Iterative Decoding and Equalization for 2-D Recording Channels

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***Abstract*—We study iterative decoding and equalization for information storage systems that have two-dimensional (2-D) intersymbol interference (ISI) during read-back. Two iterative schemes for 2-D equalization are introduced and evaluated. The first is based on minimum mean squared error (MMSE) estimation and the second is based on message passing on the combined graph of the ISI and the error correction code. Low-density parity-check codes are used for error correction. For the form of the ISI considered in our simulations the best performance is achieved by using the iterative decoding and MMSE equalization method.**

***Index Terms*—Equalization, intersymbol interference, iterative decoding, message passing.**

I. INTRODUCTION

AS CONVENTIONAL magnetic recording challenges physical limits, we are motivated to consider alternative data storage techniques. These techniques include the use of alternative magnetic recording techniques, like patterned magnetic media, or totally different media and techniques such as local probes, multitrack, two-dimensional (2-D), or holographic methods. The focus of this paper is iterative decoding and equalization for information storage systems with 2-D intersymbol interference (ISI). ISI is the primary performance degrading factor for magnetic recording channels. For a 2-D recording medium the ISI is also 2-D. In conventional magnetic media, as inter-track spacing is reduced, ISI and inter-track interference taken together give rise to a 2-D ISI model.

Low-density parity-check (LDPC) codes have been shown to have "good" performance on a large class of channels [1]. In particular, application of LDPC codes on magnetic recording channels has been studied extensively [2] and encouraging results have been presented. An LDPC code is described completely by a random, sparse parity check matrix $\mathbf{H}$; a vector is a codeword if and only if it is in the null space of $\mathbf{H}$. An equivalent way to describe LDPC codes is through a bipartite graph, consisting of variable nodes, check nodes, and the edges connecting them. The edges represent how the codeword bits are connected through the parity-check matrix. The degree of a node is the number of edges emanating from it. An ensemble of bipartite graphs can be specified in terms of the block length and degrees of the variable nodes and check nodes. Regular LDPC codes have fixed, although different, variable node and check node degrees, whereas degree distributions for irregular codes can be described in terms of a degree distribution polynomial [3].

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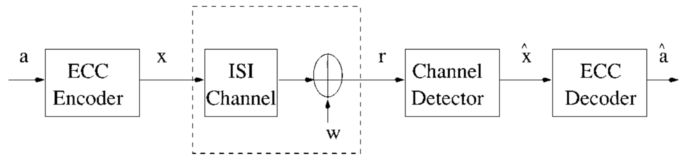

Fig. 1. Discrete-time system.

The random interconnection between the codeword bits on a global scale makes the LDPC codes a good choice for an error correction code (ECC) over a channel where ISI affects bits locally. Further motivation for using LDPC codes is provided by the fact that data in 2-D recording may be arranged in bigger sectors than in one dimension and LDPC codes get close to channel capacity for large enough block lengths.

Equalization is traditionally used to mitigate the effects of ISI on magnetic recording media. Some equalization techniques for one-dimensional (1-D) ISI are discussed by Tüchler *et al.* [4], while others have studied equalization for 2-D recording [5]–[7]. Read and write on 2-D media can be accomplished by the use of a 2-D array of heads. Vettiger *et al.* [8] describe the use of a 2-D array of atomic force microscope tips for reading and writing, which potentially can be modified to magnetic force microscope tips. Multitrack recording [9], [10] provides another way of simultaneously reading or writing on several tracks. The organization of the paper is as follows. Section II describes the channel model being considered and the equalization methods used along with the LDPC codes. Simulation results are provided in Section III and concluding remarks in Section IV.

II. EQUALIZATION METHODS

A. Channel Model

The recording system can be represented by the discrete time system shown in Fig. 1.

Here, $\mathbf{a}$ is the uncoded user data, $\mathbf{x}$ is the encoded data, and $\mathbf{r}$ is the channel output. $\mathbf{x}$ is an $l \times l$ matrix, where l is the square root of the block length, with elements $x(i,j) \in \{\pm 1\}$. The noise $\mathbf{w}$ is assumed to be additive white Gaussian noise (AWGN) with zero mean and variance σ^2. The channel output $\mathbf{r}$ is an $l \times l$ matrix with elements

$$r(i,j) = \sum_{k_1=0}^{L} \sum_{k_2=0}^{L} x(i-k_1, j-k_2)\, h(k_1,k_2) + w(i,j) \quad (1)$$

