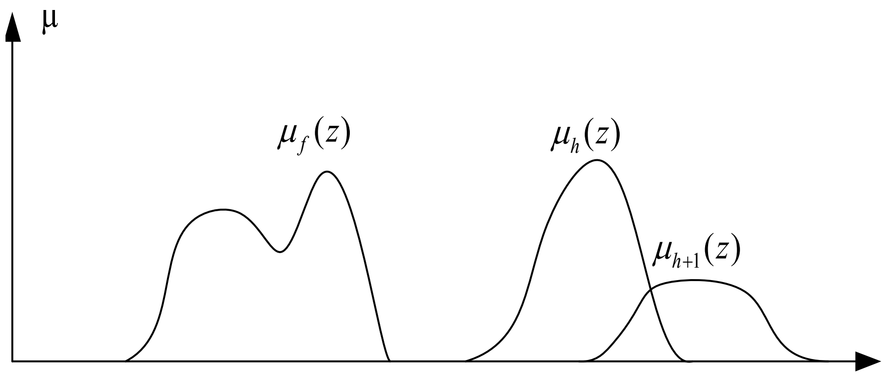


Figure 1. Membership functions for the branches of a decision tree

It is obvious, that, in general, the membership functions for the estimation of the relevant branches of the tree can be arbitrary. Therefore, choosing the best alternatives should be performed on the coordinate of center of gravity:

$$CG_f = \frac{\sum_i \mu_f(z_i) z_i}{\sum_i \mu_f(z_i)} \tag{2}$$

According to this estimate (2) as the best alternative would be considered an alternative with the relevant branch of the tree with the number *h +1*.

However, this solution cannot be regarded as a quite reasonable one.

The fact, the membership function can be interpreted as estimates of the distribution function of the truth of the decision. It is easy to see (Fig. 1) that the estimate of the true for solution on a tree branch with index *h* will be higher than at on branch with index *h+1*.

For solving this situation, we can use the multiplicative estimate $R_f = CG_f \mu_f (CG_f)$, multiplication of the coordinates of the gravity center and the truth-value at this point. It is possible that for the situation, shown in Fig. 1, $R_h > R_{h+1}$, and then the solution corresponding to the branch with index *h*, can be seen as more preferable.

We can also use the reliability of the decision [3], which takes into account the degree of vagueness of fuzzy estimates.

In decisions making practice there may be the situations, when some or all of the states of nature and evaluation of alternatives may have only qualitative views. Since fuzzy numbers – there are, in essence, the fuzzy sets, whose elements are defined on the set of real numbers, the processing of fuzzy decision tree in the presence of quantitative and qualitative assessments can be based on operations on fuzzy sets without using arithmetic operations. In case the decision tree contains both quantitative and qualitative assessments, they all must be reduced to a single universal set *U = [0,1]*.

3.2. Example of Problem Formulation for the Decision Tree with Fuzzy Estimates

Some examples (fragments) of the fuzzy estimates for the decision tree, presented on Fig. 2, are show on Fig. 3.

