Distributed Differential Space-Time Coding for Broadband Cooperative Networks

Sami (Hakam) Muhaiedat∗, Paul Ho∗ and Murat Uysal∗∗

∗ School of Engineering Science, Simon Fraser University, Burnaby, BC, Canada, V5A 1S6
E-mail: muhaiedat@ieee.org, paul@cs.sfu.ca

∗∗Department of Electrical and Computer Engr., University of Waterloo, Waterloo, Canada, N2L 3G1
E-mail: muysal@ece.uwaterloo.ca

Abstract—In this paper, we investigate distributed differential space-time block coding (STBC) for cooperative communications over frequency-selective fading channels. We carefully exploit the unitary structure of the orthogonal STBC to design a low complexity differential STBC receiver for multi-carrier broadband cooperative networks. We consider amplify-and-forward relaying and assume a single-relay scenario. Under the assumption of perfect power control for the relay terminal and high signal-to-noise ratio for the underlying links, our performance analysis demonstrates that the considered scheme is able to exploit fully the spatial diversity. We further present Monte Carlo simulation results to confirm our analytical observations.

I. INTRODUCTION

There has been a growing demand for high data rate services for wireless multimedia and internet services. Spatial diversity offers significant improvement in link reliability and spectral efficiency through the use of multiple antennas at the transmitter and/or receiver side [1]–[4]. Recently, cooperative communications have gained much attention due to the ability to explore the inherent spatial diversity in relay channels [5]–[9]. The idea behind cooperative diversity is that in a wireless environment, the signal transmitted by the source nodes is overheard by other nodes, which are also known as partners. The source and its partners jointly process and transmit their information, creating a "virtual antenna array" although each of them is equipped with only one antenna.

Most the current works on cooperative diversity consider coherent detection and assume the availability of perfect channel state information (CSI) at the receiver. In fading channels where the coherence time is large enough, the channel estimation can be carried out through the use of pilot symbols [10]. For fast fading channels where the phase carrier recovery is more difficult, differential detection provides a more practical solution. In [10]–[14], differential detection has been investigated for cooperative transmission scenarios. The works in [10]–[14] assume an idealized transmission environment with an underlying frequency-flat fading channel. This assumption can be justified for narrow-band cooperative scenarios with fixed infrastructure; however, it is impractical if wideband cooperative networks are considered. Motivated by this practical concern, in this paper, we investigate the applicability of differential STBC for broadband cooperative transmission over frequency-selective channels.

Related work and contributions: Although there have been considerable research efforts on differential STBC (conventional and distributed) for frequency flat fading channels (see for example [10]–[20]), only a few isolated results have been reported on conventional differential STBC for frequency-selective channels [18]–[20] and not yet any published in the context of cooperative transmission. In this paper, we aim to fill this research gap and investigate distributed differential STBC multi-carrier transmission for broadband cooperative networks. Our contributions in this work are summarized as follows:

• We propose a distributed differential OFDM (DD-OFDM) STBC scheme for broadband cooperative systems with amplify-and-forward (AaF) relaying. The proposed scheme can be considered an extension of differential OFDM-STBC scheme proposed in [18] for point-to-point links. Carefully exploiting the underlying orthogonality of distributed STBC, the proposed scheme is able to preserve low-decoding complexity.

• Our performance analysis through the derivation of pairwise error probability (PEP) reveals that the proposed scheme is able to exploit fully the available spatial diversity.

• We present a comprehensive Monte Carlo simulation study to confirm the analytical observations and give insight into system performance.

We also note that outer coding with frequency interleaving can be further combined with OFDM to extract the multipath diversity available in the considered cooperative scenario to improve the performance. Extension to coded DD-OFDM is straightforward, but, due to space limitations, is not pursued in this work.

The rest of the paper is organized as follows: In Section II, the transmission model is introduced. The differential scheme under consideration for distributed STBC is described in Sections III, followed by the corresponding diversity gain analysis in Section IV. Numerical results are presented in Section V and the paper is concluded in Section VI.