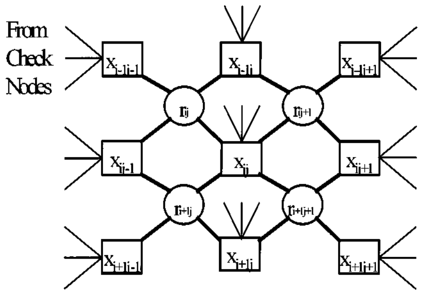

Fig. 2. The full graph includes edges connecting variable nodes (x_{ij}) to the check nodes (not shown) and to the data nodes (r_{ij}). The 2-D ISI yields many length four cycles in this graph for example $r_{ij} \to x_{ij} \to r_{ij+1} \to x_{i-1j} \to r_{ij}$.

where $L+1$ represents the number of elements over which the ISI extends in each dimension and $\mathbf{h}$ is the 2-D channel point spread function. For our simulations, we use

$$h = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{pmatrix}.$$

For error correction, we use LDPC coset codes [11] with the code graph chosen uniformly at random from the ensemble of regular graphs.

B. Equalization Methods

Two equalization methods are studied, the first based on minimum mean-squared error (MMSE) estimation, and the second based on message passing on the ISI and LDPC code graph. For the first scheme, equalization was performed using a Wiener filter which is designed subject to the input power constraint and assuming the input to be Gaussian. 2-D MMSE equalization has been shown to be very effective for detection on 2-D ISI channels [7]. The Wiener filter is applied iteratively if decoding failed after a single application with either soft or hard information being passed from the LDPC decoder to the Wiener filter. The soft information passed is the estimated mean of the codeword and the hard information is the estimated codeword. At each iteration, the Wiener filter operation is

$$\hat{x}_w^{k+1} = \hat{x}_p^k + W^{**}\left(r - h^{**}\hat{x}_p^k\right) \quad (2)$$

where k is the current iteration number, $\hat{x}_{\mathbf{p}}^k$ is the information passed to the Wiener filter by the LDPC decoder at the kth iteration, W is the Wiener filter, $\hat{x}_{\mathbf{w}}^{k+1}$ is the MMSE estimate which is used by the LDPC decoder for initializing the posteriors, and ** represents 2-D convolution.

The second equalization method studied is a pure *a posteriori* probability (APP)-based algorithm that is used for both equalization and decoding. This algorithm computes approximate APPs of the codeword bits given the observations by performing message passing on a three-level graph of the LDPC code and the channel ISI. The APPs would be exact if there were no loops in the graph topology, but as we see in Fig. 2, this is not the case for a 2-D ISI channel.

The upper two levels in the graph represent the LDPC code bipartite graph. The lower two levels represent the channel ISI graph showing how the ISI connects the codeword bits to the observed data. Messages passed on the graph are probabilities calculated using the observed data to approximate the APPs of the codeword bits. The message-passing algorithm is described in [1], [12]. The decoding complexity of the message-passing algorithm, per iteration, is linear in the block length.

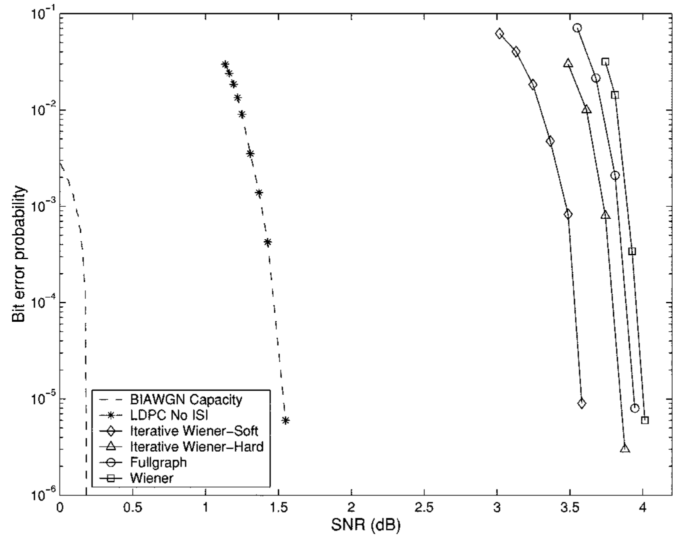

Fig. 3. Performance curves for the proposed schemes.

The APP of the code bits is initialized using the observed data. After initialization the APP algorithm first passes messages on the LDPC bipartite graph for a fixed number of iterations and if decoding fails then message passing is done on the three-level graph again for a fixed number of iterations. The message-passing schedule used on this "full graph" is $\mathbf{x} \to \mathbf{c} \to \mathbf{x} \to \mathbf{r} \to \mathbf{x}$, where $\mathbf{x}$ are the variable nodes, $\mathbf{c}$ are the parity check nodes, and $\mathbf{r}$ are the observed data nodes. While performing message passing for the variable to check and variable to data messages, we also have to consider the messages sent to the variable nodes from the data nodes and check nodes, respectively.

