Inhomogeneous reionization and the polarization of the cosmic microwave background

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ABSTRACT

In a universe with inhomogeneous reionization, the ionized patches create a second order signal in the cosmic microwave background polarization anisotropy. This signal originates in the coupling of the free electron fluctuation to the quadruple moment of the temperature anisotropy. We examine the contribution from a simple inhomogeneous reionization model and find that the signal from such a process is below the detectable limits of the Planck Surveyor mission. However the signal is above the fundamental uncertainty limit from cosmic variance, so that a future detection with a high accuracy experiment on sub-arcminute scales is possible.

Subject headings: cosmic microwave background – cosmology:theory

1. Introduction

The Microwave Anisotropy Probe and the Planck Surveyor (MAP; Planck Surveyor; Bersanelli et al. 1996) will provide a precise measurement of the cosmic microwave background (CMB) temperature anisotropy. While existing measurements of the polarization of the CMB radiation only give crude upper limits to its anisotropy (Wollack et al. 1993; Partridge et al. 1997), forthcoming experiments will be more sensitive to the power in the polarization (Staggs, Gundersen & Church 1999). MAP is expected to make only a statistical detection of the polarization anisotropy, while Planck will finally measure the polarization power spectrum to high accuracy with an average pixel sensitivity for the polarization fluctuation of around $\Delta T/T = 5 \times 10^{-6}$, full sky coverage and an integration time of approximately one year (Planck Surveyor). To extract the anisotropy power spectrum from the measured polarization, it is essential to understand the polarization of the foregrounds. This will be one of the main tasks of the near future. Since Planck will measure the power spectrum up to arcminute scales (Planck Surveyor), second order effects in the fluctuation of the CMB radiation could become important. The study of second order effects is well established for the temperature anisotropies (Sunyaev & Zel'dovich 1970, 1980; Kaiser 1984; Ostriker & Vishniac

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1986; Vishniac 1987; Dodelson & Jubas 1995; Aghanim et al. 1996; Gruzinov & Hu 1998; Peebles & Juskiewicz 1998; Knox, Scoccimarro & Dodelson 1998; Haiman & Knox 1999), however for the polarization power spectrum these effects have not been examined extensively in the past (Sunyaev & Zel'dovich 1980) and are only the subject of very recent investigations (Seshradi & Subramanian 1998; Hu 1999). It is well known that the universe is ionized at least out to redshifts of $z \approx 5$ (Gunn & Peterson 1965) and any *realistic* model of how this reionization might have taken place is thought to be inhomogeneous. The hot ionized gas interacts with the CMB photons and results in features in the anisotropy power spectrum. In this *Letter* we will focus on the second order effects from inhomogeneous reionization. The first order effect of reionization is an enhanced polarization anisotropy on large scales and a suppression on small scales (Zaldarriaga 1997). The second order effect due to homogeneous reionization is a Ostriker-Vishniac-type effect for the polarization, where the fluctuation in the free electron density follows the linear density variations of the overall matter (Seshradi & Subramanian 1998). The second order contribution we are going to discuss in this *Letter* is very similar to this effect, only that the source of the fluctuation in the free electron density is different. In the following section we will present the model of inhomogeneous reionization and derive the polarization anisotropy in the CMB which is caused by these fluctuations.

2. Second order polarization anisotropy from inhomogeneous reionization

The source of reionization is thought to be the UV radiation of early objects like quasars and proto-galaxies as hosts of an early generation of stars. The nuclear and gravitational energy of these objects is transformed into radiation which subsequently ionizes the hydrogen in spheres which surround them (Tegmark, Silk & Blanchard 1994; Rees 1996; Aghanim et al. 1996; Haiman & Loeb 1997, 1998; Loeb 1997; Silk & Rees 1998; Haiman & Knox 1999). Another way to study the consequences of inhomogeneous reionization is to use *effective* models which describe the distribution of ionized regions by a small number of free parameters (Gruzinov & Hu 1998; Knox, Scoccimarro & Dodelson 1998; Haiman & Knox 1999). We adopt the model by Gruzinov & Hu (1998), which describes inhomogeneous reionization as a set of uncorrelated patches of a certain fixed size R. The number density of these patches grows with time and finally the whole universe becomes reionized in a homogeneous way. More realistic models include varying patch sizes Aghanim et al. (1996) and contain correlations of the ionized regions. It turns out that these correlations lead to a somewhat different signal on smallest scales for the temperature anisotropies (Knox, Scoccimarro & Dodelson 1998). However, the naive uncorrelated model by Gruzinov & Hu (1998) gives a good estimate of the effect on the CMB (Haiman & Knox 1999). Below we will discuss the differences we expect for the polarization anisotropy for a model with correlated patches.

