A new algorithm based on copulas for VaR valuation with empirical calculations

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Abstract

This paper concerns the application of copula functions in VaR valuation. The copula function is used to model the dependence structure of multivariate assets. After the introduction of the traditional Monte Carlo simulation method and the pure copula method we present a new algorithm based on mixture copula functions and the dependence measure, Spearman’s rho. This new method is used to simulate daily returns of two stock market indices in China, Shanghai Stock Composite Index and Shenzhen Stock Composite Index, and then empirically calculate six risk measures including VaR and conditional VaR. The results are compared with those derived from the traditional Monte Carlo method and the pure copula method. From the comparison we show that the dependence structure between asset returns plays a more important role in valuating risk measures comparing with the form of marginal distributions.

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1. Introduction

This paper presents an algorithm using copula functions to simulate random variables and further to valuate the Value-at-Risk (VaR) of a portfolio composed of two financial assets. The problem of modeling asset returns is one of the most important issues in finance. People generally use Gaussian processes because of their tractable properties of easy computation. However, it is well known that asset returns are fat-tailed. For the multivariate case, the joint normal distribution and more generally the elliptical distribution restricts the type of association between margins to being linear, but other dependence structures such as rank correlation and tail dependence should also be considered by risk managers. These two difficulties, Gaussian assumption and dependence structure, can be effectively solved by copulas.

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As we know, linear correlation has the serious deficiency that it is not invariant under non-linear strictly increasing transformation, while the dependence measures derived from copulas can overcome this shortcoming and have broader applications [11,16,17]. Furthermore, copulas can be used to describe more complex multivariate dependence structures, such as non-linear and tail dependence [6]. With the development of computer software and information technology, the theory of copulas has experienced rapid development since the end of 1900s [5,13,14].

But copulas were not used in finance until 1999. After that, copulas were often cited in the financial literature. Refs. [7] and [8] investigated the problem of fitting multivariate distributions to financial data. Refs. [2,9] and [10] studied the problem of calculating financial risk using copulas.

In this paper, we use copulas to model the dependence structure of multivariate financial assets, and design an algorithm based on the so-called mixture copulas and Spearman’s rho to simulate two random variables, then demonstrate the usefulness of this algorithm with an example in Chinese stock markets to calculate the financial risk of a portfolio composed of two stock indices simulated from the algorithm. We first give the definition of copulas and several important dependence measures in Section 2, then introduce six financial risk measures which are always used by risk managers in Section 3. Our focus is on the valuation of VaR. In Section 4 we first introduce briefly the traditional Monte Carlo method and the pure copula method for VaR calculation, then present our algorithm based on mixture copula and Spearman’s rho. In some sense, the above three methods embody the parametric, nonparametric and semi-parametric methods, respectively. Section 5 demonstrates the use of our algorithm to value the financial risk in Chinese stock markets, and also give the results obtained from the other two methods. The comparison of the results derived from the three methods with the real change of the portfolio’s value shows that for lower confidence level, the traditional Monte Carlo and pure copula method perform better than the mixture copula method, while for higher confidence level, the mixture copula method is a better choice. Section 6 concludes the paper.

2. Copulas and dependence measures

2.1. Copulas

In what follows we give the definition of copula functions and some related dependence measures. Readers interested in more details can refer to [12]. Here we consider the bivariate case; nevertheless, all the results carry over to the general multivariate setting.

**Definition 1.** A two-dimensional copula is a real function defined on \( I^2 = [0, 1] \times [0, 1] \), with range \( I = [0, 1] \), such that

1. \( \forall (u, v) \in I^2, C(u, 0) = 0 = C(0, v), C(u, 1) = u, C(1, v) = v \);
2. \( \forall [u_1, u_2] \times [v_1, v_2] \in I^2 \) with \( u_1 \leq u_2, v_1 \leq v_2 \),

\[
C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0.
\]

Thus a copula can represent the joint distribution function of two standard uniform random variables \( U, V \):

\[
C(u, v) = P(U \leq u, V \leq v).
\]

We can use this feature to rewrite via copulas the joint distribution function of two (even non-uniform) random variables. The most interesting fact about copulas in this sense is the following Sklar’s theorem [15].