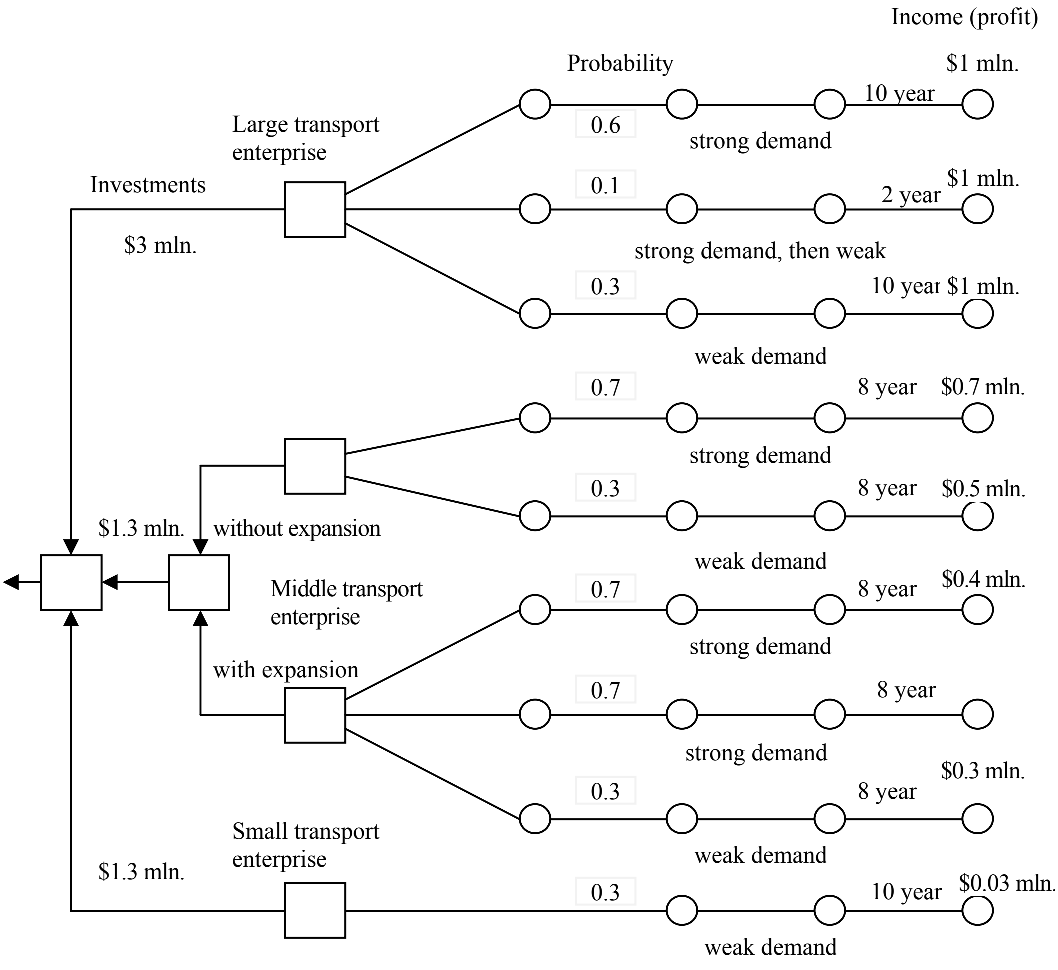
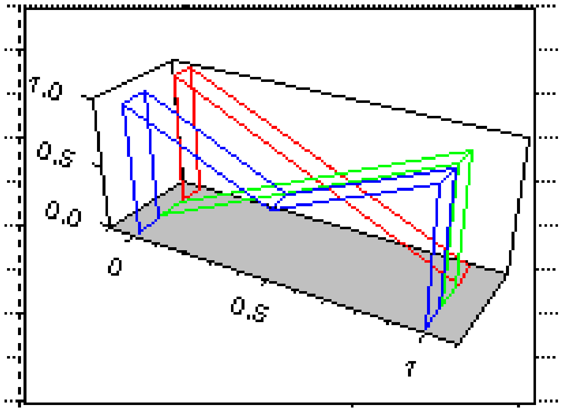


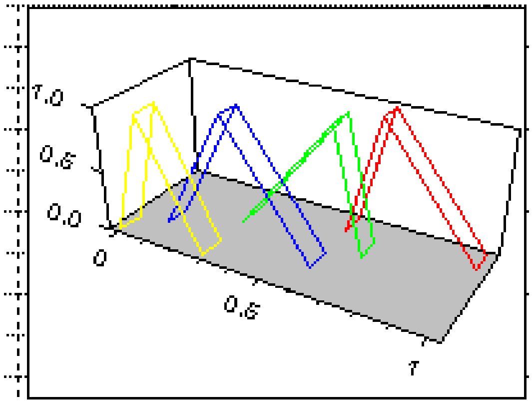
Figure 2. Decision tree for calculations (example, numerical values are approximate)

The problem for solving is to make investment decision for transport enterprise in the situation of the uncertainly of the market conditions and states of nature (the value of the expected demand, the size of the enterprise in which investments are assumed etc).