We define the ionization fraction $x_{\rm e}$ to be the ratio of the number density of free electrons $n_{\rm e}$ and the overall (free and bound) number density of electrons n, i.e. $x_{\rm e} = n_{\rm e}/n$. Since we want to study the effects of inhomogeneous reionization on the CMB to second order, we not only need the mean background ionization fraction $\bar{x}_{\rm e}$, we also have to know the variance of the distribution.

Gruzinov & Hu (1998) give the second moment to be

$$\langle x_{\rm e}(\eta_1,\mathbf{x}_1)x_{\rm e}(\eta_2,\mathbf{x}_2)\rangle = \bar{x}_{\rm e}(\eta_{\rm min})\bar{x}_{\rm e}(\eta_{\rm max}) + \bar{x}_{\rm e}(\eta_{\rm min})\left[1-\bar{x}_{\rm e}(\eta_{\rm max})\right]{\rm e}^{-\frac{(\mathbf{x}_1-\mathbf{x}_2)^2}{2R^2}}\,, \tag{1}$$

where $\eta \equiv \int dt/a$ is the conformal time, with a the scale factor and the subscripts min and max refer to the minimum, respectively maximum of η_1 and η_2. The correlation (1) drops off exponentially if the distance between two points is larger than the size of a patch R. If η_1 or η_2 is in one of the homogeneous regimes (nearly neutral or complete ionization) the correlation is just the product of the mean values. In the following we will describe the inhomogeneity as the fluctuation of the free electron number density $\delta_{\rm e}(\eta,\mathbf{x}) = (n_{\rm e}(\eta,\mathbf{x}) - \bar{n}_{\rm e}(\eta))/\bar{n}_{\rm e}(\eta)$. We study a universe with a 5% baryon content, a Hubble constant of $H_0 = 100h\,$km/sec/Mpc, with $h = 0.5$, critical matter density and no cosmological constant, but the results are easy to generalize to an open or Λ universe. The matter fluctuations are taken to be adiabatic with an initial spectral index of $n = 1.0$.
We are not including tensor perturbations and therefore the magnetic component of the polarization fluctuation is zero (Zaldarriaga & Seljak 1997). It is then sufficient to study the perturbations in the Stokes parameter Q. If we assume the wave is traveling into the z direction with E_x and E_y the amplitudes of the electric field in x and y direction respectively, this parameter is given by $\mathrm{Q} = \langle E_x^2\rangle - \langle E_y^2\rangle$. The Boltzmann equation for the fluctuations in Q is (Bond & Efstathiou 1987; Zaldarriaga & Seljak 1997)

$$\dot{\Delta}_{\rm Q} + \gamma_i\partial_i\Delta_{\rm Q} = n_{\rm e}\sigma_{\rm T}a\left(-\Delta_{\rm Q} + \frac{1}{2}\left[1-P_2(\mu)\right]\Pi\right)\,, \tag{2}$$

with γ_i the direction of the photon momentum, $P_2(\mu)$ the second Legendre polynomial, $\Pi = \Delta_{\rm T2} + \Delta_{\rm Q2} + \Delta_{\rm Q0}$ the polarization tensor and the overdot refers to the derivative with respect to conformal time. We expand this equation to second order and use $n_{\rm e}(\eta,\mathbf{x}) = \bar{n}_{\rm e}(\eta)\left[1+\delta_{\rm e}(\eta,\mathbf{x})\right]$. The integral solution of the second order contribution in Fourier space is then

$$\Delta_{\rm Q}^{(2)}(\eta_0,\mathbf{k},\mu) = \frac{3}{4}(1-\mu^2)\int_0^{\eta_0} g(\eta_0,\eta)e^{ik(\eta-\eta_0)\mu}\mathcal{S}(\eta,\mathbf{k})d\eta\,, \tag{3}$$