Notation: (·), (·)T, and (·)H denote conjugate, transpose, and conjugate transpose (i.e. Hermitian) operations, respectively. E[·] denotes expectation, [·]k,l denotes the (k, l)th entry
of a matrix, \([\cdot]_k\) denotes the \(k^{th}\) entry of a vector and \(\|\|\) denotes the Euclidean norm of a vector, \(I_N\) denotes the identity matrix of size \(N\). \(Q\) represents the \(N \times N\) FFT matrix whose \((l,k)\) element is given by \(Q(l,k) = (1/\sqrt{N}) \exp(-j2\pi lk/N)\) where \(0 \leq l,k \leq N-1\). Bold upper-case letters denote matrices and bold lower-case letters denote vectors.

II. SIGNAL MODEL

A wireless communication scenario is considered where the source terminal \(S\) transmits information to the destination terminal \(D\) with the assistance of a relay terminal \(R\) (See Fig.1). The transmitted data is organized into frames, with each frame comprising of two OFDM blocks. The size of each OFDM block is \(N+l\) samples, where \(N\) is the number of information symbols and \(l\) is the size of the cyclic prefix (CP). The channel impulse responses (CIR) in the \(S \rightarrow R\), \(S \rightarrow D\) and \(R \rightarrow D\) links are, respectively, given by the random vectors \(h_{SR} = [h_{SR}[0],...,h_{SR}[LSR]]^T\), \(h_{SD} = [h_{SD}[0],...,h_{SD}[LSD]]^T\), and \(h_{RD} = [h_{RD}[0],...,h_{RD}[LRD]]^T\), where \(LSR\), \(LSD\), and \(L_{RD}\) denote the corresponding channel memory lengths, and \(l = \max(L_{SR}+L_{RD},L_{SD})\) is the size of the cyclic prefix. The entries of \(h_{SR}\), \(h_{SD}\), and \(h_{RD}\) are assumed to be independent zero-mean complex Gaussian with variances \(1/(LSR+1)\), \(1/(LSD+1)\), and \(1/(L_{RD}+1)\) respectively. We further assume that the channel coherent time is substantially greater than four frames (i.e., eight OFDM blocks) which can be easily justified for a quasi-static fading channel. This is a requirement to avoid the irreducible error floor in the differential space-time detector at the destination terminal as will be further discussed in the next section. As in any OFDM system, the receiver in the destination terminal first removes the cyclic prefix before performing FFT demodulation. The operation eliminates interblock interference (IBI) induced by the FIR channel and make the channel matrices circulant [21].

We assume the user cooperation protocol proposed by Nahar et al. [9]. Specifically, the source terminal communicates with the relay terminal during the first signaling interval of each frame. There is no transmission from source-to-destination within this period. In the second signaling interval, both the relay and source terminals communicate with the destination terminal. For the link, AF relaying is used, in which the relay terminal amplifies and re-transmits the signal received from the source terminal in the first signaling interval. All terminals are equipped with single transmit and receive antennas.

Let \(y_{SRD}\) represent the data vector being transmitted to the relay terminal during the first block of each frame. This vector is of size \(N \times 1\) and all its entries are complex MPSK symbols generated through differential space-time (ST) encoding. Before actual transmission, the vector \(y_{SRD}\) is first precoded to the discrete-time signal \(Q^Hy_{SRD}\), where \(Q^H\) is the matrix representation of the inverse FFT (IFFT). A cyclic prefix which consists of the last \(l\) symbols of \(Q^Hy_{SRD}\) itself to form the transmitted OFDM block. The corresponding received signal at the relay terminal, after CP removal, is

\[
r_R = \sqrt{E_{SR}}H_{SR}Q^Hy_{SRD} + n_R.
\]

where \(H_{SR}\) is an \(N \times N\) circulant matrix with entries \([H_{SR}]_{k,l} = h_{SR}(k-l) \mod N\), and \(E_{SR}\) is the average energy available at the relay terminal considering the path loss and possible shadowing effects in the \(S \rightarrow R\) link. \(n_R\) is the additive white Gaussian noise vector with each entry having zero-mean and variance of \(N_0/2\) per dimension.