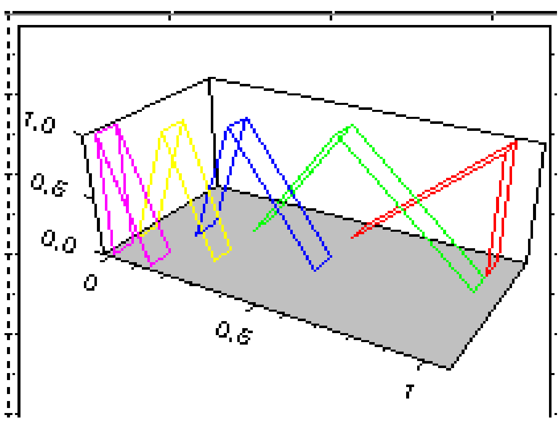
In this example, using triangular membership functions, can be explained only by considerations of simplicity of calculations.



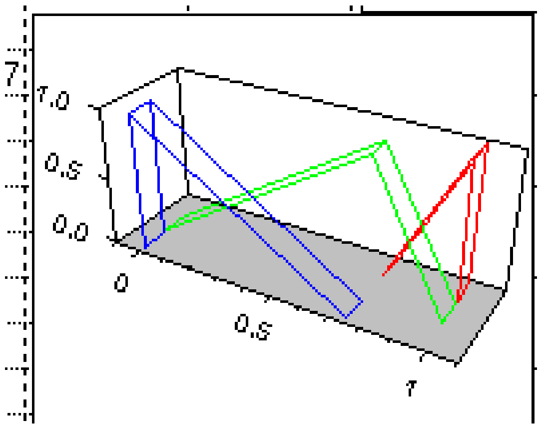
Demand: weak (0.293), strong (0.707),
Strong then weak (0.5)



Probability: 0.7 (0.76), 0.6 (0.54), 0.3 (0.33), 0,1 (0.13)



Expected revenue US$, mln.: 1 (0.87),
0.7 (0.61), 0.5 (0.33), 0.4 (0.14), 0.3 (0.42)



The timing of the project's realization:
10 years (0.93), 8 years (0.61), 2 years (0.22)

Figure 3. Fragments of the fuzzy estimates for the decision tree

3.3. General Approaches to Problem Solution for the Described Example

To solve this variant of the problem, we propose to use the following approach. Let us assume that the decision tree consists of a set of nature states $S = \{s_i : i = \overline{1, I}\}$ and a variety of assessment alternatives $Q = \{q_j : j = \overline{1, J}\}$. For each nature state there are many linguistic assessments $L(s_i) = \{l_k(s_i) : k = \overline{1, K}\}$ and the corresponding fuzzy sets defined by membership functions $\mu_{k,i}(z), k \in [1, K], i \in [1, I]$, where $z \in U = [0,1]$ is a formal variable.

The sets of alternative assessment are also the fuzzy sets $L(q_j) = \{l_p(q_j) : p = \overline{1, P}\}$ and $\mu_{p,j}(y), p \in [1, P], j \in [1, J]$, where $y \in U = [1,0]$ is a formal variable.

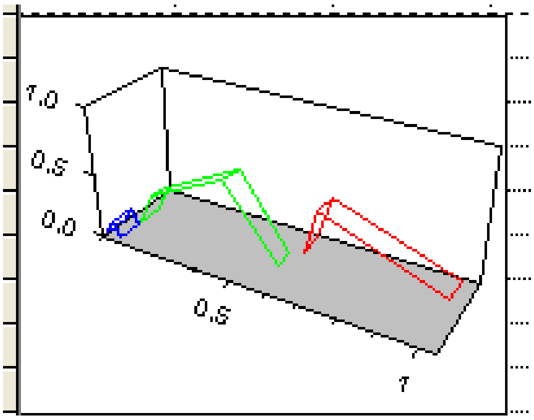
Suppose that a branch of fuzzy decision tree $\tau_f \in T$ (where T is the set of branches, $f = \overline{[1, F]}$), contains N_f nature states and H_f assessments of alternatives. Then a fuzzy set, that represents the evaluation of the alternative solutions, corresponding with the *f* branches number, may be defined as:

$$M_f = \bigcap_{\text{for all } j\in[1,N_f],\ \ i\in[1,H_f]} (\mu_{p,j}, \mu_{k,i}), \tag{3}$$

where M_f is a membership function, corresponding to the fuzzy set that represents an integral estimate of *f*-th branch of the tree.