where $\mu = \cos\theta = \gamma_i k_i/k$ and η_0 is the conformal time today. We have neglected in this solution the couplings of the first order polarization fluctuations to $\delta_{\rm e}$, since the first order temperature quadrupole will dominate these terms (Seshradi & Subramanian 1998). The homogeneous background ionization history is encoded in the visibility function $g(\eta_0,\eta) = \dot{\tau}\exp\{-\tau(\eta_0)+\tau(\eta)\}$ with the differential optical depth $\dot{\tau} = \bar{x}_{\rm e}n\sigma_{\rm T}a$, where $\sigma_{\rm T}$ is the Thomson scattering cross section. The source $\mathcal{S}$ of the second order fluctuation is the mode coupling term between the fluctuation in the free electron density and the *first* order quadrupole fluctuation of the temperature,

$$\mathcal{S}(\eta,\mathbf{k}) = \frac{1}{(2\pi)^{3/2}}\int\delta_{\rm e}(\eta,\mathbf{k}-\mathbf{p})\Delta_{\rm T2}^{(1)}(\eta,\mathbf{p})\,d^3p \approx \frac{1}{(2\pi)^{3/2}}\delta_{\rm e}(\eta,\mathbf{k})\int\Delta_{\rm T2}^{(1)}(\eta,\mathbf{p})\,d^3p\,. \tag{4}$$

The approximation in (4) needs explanation and we follow the argument of Seshradi & Subramanian (1998). The free streaming solution of the first order quadrupole of the temperature anisotropy is proportional to the second spherical Bessel function, $\Delta^{(1)}_{\rm T2}(\eta, \mathbf{p}) \propto j_2(p[\eta - \eta_{\rm rec}])$, with $\eta_{\rm rec}$ the time of recombination (Hu & Sugiyama 1995). The second spherical Bessel function can be approximated by a Gaussian with a peak around $p \approx p_0 = 3.345/(\eta - \eta_{\rm rec})$ and since we are only interested in reionization times below $z = 100$ we get $p_0 < (250h^{-1}{\rm Mpc})^{-1}$. The typical size of a reionized patch is of the order $10h^{-1}$Mpc, which defines the scale where the free electron fluctuation $\delta_{\rm e}$ varies the most. Therefore $\delta_{\rm e}$ is nearly constant where the quadrupole has the largest contribution to the integral in (4), i.e. $\delta_{\rm e}(\eta, \mathbf{k} - \mathbf{p}) \approx \delta_{\rm e}(\eta, \mathbf{k} - \mathbf{p}_0)$ in the relevant integration range. Furthermore the dominant k-range is of the order $(10h^{-1}{\rm Mpc})^{-1}$ and much larger than p_0, therefore we can write $\delta_{\rm e}(\eta, \mathbf{k} - \mathbf{p}_0) \approx \delta_{\rm e}(\eta, \mathbf{k})$.

To calculate the power spectrum we expand the fluctuation in Q into spin-2 spherical harmonics and calculate the correlator of the expansion coefficients (Zaldarriaga & Seljak 1997). The Stokes parameter Q is not invariant under rotation and therefore dependent on the coordinate system we choose. However we can construct an invariant quantity by applying a spin raising operator on Q (Zaldarriaga & Seljak 1997). In our case the resulting quantity is just the electrical field E-type component of the polarization fluctuation. To calculate the two-point function of the expansion coefficients we need to know the correlator of the source $\mathcal{S}(\eta, \mathbf{k})$, which is

$$\langle \mathcal{S}(\eta_1, \mathbf{k}_1)\mathcal{S}(\eta_2, \mathbf{k}_2)\rangle \approx 4\pi \, \langle \delta_{\rm e}(\eta_1, \mathbf{k}_1)\delta_{\rm e}(\eta_2, \mathbf{k}_2)\rangle \int_0^\infty dp\, p^2 P_i(p) \Delta^{(1)}_{\rm T2}(\eta_1, p)\Delta^{(1)}_{\rm T2}(\eta_2, p)\,, \tag{5}$$