**Theorem 1.** Let \( F(x, y) \) be a joint distribution with continuous margins \( F_1(x) \) and \( F_2(y) \), then there exists a unique copula \( C \) such that

\[
F(x, y) = C(F_1(x), F_2(y)).
\]  

We can see from this theorem that any copula \( C \) pertaining to function \( F \) can be expressed as

\[
C(u, v) = F(F_1^{-1}(u), F_2^{-1}(v)).
\]

Sklar’s theorem is very important because it provides a way to analyze the dependence structure of random variables without studying their marginal distributions.
An often-used copula function is the following Gaussian copula:

\[
C_{\rho}(u, v) = \int_{-\infty}^{\phi^{-1}(u)} ds \int_{-\infty}^{\phi^{-1}(v)} dt \frac{1}{2\pi(1-\rho^2)^{1/2}} \exp \left\{ -\frac{(s^2 - 2\rho st + t^2)}{2(1-\rho^2)} \right\} dt.
\]  (3)

**Proposition 1.** For every copula the well-known Fréchet bounds apply [4]:

\[C_W(x_1, x_2) \equiv \max(x_1 + x_2 - 1, 0) \leq C(x_1, x_2) \leq C_M(x_1, x_2) \equiv \min(x_1, x_2).\]

For a bivariate case, the bounds \(C_W\) and \(C_M\) themselves are copulas since, if \(U\) is \((0,1)\)-uniformly distributed, then

\[C_W(x_1, x_2) = P(U \leq x_1, 1 - U \leq x_2), \quad C_M(x_1, x_2) = P(U \leq x_1, U \leq x_2),\]

so that \(C_W\) and \(C_M\) are the bivariate distribution functions of the vectors \((U, 1 - U)^T\) and \((U, U)^T\), respectively.

### 2.2. Dependence measures

Different copulas capture different types of dependence between variables. Here we present two dependence concepts which will be used in this paper.

(i) **Pearson correlation**

The Pearson correlation coefficient \(\rho(X, Y)\) for random variables \(X\) and \(Y\) is a measure of linear dependence:

\[\rho(X, Y) = \frac{\text{cov}(X, Y)}{\sqrt{\text{var}(X)\text{var}(Y)}}.\]

If \(X\) and \(Y\) are independent, \(\rho(X, Y) = 0\); if they are perfectly linearly dependent, \(\rho(X, Y) = \pm 1\). Linear correlation is a natural dependence measure for multivariate normally, or more generally, elliptically distributed random variables, but it is not invariant under non-linear strictly increasing transformations.

(ii) **Spearman’s rho**

Let \(X\) and \(Y\) be two random variables with marginal distribution functions \(F_1\) and \(F_2\) and joint distribution function \(F\), and assume that \((X_1, Y_1), (X_2, Y_2)\) and \((X_3, Y_3)\) are three independent samples of \((X, Y)\); then the Spearman’s rho of \((X, Y)\) is defined to be

\[\rho_s = 3[\Pr((X_1 - X_2)(Y_1 - Y_3) > 0) - \Pr((X_1 - X_2)(Y_1 - Y_3) < 0)].\]

That is, Spearman’s rho is proportional to the difference between the probability of concordance and the probability of discordance for two vectors \((X_1, Y_1)\) and \((X_2, Y_3)\), i.e., a pair of vectors with the same margins, but one vector has the joint distribution \(F\), while the components of the other are independent.

If we further assume that the copula of \((X, Y)\) is \(C\), then Spearman’s rho can be expressed as [12]

\[\rho_s(X, Y) = 12 \int_0^1 \int_0^1 (C(x, y) - xy) dx dy,\]  (4)

and one can also easily show that

\[\rho_s(X, Y) = \rho(F_1(X), F_2(Y)),\]

where \(\rho\) is the linear correlation coefficient. In this sense Spearman’s rho is a kind of rank correlation.