During the second transmission block of each frame, both the relay and the source transmit to the destination terminal. At the relay, the received signal is first normalized as \(\tilde{r}_R = r_R/\sqrt{E_{SR} + N_0}\). A CP of length \(l\) samples is then added to \(\tilde{r}_R\) to form the actual transmitted signal over the \(R \rightarrow D\) link. The direct transmission from the source \(S\) to the destination terminal \(D\), on the other hand, is obtained by adding a proper cyclic prefix to the pre-coded data vector \(Q^Hy_{SD}\), where \(y_{SD}\) represents in general the \(N \times 1\) data vector that \(S\) communicates to \(D\) during in the second transmission interval of each frame. The signal arriving at \(D\), after CP removal, is

\[
z = \sqrt{E_{RD}}H_{RD}\tilde{r}_R + \sqrt{E_{SD}}H_{SD}Q^Hy_{SD} + n_D
\]

where \(n_D\) is an additive white Gaussian noise vector with each entry having zero-mean and variance of \(N_0/2\) per dimension; \(H_{SD}\) and \(H_{RD}\) are \(N \times N\) circulant matrices with entries, \([H_{SD}]_{k,l} = h_{SD}(k-l) \mod N\) and \([H_{RD}]_{k,l} = h_{RD}(k-l) \mod N\). Here, \(E_{SD}\) and \(E_{RD}\) are the average energies available at the destination terminal which take into account possibly different path loss and shadowing effects between the \(S \rightarrow D\) and \(R \rightarrow D\) links, respectively. Combining (1) and (2), we obtain

\[
z = \sqrt{E_{RD}}E_{SR}H_{RD}H_{SR}Q^Hy_{SRD} + \sqrt{E_{SD}}H_{SD}Q^Hy_{SD} + \tilde{n}
\]

where

\[
\tilde{n} = \sqrt{E_{RD}}H_{RD}n_R + n_D
\]

is a composite noise term. Each entry of \(\tilde{n}\) (conditioned on \(h_{RD}\)) has zero mean and a variance of

\[
E[|\tilde{n}_m|^2 | h_{RD}] = N_0 \left(1 + \frac{E_{RD}}{E_{SR} + N_0} \sum_{m=0}^{L_{RD}} |h_{RD}(m)|^2\right)
\]

\[
= N_0(1 + \alpha)\]

Fig. 1. Schematic representation of relay-assisted transmission.
\[ r(2t-1) = \sqrt{\gamma} H_{RD} H_{SR} Q^I y_{SRD}(2t-1) + \sqrt{\gamma_2} H_{SD} Q^I y_{SD}(2t-1) + v(2t-1), \]
\[ r(2t) = -\sqrt{\gamma} H_{RD} H_{SR} Q^I y_{SRD}(2t) + \sqrt{\gamma_2} H_{SD} Q^I y_{SD}(2t) + v(2t), \]

(13)

\[ \mathbf{R}(2t-1) = \sqrt{\gamma} \mathbf{A}_{SR} \mathbf{A}_{RD} \mathbf{y}_{SRD}(2t-1) + \sqrt{\gamma_2} \mathbf{A}_{SD} \mathbf{y}_{SD}(2t-1) + \mathbf{V}(2t-1), \]
\[ \mathbf{R}(2t) = -\sqrt{\gamma} \mathbf{A}_{SR} \mathbf{A}_{RD} \mathbf{y}_{SRD}(2t) + \sqrt{\gamma_2} \mathbf{A}_{SD} \mathbf{y}_{SD}(2t) + \mathbf{V}(2t), \]

(14)

where \( \alpha = E_{RD}/(E_{SR} + N_0) \sum_{m=0}^{L_{RD}} |h_{RD}(m)|^2 \). It should be pointed out that, conditioned on \( h_{RD} \), the different entries of the composite noise vector are correlated. However, as far as the average bit-error-probability (BEP) of the entire system is concerned, we can treat them as independent variables, as only the burst error characteristics of the system will be affected by these correlations.