where we have applied the approximation from equation (4). Further we exploit the fact that the first order temperature quadrupole anisotropy is *uncorrelated* to the free electron fluctuation from inhomogeneous reionization. This is not necessarily true in a universe where the patches are correlated but the expression can be worked out as long as the underlying fluctuations are Gaussian. The correlator in the free electron fluctuation is given by (1) and the integral in (5) is the correlator of the first order quadrupole fluctuation at unequal times. The factor $P_i(p)$ is the initial power spectrum in the metric fluctuations and is given in inflationary, adiabatic models by a power law $P_i(p) \propto p^{n-4}$, with n the spectral index. The second order anisotropy power spectrum for the E-mode polarization is then given by

$$\begin{aligned} C^{(2)}_{{\rm E},l} \;=\; & \tfrac{9}{2}(2\pi)^{9/2} R^3 \tfrac{(l+2)!}{(l-2)!} \int dk\, d\eta_1\, d\eta_2\, g(\eta_0, \eta_1) g(\eta_0, \eta_2) Q_{\rm P}(\eta_1, \eta_2) I(\eta_1, \eta_2) \times \\ & k^2 \exp\left[-\frac{k^2R^2}{2}\right] \frac{j_l(x_1)}{x_1^2}\frac{j_l(x_2)}{x_2^2}\,, \end{aligned} \tag{6}$$

with $I(\eta_1, \eta_2) = \bar{x}_{\rm e}^{-1}(\eta_{\rm min}) - 1$, $j_l(x)$ the spherical Bessel functions, $x_i = k(\eta_0 - \eta_i)$ and the correlation in the quadrupole at unequal times $Q_{\rm P}(\eta_1, \eta_2) \equiv \int dpp^2 P_i(p)\Delta^{(1)}_{\rm T2}(\eta_1, p)\Delta^{(1)}_{\rm T2}(\eta_2, p)$, i.e. $Q_{\rm P}(\eta_0, \eta_0) \propto C_{T,2}$. We should notice that the limits of the time integrals are the start and end of the reionization process, i.e. we integrate over the time during which the inhomogeneities appear. For the background reionization history we assume that the mean ionization fraction $\bar{x}_{\rm e}$ grows from zero to unity between the redshifts $z^* - \delta z^*/2$ and $z^* + \delta z^*/2$, which we will choose appropriately.

3. Results

We have calculated (6) with a modified version of CMBFAST (Seljak & Zaldarriaga 1996). The COBE normalized polarization power spectrum is shown in figs.1 and 2 for different parameters. In fig.1 the background reionization is given by $z^* = 50$ and $\delta z^* = 20$. The dotted line refers to the *first* order contribution. We recognize the feature on large scales which appears because of the homogeneous reionization background. The thick long-dashed, solid and short-dashed lines refer to the second order contribution due to inhomogeneous reionization. The long-dashed line is a model with a patch size of $R = 20h^{-1}$Mpc, the solid line for $R = 10h^{-1}$Mpc and the short-dashed line for $R = 5h^{-1}$Mpc.

To understand the behavior of the second order effect we will perform two approximations to the integral in (6). First we realize that the expression $g(\eta_0, \eta_1)g(\eta_0, \eta_2)I(\eta_1, \eta_2)$, for reasonable parameters of the reionization history, is a 2-dimensional function with a narrow peak. This allows us to perform the two time integrations in (6) and calculate the integrand at a certain time η^* multiplied by an area factor $(\delta\eta)^2$. The second feature we exploit is that the spherical Bessel function $j_l(x)$ has a tight peak at $l = x$ for large multipole moments l, so we can approximate it with a δ-function. Therefore we get

$$C^{(2)}_{\mathrm{E},l} \approx \frac{9}{2}(2\pi)^{9/2}\frac{(l+2)!}{(l-2)!}\Theta_0^3 l^{-2} Q_{\mathrm{P}}(\eta^*, \eta^*) I(\eta^*, \eta^*) g^2(\eta_0, \eta^*)(\delta\eta)^2 \mathrm{e}^{-\frac{l^2\Theta_0^2}{2}} j_l(l)\,, \tag{7}$$

with $\Theta_0 = R/(\eta_0 - \eta^*)$ as the angular size of a patch as seen today. The only unknown quantity in this expression is the effective time period $\delta\eta$. For the case with $R = 10h^{-1}$Mpc we get a good fit with $\delta\eta = 6.5h^{-1}$Mpc. Although we can not predict the amplitude of the second order power spectrum in (6) analytically, the shape is well approximated by $l(l+1)C^{(2)}_{\mathrm{E},l} \propto l^4 \mathrm{e}^{-l^2\Theta_0^2/2} j_l(l)$. This describes essentially an l^4 rise with some cut-off around the scale $l \approx \sqrt{2}/\Theta_0$ and we see in fig.1 how the peak of the signal moves to larger multipoles l when we decrease the patch size R. We also recognize that the amplitude is proportional to Θ_0^3 and therefore the larger the patch size R the larger the power in the second order anisotropy from inhomogeneous reionization.