Spearman’s rho has obvious advantages over linear correlation. The former can be considered as a measure of the degree of monotonic dependence between \(X\) and \(Y\), whereas the latter only measures the degree of linear dependence. The linear correlation coefficient depends on margins and is affected by non-linear increasing transformations, while Spearman’s rho is not affected and depends only on copulas.

### 3. Risk measures

In practice, many risk managers employ VaR (Value-at-Risk) as a tool of risk measurement. Briefly speaking, VaR is the maximal potential loss of a position or a portfolio in some investment horizon under a given confidential level. To be precise, let \(\{P_t\}_{t=1}^n\) be the market values of an asset or a portfolio of assets over \(n\) periods, \(X_t = -\frac{P_t - P_{t-1}}{P_{t-1}}\) (or
\(X_t = -\log \frac{P_t}{P_{t-1}}\); in our paper we will employ the former one) be the negative return (loss) over the \(t\)-th period. Given a positive value \(\alpha\) close to 0, the VaR of \(X\) at confidence level \((1 - \alpha)\) is given by

\[
\text{VaR}_\alpha = \inf\{x \in \mathbb{R} | Pr(X \leq x) \geq 1 - \alpha\}.
\]  

(5)

While VaR is a powerful tool for risk management, it is not a coherent risk measure, since it is not sub-additive. For this reason, a modified risk measure based on VaR – conditional VaR (CVaR for short) – is brought out to overcome this problem. In short, CVaR gives the mean loss that exceeds VaR, that is,

\[
\text{CVaR}_\alpha = E[X | X > \text{VaR}_\alpha(X)].
\]

(6)

In some papers the above risk measure is also called the expected shortfall (ES) of \(X\); see for example [1].

In this paper we will also calculate the following four risk measures [10]:

\[
e_X(\alpha) = E[X - \text{VaR}_\alpha(X)|X > \text{VaR}_\alpha(X)]
\]

(7)

\[
e^*_X(\alpha) = \text{Median}[X - \text{VaR}_\alpha(X)|X > \text{VaR}_\alpha(X)]
\]

(8)

\[
m_X(\alpha) = \frac{\text{VaR}_\alpha(X) + e_X(\alpha)}{\text{VaR}_\alpha(X)}.
\]

(9)

\[
m^*_X(\alpha) = \frac{\text{VaR}_\alpha(X) + e^*_X(\alpha)}{\text{VaR}_\alpha(X)}.
\]

(10)

For any fixed \(\alpha\), the former two measures are one-step-ahead predictions based on the unconditional distribution of the portfolio and represent the expected and median excess loss beyond \(\text{VaR}_\alpha(X)\); while the latter two represent the expected and median total loss of a portfolio standardized by its VaR.

From the definitions of the above six risk measures we can see that the key point for the valuation is VaR, which is the focus of our algorithm. In next section we will give three methods for the calculation of VaR.

4. Methods for VaR valuation

In this section we first briefly introduce the traditional Monte Carlo method for VaR valuation, then present two algorithms based on copulas and Monte Carlo method.

4.1. Traditional Monte Carlo method

In the Monte Carlo method for calculating VaR, one first gets the possible distribution from assets’ historical data, then generates variables according with this distribution and construct portfolio’s possible payoff, from which one can obtain the estimate of VaR for a given confidence level. In traditional Monte Carlo method one always assumes that the marginal and joint distributions for asset returns are normal distributions.

The following algorithm generates variables \(X_1\) and \(X_2\) with normal distributions \(N(\mu_1, \sigma^2_1)\) and \(N(\mu_2, \sigma^2_2)\) and linear correlation \(\rho\) from two \([0, 1]\)-uniform variables \(U_1\) and \(U_2\):

(1) Put \(S_1 = \sqrt{-2 \ln U_1} \sin(2\pi U_2), S_2 = \sqrt{-2 \ln U_1} \cos(2\pi U_2)\);

(2) Put \(X_1 = \sigma_1 S_1 + \mu_1, X_2 = \sigma_2(\rho S_1 + \sqrt{1 - \rho^2} S_2) + \mu_2\).