For convenience of the later analytical derivations as discussed in [9], we normalize the received vector in (3) by the factor \( 1/\sqrt{1+\alpha} \). This yields

\[ r = \sqrt{\gamma} H_{RD} H_{SR} Q^I y_{SRD} + \sqrt{\gamma_2} H_{SD} Q^I y_{SD} + v, \]

(6)

The entries of the noise component \( v \), in the resultant vector become zero mean complex Gaussian random variables with variance of \( N_0/2 \) per dimension. The terms \( \gamma_1 \) and \( \gamma_2 \) in (6) are defined as

\[ \gamma_1 = \frac{(E_{SR}/N_0) E_{RD}}{1 + E_{SR}/N_0 + (E_{RD}/N_0) \sum_{m=0}^{L_{RD}} |h_{RD}(m)|^2} \]

and

\[ \gamma_2 = \frac{(1 + E_{SR}/N_0) E_{SD}}{1 + E_{SR}/N_0 + (E_{RD}/N_0) \sum_{m=0}^{L_{RD}} |h_{RD}(m)|^2} \]

For large \( L_{RD} \), the term \( \sum_{m=0}^{L_{RD}} |h_{RD}(m)|^2 \) in the above equations approaches \( L_{RD} + 1 \) with a high probability. Consequently, we can rewrite \( \gamma_1 \) and \( \gamma_2 \) as

\[ \gamma_1 \approx \frac{(E_{SR}/N_0) E_{RD}}{1 + E_{SR}/N_0 + (L_{RD} + 1) E_{RD}/N_0}, \]

(7)

\[ \gamma_2 \approx \frac{(1 + E_{SR}/N_0) E_{SD}}{1 + E_{SR}/N_0 + (L_{RD} + 1) E_{RD}/N_0} \]

(8)

In summary, the vector \( r \) in (6) represents the time-domain observation available to the destination terminal \( D \) at the end of each frame. For a \( 2 \times 2 \) Alamouti scheme [4], \( r \) is analogous to the composite scalar received over one time slot, and the transmission of \( y_{SRD} \) and \( y_{SD} \) through the \( S \rightarrow R \rightarrow D \) and the \( S \rightarrow D \) links is equivalent to transmitting two MPSK symbols using two transmit antennas. In the following, we will discuss in detail the differential ST encoding and decoding procedures employed in the distributed differential OFDM-STBC system.

III. DISTRIBUTED DIFFERENTIAL (DD) ENCODING/DECODING

Since we assume a single relay scenario, the information data symbols are parsed to two streams of \( N \times 1 \) blocks \( x_i(t) = [x_i^1(t), ..., x_i^{N-1}(t)]^T, i = 1, 2 \), where \( t \) is the time-index for the OFDM-STBC symbols, and the individual \( x_i^n(t) \)'s are complex symbols drawn from an unit-energy \( M \)-ary PSK (MPSK) constellation. It should be pointed out that one OFDM-STBC symbol spans over two frames.

A. DIFFERENTIAL ENCODING FOR DD-OFDM STBC

The data vector \( x_1(t) \) and \( x_2(t) \) are differentially encoded into the (frequency domain) OFDM vectors \( y_{SRD}(k) = [y_{SRD}^0(k), ..., y_{SRD}^{N-1}(k)]^T \) and \( y_{SD}(k) = [y_{SD}^0(k), ..., y_{SD}^{N-1}(k)]^T, k = 2t - 1, 2t \) according to

\[ Y^n(t) = X^n(t)Y^n(t-1), \]

(9)

where

\[ X^n(t) = \frac{1}{\sqrt{2}} \left[ \begin{array}{c} x_1^n(t) - (x_2^n(t))^* \\ (x_1^n(t))^* \end{array} \right], \]

(10)

and

\[ Y^n(t) = \left[ \begin{array}{c} y_{SRD}^0(2t-1) \\ y_{SRD}^0(2t) \\ y_{SD}^0(2t-1) \\ y_{SD}^0(2t) \end{array} \right]. \]

(11)

Here, we set \( Y^n(0); n = 0, 1, ..., N-1, to \( I_2 \). Consequently, all the \( Y^n(t) \)'s are unitary matrices, with the properties

\[ y_{SRD}^0(2t) = -(y_{SD}^0(2t-1))^*, \]

\[ y_{SD}^0(2t) = (y_{SRD}^0(2t-1))^* \]

(12)

This means at the sequence level, the OFDM blocks in frames \( 2t-1 \) and \( 2t \) have the relationships \( y_{SRD}(2t) = -y_{SD}(2t-1) \) and \( y_{SD}(2t) = y_{SRD}(2t-1) \). In substituting the signals in (9) into (6), we obtain (13) on top of this page which represents the received signals at the destination terminal over two consecutive frames.
B. DEMODULATION OF DD-OFDM STBC