The considered variant for finding the best alternative solutions by using fuzzy decision tree allows to realize fully the concept of the expected values. However, it is possible that in some nature states and assessment of alternatives, especially when they use only quality assessments, for some tree branches fuzzy sets, obtained from equation (3), will be empty.

At the same time, this situation will not necessarily take place only for the previously known bad alternatives. In addition, it can take place for a large number of alternatives from the considered set that would naturally reduce the level of validity of the constructed decisions. Fig. 4 shows the corresponding example of this situation for fuzzy decision tree, shown in Fig. 2. Here, presented alternative solutions for 1,2,3,6,7,9 – branches, which got zero estimates not because they are automatically bad, but as results of an empty crossings.



crossings			
branch 1	0	0	0
branch 2	0	0	0
branch 3	0	0	0
branch 4	0.73	0.27	0.19
branch 5	0.36	0.48	0.17
branch 6	0	0	0
branch 7	0	0	0
branch 8	0.12	0.11	0
branch 9	0	0	0

Figure 4. Results of crossing for branches of decision tree

The problem of empty crossings may be overcoming as follows steps [1]. Segments of each branch of the tree (except the last one representing the solution) are divided into subsets, giving a non-empty crossing. As a result, for each branch there will be received certain groups of non-empty fuzzy sets with corresponding membership functions: $M_f = \{\mu_{f1}, \mu_{f2}, ..., \mu_{fr}\}$, where r is the number of not empty subsets, which can be obtained on the set of segments of a tree branch with *f* number; $\mu_{fi} \neq 0$, but $\bigcap_i \mu_{fi} = \varnothing$.

3.4. Operation of Constructing the Fuzzy Set Geometric Projection

To obtain a solution, an operation on fuzzy sets proposed in [4] and called "geometric projection of fuzzy sets" is used.

It should be noted that this title is not entirely successful, because it is call an analogy with the famous "Operation of projection of fuzzy sets", which led to various misunderstandings.

Therefore, in the future we propose to use the name "shadow of a fuzzy set" for this operation and define it as a shadow (shade, *Sh*).

We define this operation as follows.

Shadow of a fuzzy set $\widetilde{A}$ to fuzzy set $\widetilde{B}$ must satisfy the following conditions: $Sh(\widetilde{A},\widetilde{B})$ also will be fuzzy set, $Sh(\widetilde{A},\widetilde{A}) = \widetilde{A}$, $Sh(\widetilde{A},\widetilde{B}) = 0$ if ,at least, one of the sets $\widetilde{A}$ or $\widetilde{B}$ is empty, or sets $\widetilde{A}$ and $\widetilde{B}$ are orthogonal.

Procedure for constructing a shadow of a fuzzy set $\widetilde{A}$ on fuzzy set $\widetilde{B}$ is defined as follows (Fig. 5):

$$Sh_{\varphi}(\widetilde{A},\widetilde{B}) = \{\varphi[\mu_{\widetilde{A}}(y), \mu_{\widetilde{B}}(x)]/[y, x' = f(y)]\},$$

where $f(y) = \dfrac{CG[\mu_{\widetilde{B}}(x)]}{CG[\mu_{\widetilde{A}}(y)]} y$ – is a projection function; $CG[\mu_{\widetilde{B}}(x)]$ and $CG[\mu_{\widetilde{A}}(y)]$ are coordinates of the centers of gravity of the figures, which are limited by membership functions $\mu_{\widetilde{A}}(y)$, $\mu_{\widetilde{B}}(x)$; φ – functional, giving the appearance for transformations of the membership functions.