This algorithm can be considered as a kind of parametric method.

4.2. Pure copula method

From the above algorithm we can see that the traditional Monte Carlo method restricts the joint distribution for asset returns to being normal, but it is well known that in practice asset returns are not normal. To overcome this problem one can use a copula function to give the joint distribution properly characterizing the dependence structure of asset returns.
The following algorithm generates variables \( X_1 \) and \( X_2 \) with a given copula function \( C \) being the joint distribution:

1. Generate random variables \( U, W \) with \([0, 1]\)-uniform distribution;
2. For a given copula function \( C \), calculate \( C^{-1}(W) \), and put \( V = C^{-1}(W) \);
3. Put \( X_1 = F_1^{-1}(U) \), \( X_2 = F_2^{-1}(V) \), where \( F_1 \) and \( F_2 \) are the given marginal distributions of assets’ returns.

This algorithm can be considered as a kind of nonparametric method. With the aim of comparison we call this algorithm a pure copula method. Here one can choose any copula function meeting his demand.

4.3. Mixture copula method

In the following we will present our method, which we call a mixture copula method, since in this method a mixture distribution, or equivalently, a mixture copula, is used to describe the dependence structure between asset returns. Here the mixture distribution means the linear combination of two distributions, with Spearman’s rho \( \rho_s \) being the combination coefficient. We can see from the following algorithm that this method is in fact a kind of semi-parametric method.

Before giving the algorithm, we first give the following lemma \([3]\) which will help to get the main result proved.

**Lemma 1.** Let \( F_1 \) and \( F_2 \) be two univariate distributions, \( \rho_{\text{min}} \) and \( \rho_{\text{max}} \) the corresponding minimum and maximum linear correlations. Let \( \rho \in [\rho_{\text{min}}, \rho_{\text{max}}] \); then the bivariate mixture distribution given by

\[
F(x, y) = \lambda F_W(x, y) + (1 - \lambda) F_M(x, y),
\]

where

\[
\lambda = \frac{\rho_{\text{max}} - \rho}{\rho_{\text{max}} - \rho_{\text{min}}},
\]

\[
F_W(x, y) = \max(F_1(x) + F_2(y) - 1, 0), \quad F_M(x, y) = \min(F_1(x), F_2(y)),
\]

has margins \( F_1 \) and \( F_2 \) and linear correlation \( \rho \).

To go further we put

\[
\lambda_s = \frac{\rho_{\text{max}} - \rho_s}{\rho_s - \rho_{\text{min}}},
\]

\[
\tilde{F}(x, y) = \lambda_s F_W(x, y) + (1 - \lambda_s) F_M(x, y),
\]

where \( \rho_s \) is the Spearman’s rho corresponding to distributions \( F_1 \) and \( F_2 \). Then we get the following proposition from the above lemma.

**Proposition 2.** The random vector generated by the following algorithm has the joint distribution \( \tilde{F}(x, y) \) with margins \( F_1, F_2 \) and Spearman’s rho \( \rho_s \):

1. Simulate \( U \) and \( V \) independently from standard uniform distribution;
2. If \( U \leq \lambda_s \), take \((X, Y)^T = (F_1^{-1}(V), F_2^{-1}(1 - V))^T\);
3. If \( U > \lambda_s \), take \((X, Y)^T = (F_1^{-1}(V), F_2^{-1}(V))^T\), where superscript \( T \) means the transpose of a vector.