III. Results

A. Signal-to-Noise Ratio (SNR) Definition

For a channel with ISI, the SNR is defined as

$$SNR = 10 \cdot \log\left(\frac{||h||^2 E_b}{2R\sigma^2}\right) \text{dB} \quad (3)$$

where E_b is the energy per bit at the user side, $||h||$ is the $\mathcal{L}_2$ norm of the channel ISI, R is the code rate (the ratio of the number of user bits to the number of code bits), and σ^2 is the noise variance.

B. Simulation Results

The results for the proposed iterative decoding schemes are shown in Fig. 3. The ECC used was a block length 10000, rate 1/2 regular (3,6) LDPC code. The input data $\mathbf{x}$ is obtained by scanning the codeword bits into a 100×100 matrix. Prior to transmission, a guard band of all 1's is added around the codeword matrix, the purpose of which can be to isolate sectors in 2-D and also to provide termination for our algorithms. The Wiener filter as implemented here has infinite support.

Capacity computation for a discrete input 2-D ISI channel is still an open problem. In Fig. 3, we compare the performance of

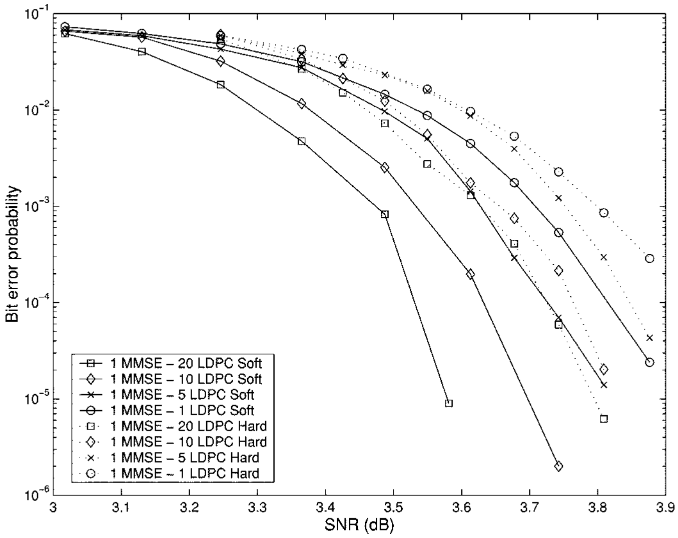

Fig. 4. Performance comparison of different MMSE-LDPC schedules.

our decoding schemes to the performance of the LDPC code on the AWGN channel without ISI. The figure also shows the capacity of a binary input AWGN channel with no ISI. The Wiener filter applied iteratively for ten iterations using soft information from the LDPC decoder has the best performance among the three schemes at about 2 dB from the LDPC code performance at a bit-error rate of 10^{-5}. The full-graph algorithm iterates for 50 iterations on the LDPC bipartite graph, and if decoding fails, then it iterates for ten iterations on the three-level graph. The performance of the full graph algorithm is degraded due to the presence of length four cycles in the channel ISI graph. As noted in the literature [12], the presence of many short cycles, as in this case, deteriorates the performance of the message passing algorithm. The curve labeled "Wiener" in Fig. 3 is the performance when MMSE equalization is done only once. No undetected errors were observed in the simulations.

The plots in Fig. 4 show how the MMSE equalizer performance varies with different message passing schedules between the equalizer and LDPC decoder. At the first iteration, an MMSE equalization is done on the observed data and the filtered data is passed on to the LDPC decoder. The LDPC decoder iterates for a fixed number of iterations for each iteration of the equalizer and then passes information, soft or hard, to the equalizer. This is continued until the decoder converges to a codeword or a preset number of maximum iterations of the equalizer are exhausted, ten for our simulations.

Irregular LDPC codes, when optimized, perform better than regular LDPC codes, as shown in [3], so we expect better performance on a 2-D ISI channel also. For the block lengths being considered here, however, the improvement is not significant. The reason for this is that irregular LDPC codes are optimized assuming a large block length, so significant gain in performance over regular codes is observed only when the block length is large [3].

IV. Conclusion

Iterative decoding and equalization schemes for information storage systems with 2-D ISI were introduced and evaluated. The Wiener filter applied iteratively with soft information passed to the LDPC code decoder gives the best performance. An algorithm that does message passing on the three-level graph of the LDPC code and channel ISI is introduced. Its performance is degraded due to the presence of short cycles in the channel ISI graph. Other message-passing schedules are being considered to pass messages on the full graph in order to avoid short cycles, which would enhance the performance significantly.

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