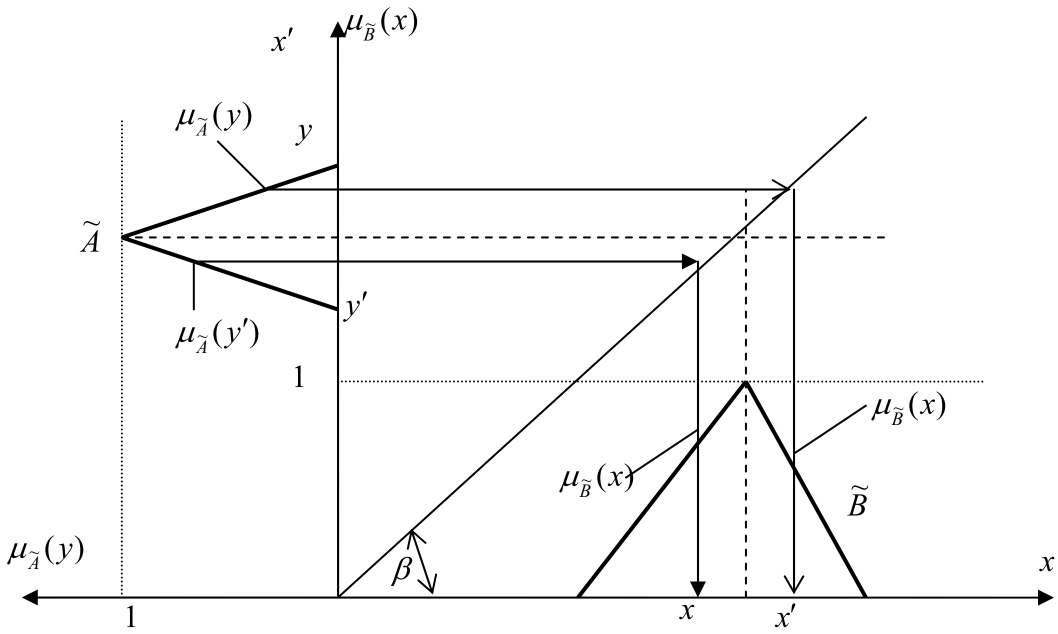


Figure 5. Procedure of fuzzy set shadow constructing

The meaning of this operation is the following: depending on the relative position of fuzzy sets and consequently, the slope of the projection line change the "shadow" of a fuzzy set, is superimposed on another one.

Results of this operation will represent a degree of interaction of estimations of the concepts represented by fuzzy sets. We shall name a fuzzy set $\widetilde{A}$, which is projected on other fuzzy set $\widetilde{B}$ as a source of a shadow. We shall name a fuzzy set $\widetilde{B}$ on which the shadow of fuzzy set $\widetilde{A}$ is projected, as a receiver of the shadow.

The further transformations are carried out in the following sequence. For each indistinct set represented by membership function μ_{fi}, the shadow on the indistinct set representing the required decision is under construction.

As a result for every μ_{fi} we shall receive a corresponding shadow Sh_{f_i}, which, by definition ,will be indistinct set with corresponding membership function $\mu_{Sh_{f_i}}$.

The integrated decision for a branch with number *f* we shall define as crossing $\mathrm{M}_{Sh_f} = \bigcap_i \mu_{Sh_{f_i}}$. Similar transformations are carried out for other branches of a tree.

If to consider $\mu_{Sh_{f_i}}$ as a function of validity distribution, we shall define the best decision as $\max_f M_{Sh_f}$. If unequivocal definition of a maximum is impossible, the value of corresponding coordinate of the centre of gravity CG of membership function M_{Sh_f} is determined and then we find a maximal value as $\max_f M_{Sh_f}(CG)$.

3.5. Graphical and Numerical Results for the Described Example

Below (figs. 6-8 and tab. 1) we show (not losing a generality) the separate graphic and numerical results of calculations for analyzed decision tree from the example.

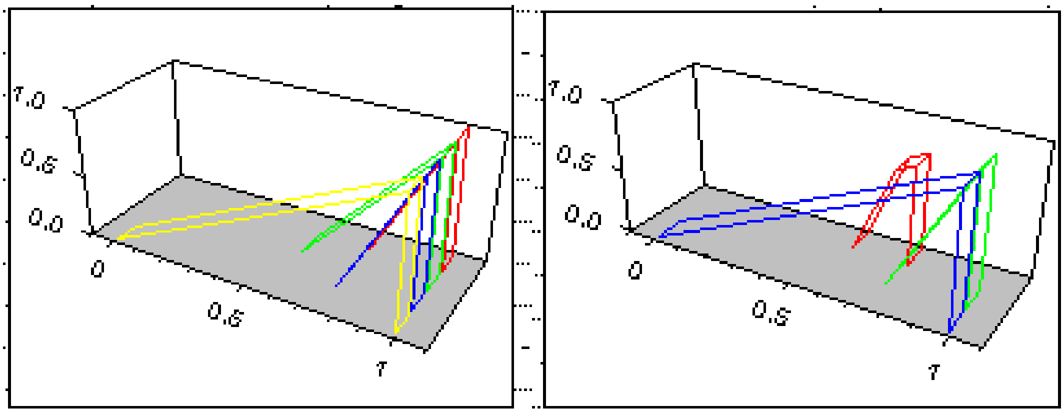


Figure 6. The first individual convolution of conditions for branch B1 (on the left) and its shadow (on the right)