**Proof.** From Sklar’s theorem, the random vector with joint distribution \( \tilde{F}(X, Y) \) has copula

\[
C(F_1(x), F_2(y)) = \lambda_s C_W(F_1(x), F_2(y)) + (1 - \lambda_s) C_M(F_1(x), F_2(y))
\]

(11)

where \( C_W \) and \( C_M \) are copulas corresponding to joint distributions \( F_W(x, y) \) and \( F_M(x, y) \), respectively:

\[
C_W(F_1(x), F_2(y)) = \max(F_1(x) + F_2(y) - 1, 0),
\]

\[
C_M(F_1(x), F_2(y)) = \min(F_1(x), F_2(y)).
\]

Let \( U = F_1(X), \ V = F_2(Y), \ u = F_1(x), \ v = F_2(y) \); then from equation (11) we have

\[
C(u, v) = \lambda_s C_W(u, v) + (1 - \lambda_s) C_M(u, v).
\]
Since \( C(u, v) \) can be thought of as the joint distribution of two standard uniform random variables \( U \) and \( V \), and the Spearman’s rho \( \rho_s \) between \( X \) and \( Y \) is the linear correlation between \( U \) and \( V \), we conclude the proof from the above lemma.

\[\Box\]

5. An application to Chinese stock markets

In this section, we will use the above three methods to valuate the risk in Chinese stock markets. We choose Shanghai Stock Composite Index and Shenzhen Stock Composite Index to form an equally weighted portfolio and compute the six risk measures defined in Section 3 for this portfolio.

The data of daily closing prices for the two market indices are downloaded from Yahoo finance (http://cn.finance.yahoo.com/). We choose the prices from January 3, 2001 to December 31, 2004, which are 1200 data points in sum, to calculate the six risk measures, then use the price of January 1, 2005 as out-of-sample data to do a back-test. Simple statistics of the two indices and the portfolio are given in Table 1.

To calculate the six risk measures, we first model the margins for the asset returns, then adding an appropriate dependence structure. Using copulas we can take into account the leptokurtic property of asset returns shown in Table 1. In order to choose a suitable distribution we check the fitness effect of the margins of asset returns. The following Fig. 1 displays the fitness effect tested by constructing P–P plots using Laplace and normal distributions.

In Fig. 1, the above two plots are the fitness effect test for Laplace distributions fitted to Shanghai and Shenzhen Index data, respectively, while the other two plots are for normal distributions. From the figure we can see that the normal distribution which is usually fat-tailed is not so good a description for asset returns as the Laplace distribution which represents the leptokurtic property of asset returns in a better way. For this reason we will use Laplace distribution to fit the margins.

In valuating the risk measures, we make two simplifications: first, we do not consider the time dependence of the financial data and just use the daily index returns, since our main purpose is to demonstrate the effect of the new algorithm on the evaluation of risk measures; second, only the empirical estimates of the risk measures are computed, which works well as long as \( \alpha \) is not too small.

In Monte Carlo simulation, the classical bivariate normal distribution assumption is used to valuate the risk measures. In the pure copula method, the Laplace distribution is used for the asset return margins and the Gaussian copula is used to describe the dependence structure. From the introduction of the methods in Section 4 it is easy to see that the traditional Monte Carlo method is equivalent to the pure copula method if the Gaussian copula is used to model the bivariate distribution for asset returns. Different copulas may be employed in the pure copula method to describe the dependence structure between asset returns to investigate the effect of different copulas on risk measures. But here we only use the Gaussian copula because our focus is to compare the three simulation methods and show the usefulness of copulas in measuring risk rather than to investigate the effect of different copulas on risk measures. In the mixture copula method, we use Laplace distribution for the margins and the mixture distribution (or equivalently, the mixture copula) for the dependence structure between asset returns.

The empirical estimates of VaR\( \alpha \), CVaR\( \alpha \), \( e_X(\alpha) \), \( e_X^*(\alpha) \), \( m_X(\alpha) \) and \( m_X^*(\alpha) \) for the equally weighted portfolio and for \( \alpha = 0.05, 0.01 \) and 0.005 are given in Table 2.

First we can see from the table that when \( \alpha \) equals 0.05, the estimates for all of the six measures are very close under the traditional Monte Carlo method and the pure copula method, which coincides with the foregoing conclusion that the two methods are equivalent to each other if the Gaussian copula is used to model the bivariate distribution for asset returns. This shows that the dependence structure between asset returns plays a more important role in valuating risk measures comparing with the form of marginal distributions.