To recover the data vectors $\mathbf{x}_k(t)$ and $\mathbf{x}_d(t)$ from the received signals in (13), we first perform a FFT on each of these signals. This is equivalent to multiplying $r(2t-1)$ and $r(2t)$ by the matrix $Q$ defined earlier. The resultant signal is given (14) on top of the previous page. In (14), $R(k) = Q \cdot r(k), V(k) = Q \cdot v(k), k = 2t-1, 2t$; and $\Lambda_{SR}, \Lambda_{RD},$ and $\Lambda_{SD}$ are diagonal matrices containing the $N$-point Discrete Fourier Transforms (DFT)\(^1\) of the channel impulse responses $h_{SR}, h_{RD}, h_{SD}$ respectively. The frequency response matrices $\Lambda_{SR}, \Lambda_{RD},$ and $\Lambda_{SD}$, were obtained based on the observation that any circulant matrix $H$ constructed from the channel impulse response $h$ can be written in the form $H = Q^H \Lambda Q$, where $\Lambda$ is a diagonal matrix whose $n^{th}$ element is equal to the DFT coefficient of $h$. Note that the noise terms, $\mathbf{V}(2t-1)$ and $\mathbf{V}(2t)$, are complex Gaussian noise and they all have zero-mean and variance of $N_0/2$ per-dimension.

It is evident from the structure of (14) that the FFT/IFFT operation has converted each of $h_{SR}, h_{RD}, h_{SD}$ from a frequency selective channel into independent parallel flat fading sub-channels. Considering the $n^{th}$ sub-channel, we can first define $\hat{\mathbf{Y}}^n(t) = [R^n(2t-1), R^n(2t)]^T$ and $\mathbf{W}^n(t) = [\mathbf{V}^n(2t-1), \mathbf{V}^n(2t)]^T$, where $R^n(2t-1), R^n(2t), V^n(2t-1), V^n(2t)$ are the $n^{th}$ components of $R(2t-1), R(2t), V(2t-1), V(2t)$ respectively. Then it can be easily shown that $\hat{\mathbf{Y}}^n(t)$ is related to the differentially encoded ST symbol $\mathbf{Y}^n(t)$ in (11) according to

$$\hat{\mathbf{Y}}^n(t) = \mathbf{Y}^n(t) \mathbf{F}^n + \mathbf{W}^n(t)$$

where

$$\mathbf{F}^n = [\sqrt{\Lambda_{SR}} A_{RD}^n \sqrt{\Lambda_{SD}}^T]^T$$

with $\Lambda_{SR}^n A_{RD}^n$ and $\Lambda_{SD}$ being the frequency responses of the $S \rightarrow R$, $R \rightarrow D$ and $S \rightarrow D$ links at the $n^{th}$ subcarrier. For notation brevity, we will drop the subcarrier index $n$ in all relevant parameters in the following discussion.

Because of differential encoding (9), the current input to the ST differential detector, $\hat{\mathbf{Y}}(t)$, for each sub-channel is related to the previous input, $\hat{\mathbf{Y}}(t-1)$, according to

$$\hat{\mathbf{Y}}(t) = \mathbf{X}(t) \hat{\mathbf{Y}}(t-1) + \mathbf{U}(t)$$

where

$$\mathbf{U}(t) = \mathbf{W}(t) - \mathbf{X}(t) \mathbf{W}(t-1)$$

is an effective noise vector whose entries, $U_1(t)$ and $U_2(t)$, are zero mean complex Gaussian random variables with variance $N_0$ per-dimension. Strictly speaking, $\mathbf{U}(t)$ and $\hat{\mathbf{Y}}(t)$ are correlated. However at sufficiently large SNR values, this correlation can be safely ignored. Consequently, we simply treat $\mathbf{U}(t)$ as an independent noise term and derive an approximate maximum likelihood (ML) decoder for $\mathbf{X}(t)$ based on the signal structure in (17). Note that (17) is independent of the channel vector $\mathbf{F}$.