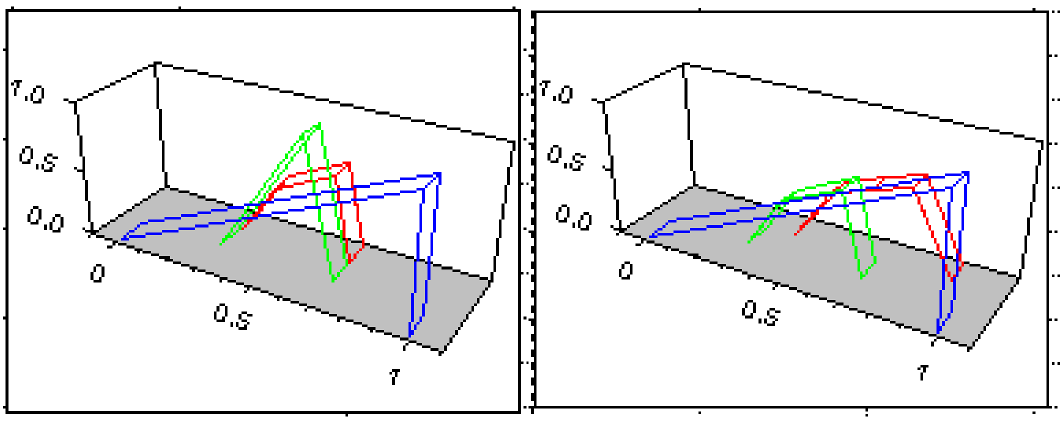


Figure 7. The second individual convolution of conditions for branch B1 (on the left) and its shadow (on the right)

On both figures (6 and 7) we can see on the left the following: red line – membership function which is a result of convolution, other colours – membership functions of convolution participants; and on the right: red line – shadow of individual convolution, green line – source of shadow, blue line – receiver of shadow.

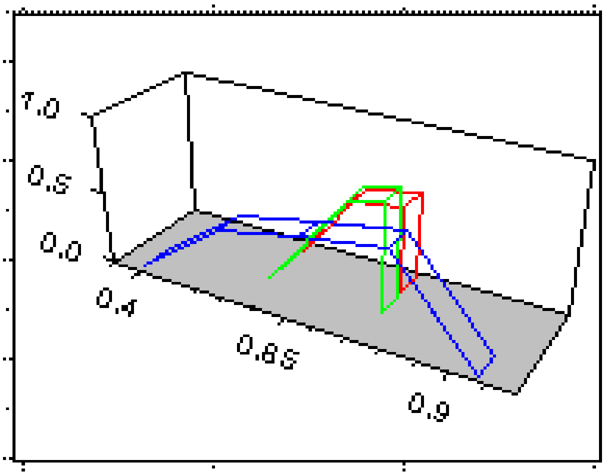


Figure 8. The crossing of shadows of individual convolutions

In this figure the red line is the result of crossing, dark blue and green lines are the sources of shadow.

Calculation results by proposed approach for the decision tree, presented in Fig. 1, are shown in tabl. 1.

Table 1. Results of ranging for branches of the decisions tree

Number of a branch of decision tree	B1	B5	B4	B6	B3	B9	B2	B7	B8
Estimation of the validity of decision	0.56	0.48	0.27	0.19	0.16	0.15	0.13	0.13	0.12

For an examined numerical example the development of a situation on the first branch (B1=0.56) is the most expected and comprehensible.

4. Conclusions

Thus, the offered method for an alternative choice of the decision on the basis of an fuzzy decision tree allows to consider the uncertainty of estimations of decisions and conditions of the nature, and also supposes the use of their qualitative representations.

The method was repeatedly used in practice at an estimation of various alternative technical and investment projects. The received results have been confirmed by the subsequent practice.

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