### Table 1

<table>
<thead>
<tr>
<th></th>
<th>( R_{SH} )</th>
<th>( R_{SZ} )</th>
<th>( R_P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N ) (valid)</td>
<td>1200</td>
<td>1200</td>
<td>1200</td>
</tr>
<tr>
<td>Mean</td>
<td>0.0000287</td>
<td>-0.001005</td>
<td>-0.000359</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.0136276</td>
<td>0.0142559</td>
<td>0.0138640</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.896</td>
<td>0.685</td>
<td>0.795</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>7.143</td>
<td>6.182</td>
<td>6.732</td>
</tr>
</tbody>
</table>
Fig. 1. P–P plots of Laplace and normal distributions fitted to Shanghai and Shenzhen index data.

Table 2
Results for six risk measures under three methods

<table>
<thead>
<tr>
<th></th>
<th>Traditional Monte Carlo</th>
<th>Pure copula method</th>
<th>Mixture copula method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.05</td>
<td>.01</td>
<td>.005</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>VaR$\alpha$</td>
<td>$-17.48$</td>
<td>$-23.45$</td>
</tr>
<tr>
<td></td>
<td>CVaR$\alpha$</td>
<td>1.24</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>$e_{X}(\alpha)$</td>
<td>18.73</td>
<td>23.83</td>
</tr>
<tr>
<td></td>
<td>$e_{X}^*(\alpha)$</td>
<td>18.43</td>
<td>23.93</td>
</tr>
<tr>
<td></td>
<td>$m_{X}(\alpha)$</td>
<td>$-0.071$</td>
<td>$-0.016$</td>
</tr>
<tr>
<td></td>
<td>$m_{X}^*(\alpha)$</td>
<td>$-0.054$</td>
<td>$-0.020$</td>
</tr>
</tbody>
</table>

In order to check the reliability of the computed VaR, one can observe the portfolio’s real value change at a certain moment, and compare the computed results with it to do the back-test. For this, we get $-14.46$ of the portfolio’s real value change using the real data from the security markets, which means that the constructed portfolio experienced a loss of 14.46 from December 31, 2004 to January 1, 2005. Comparing this with the values of VaR in Table 2, we find that for lower confidence level, the traditional Monte Carlo and pure copula method perform better than the mixture copula method which underestimates the maximum possible loss, while for higher confidence level, the mixture copula method is a better choice.

When taking a look at CVaR$\alpha$, $e_{X}(\alpha)$ and $e_{X}^*(\alpha)$, we can see that the values obtained from the mixture copula method are less than those obtained from the traditional Monte Carlo method and the pure copula method, and the values obtained from the pure copula method are the largest. The last two rows in Table 2 give the expected and median total loss of the portfolio standardized by its VaR, respectively. The values for the two measures are almost all negative, since the absolute values of VaR are almost all less than $e_{X}(\alpha)$ and $e_{X}^*(\alpha)$. 
6. Conclusions

In this paper, we design a new algorithm using copulas to valuate financial risk. Our work deals with the simulation problem in valuating the portfolio’s VaR and other risk measures.

On the basis of copula theory and the dependence measure, Spearman’s rho, a new simulation algorithm is presented and used to calculate the financial risk in Chinese stock markets. The portfolio is composed of the Shanghai Stock Composite Index and Shenzhen Stock Composite Index with equal weight. The values of six risk measures derived from this new method are compared with those obtained from the traditional Monte Carlo simulation and pure copula method. The comparison shows that the dependence structure between asset returns plays a more important role in valuating risk measures comparing with the form of marginal distributions.

From this paper one can see that the copula is a very powerful tool for risk measurement in that it fulfills one of its main goals: modeling the dependence structure between individual risks. On the other hand, this paper also indicates that the mixture copula method has advantages over the other two methods in two respects: first, it is free of choosing the best suitable margins to model asset returns, and not constrained by the distribution assumption as in the traditional Monte Carlo method; second, in terms of simulation, this method is easier to implement than the pure copula method which involves intractable computations.

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