Because of its orthogonal structure, the detection of $\mathbf{X}(t)$ from $\hat{\mathbf{Y}}(t)$ enjoys a linear decoding complexity, i.e., the search for the most likely transmitted symbol is equivalent to independent search for the most likely constituent scalar symbols. To demonstrate this point, we first re-write (17) in the following form

$$\sqrt{\left| R(2t-1) \right|^2 + \left| R(2t-3) \right|^2} \begin{bmatrix} R(2t-1) & R(2t-3) & R(2t-2) & R^*(2t-3) \\ R^*(2t-1) & R^*(2t-2) & -R^*(2t-3) & -R(2t-3) \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ U_1(t) \\ U_2(t) \end{bmatrix}$$

Multiplying both sides by

$$\frac{1}{\sqrt{\left| R(2t-1) \right|^2 + \left| R(2t-3) \right|^2}}$$

yields the decision statistics

$$\begin{bmatrix} \rho_1(t) \\ \rho_2(t) \end{bmatrix} = \begin{bmatrix} R(2t-1) & R(2t-3) & R(2t-2) & R^*(2t-3) \\ R^*(2t-1) & R^*(2t-2) & -R^*(2t-3) & -R(2t-3) \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ U_1(t) \\ U_2(t) \end{bmatrix}$$

where the newly formed noise terms, $\rho_1$ and $\rho_2$, are Gaussian with zero-mean and variance of $N_0$ per dimension. It is evident from (19) that the data streams $x_i(t), i = 1, 2$, are decoupled. Consequently, they can be detected independently using the decision rule given in (20) on top of this page. In (20), $x_k = e^{j2\pi k/M}, k = 0, 1, ..., M-1$, are the complex symbols from the MPSK signal constellation.

From the above discussion, it should be clear that in order to perform DD-OFDM STBC decoding, the receiver needs to process 4 frames, or equivalently, 8 OFDM blocks at a time. In other words, for this DD-OFDM STBC scheme to work properly, the channel coherent time should be greater than 8 OFDM blocks. Once the decisions in all the sub-channels are made, the detection window is shifted by 2 frames and the operations are repeated for the next set of symbols.

IV. DIVERSITY GAIN ANALYSIS

In this section, we investigate the diversity gain for DD-OFDM STBC scheme through the derivation of pairwise
error probability (PEP) expression. We define the transmitted codeword as \( x(t) = [x_1(t), x_2(t)]^T \) and the corresponding decisions from (20), i.e. \( \hat{x}(t) = [\hat{x}_1(t), \hat{x}_2(t)]^T \), the decoded codeword. For any \( \hat{x}(t) \neq x(t) \), the probability that \( x(t) \) is decoded into \( \hat{x}(t) \), under a set of two codewords, is known as the PEP. Conditioned on the observation \( \hat{Y}(t-1) = [R(2t-3), R(2t-2)]^T \), the PEP is given by (21) on top of this page, where

\[
\|\hat{Y}(t-1)\|^2 = \hat{Y}(t-1)^H \hat{Y}(t-1) = \sqrt{|R(2t-3)|^2 + |R(2t-2)|^2},
\]

is the squared Euclidean distance between \( x(t) \) and \( \hat{x}(t) \), and \( Q(.) \) is the Gaussian-Q function. Applying the standard Chernoff bound to (21), we obtain (24).

Using (15) and (16), and ignoring the noise term in \( W(t-1) \) due to the high SNR assumption, we can express \( \|\hat{Y}(t-1)\|^2 \) as

\[
\|\hat{Y}(t-1)\|^2 = \mathbf{F}^H \mathbf{F} = \gamma_1 \Lambda_{SR}^2 \Lambda_{RD}^2 + \gamma_2 \Lambda_{SD}^2
\]

Substituting (25) into (24), dropping the index \( t \), and following similar steps detailed in [21] to average over \( \Lambda_{SR}^2, \Lambda_{RD}^2 \), and \( \Lambda_{SD}^2 \), we obtain the PEP bound

\[
P(x, \hat{x}) \leq \left( \frac{d^2(x, \hat{x})}{16N_0} \right)^{-1} \left( 1 + \frac{d^2(x, \hat{x})}{16N_0} \right)^{-1} \times \Gamma \left( 0, \frac{1}{d^2(x, \hat{x})E_{SD}/16N_0} \right)
\]

where \( \Gamma(., .) \) denotes the incomplete gamma function [22]. To have further insight into maximum achievable diversity order, we assume perfect power control where \( S \to D \) and \( R \to D \) links are balanced and high SNRs for all underlying links, i.e. \( E_{SD}/N_0 = E_{RD}/N_0 >> 1 \). We also assume that SNR in \( S \to R \) is large enough, i.e. \( E_{SR}/N_0 > E_{SD}/N_0 \). Under these assumptions, we have \( \gamma_1 = \gamma_2 = E_{SD} \), simplifying (26) to

\[
P(x, \hat{x}) \leq \left( \frac{E_{SD}}{16N_0} d^2(x, \hat{x}) \right)^{-1} \times \Gamma \left( 0, \frac{1}{d^2(x, \hat{x})E_{SD}/16N_0} \right)
\]

It is seen from (27) that DD-OFDM STBC is able to extract the full spatial diversity order which is equal to two for the single-relay scenario under consideration. If we further consider the limiting case of \( E_{SD}/N_0 \to \infty \) (i.e. the previous assumption of \( E_{SR}/N_0 > E_{SD}/N_0 \) is therefore no longer valid), (26) takes the following form

\[
P(x, \hat{x}) \leq \left( \frac{1}{L_{RD}+1} \right)^{-1} \left( \frac{E_{SR}}{16N_0} d^2(x, \hat{x}) \right)^{-1} \times \Gamma \left( 0, \frac{1}{L_{RD}+1} \frac{1}{d^2(x, \hat{x})E_{SR}/16N_0} \right)
\]

It is observed that the performance becomes independent of \( E_{SD}/N_0 \) and is now governed by \( E_{SR}/N_0 \).

V. SIMULATION RESULTS AND DISCUSSION

In this section, we present Monte-Carlo simulation results for DD OFDM-STBC system under consideration. We assume 4-PSK modulation, and \( N=64 \) subcarriers. We model \( S \to R \), \( S \to D \) and \( R \to D \) links as frequency-selective channels with
memory lengths $L_{SR} = L_{SD} = L_{RD} = 1$ and a uniform delay power profile. We assume $S \rightarrow D$ and $R \rightarrow D$ links are balanced, i.e., $E_{SR}/N_0 = E_{RD}/N_0$. The power in $S \rightarrow R$ is assumed to be adjusted according to $E_{SR}/N_0 = \alpha E_{SD}/N_0$ where $\alpha > 0$. This would give insight into various power allocation scenarios.

Fig. 2. depicts the symbol error rate (SER) performance of the DD-OFDM STBC scheme assuming $\alpha = 10$, i.e., $E_{SR}/N_0 > E_{SD}/N_0$. As a benchmark, we include the performance of “genie” receiver which assumes perfect CSI. We also include the performance of non-cooperative direct transmission with differential encoding/decoding. Our simulation results indicate that the performance loss with respect to the genie receiver is 3dB at BER= $10^{-3}$. On the other hand, the performance improvement of our scheme with respect to the non-cooperative direct transmission is 5dB at BER= $10^{-3}$. Furthermore, it can be readily seen that the proposed differential scheme is able to extract a diversity order of two confirming our earlier observations through the derived PEP.

Fig. 3. illustrates the performance of DD-OFDM STBC scheme for $\alpha = 0.1, 1, 10$. The case of $\alpha = 1$ indicates that $S \rightarrow R$ and $S \rightarrow D$ links are balanced. It is observed from Fig.3 that the performance of the proposed scheme degrades for $\alpha = 0.1$ when $E_{SD}/N_0 > E_{SR}/N_0$, indicating that the performance is now governed by $E_{SR}/N_0$. It should be noted however that the diversity order is still preserved, confirming our observation from (28).

VI. CONCLUSION

We have investigated distributed differential OFDM STBC for cooperative communications over frequency-selective fading channels. We have carefully exploited the unitary structure of STBCs to design a low complexity distributed differential STBC receiver for broadband cooperative networks. Under the assumption of perfect power control where $S \rightarrow D$ and $R \rightarrow D$ links are balanced and high SNRs for all underlying links, we have demonstrated that the considered scheme is able to fully exploit the underlying spatial diversity.

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