In fig.2 we have plotted the results for a model with a more realistic background reionization history (Haiman & Knox 1999). In this case reionization takes place around $z^* = 10$ with a time period of $\delta z^* = 5$. The long-dashed line corresponds to a patch size of $R = 10h^{-1}$Mpc, the solid line to $R = 5h^{-1}$Mpc and the short dashed-line to $R = 1h^{-1}$Mpc. One clearly sees that the power in the anisotropy for this late time reionization is much smaller than in the case for earlier times in fig.1. This is because the visibilty $g(\eta_0, \eta^*)$ is much smaller for the short and late time reionization phase. Again the the peak moves to the right when the patch size is decreasing. For inhomogeneous reionization with *correlated* patches we expect the same behavior as for the temperature anisotropy power spectrum as discovered by Knox, Scoccimarro & Dodelson (1998). In these models the power in the anisotropies is much wider distributed over the multipole moments l than for an uncorrelated scenario, but the magnitude is very similar.

4. Conclusion

In figs.1 and 2 we find that the second order signal dominates over the first order signal only for very large multipoles. Even for an unrealistic reionization model like in fig.1 the second order contribution is relevant only on scales smaller than 5 arcsec. One might hope, that once the cosmological parameters are estimated by the first order temperature power spectra and other experiments, one can reveal the nature of the second order effects. The Ostriker-Vishniac effect for polarization is of the same order or smaller than the signal from inhomogeneous reionization, dependent on the ionization parameters (Seshradi & Subramanian 1998). However this effect is completely determined by the linear power spectrum and therefore can be removed like the first order contribution.
At the present polarization has not been detected in the CMB and the measurements give only crude upper limits (Staggs, Gundersen & Church 1999). We have given the 95% confidence upper limits from the Saskatoon anisotropy experiment (Wollack et al. 1993) and the VLA 8.4 GHz CBR project (Partridge et al. 1997) as a circled and a diamond point in figs.1 and 2. The most accurate future experiment which will measure the polarization anisotropy is the Planck Surveyor. Its High Frequency Instrument (HFI) is expected to measure the polarization to high accuracy in its 143 and 217 GHz channels. The average sensitivity $\Delta T/T$ to the linear polarization per pixel and the angular resolution is 3.7×10^{-6} and 8.0 arcmin, and 8.9×10^{-6} and 5.5 arcmin, respectively (Planck Surveyor). We have calculated the expected polarization signal sensitivity due to cosmic variance, beam size and instrument noise with the methods described in (Knox 1995; Bond, Efstathiou & Tegmark 1997; Bond, Jaffe & Knox 1998). In figs.1 and 2 we show the sensitivity histograms, resulting from a logarithmic bin by weighted average. One recognizes that the noise levels are all above the second order signal from inhomogeneous reionization, so that even with a fairly wide binning strategy, the sensitivity of the Planck Surveyor is not large enough to reveal these signals. The dashed horizontal lines in the histograms in figs.1 and 2 are the logarithmically binned uncertainty contributions just from cosmic variance, which are given by $(\Delta C_l)^2 = 2C_l^2/(2l+1)$, with full sky coverage. Cosmic variance is the fundamental uncertainty limit and describes the fact that we can only observe one universe with only $2l+1$ modes of a certain multipole moment l. It is clear from figs.1 and 2 that the second order contribution from inhomogeneous reionization is above these limits for large multipoles. Therefore a high accuracy polarization measurement on sub-arcminute scales could reveal such a signal. But this depends on how well one can remove polarization foregrounds and the magnitude of other foreground-type second order contributions, like the Sunyaev-Zel'dovich effect (Hu 1999). However, there might be the possibility to disentangle all these effects by exploiting small scale measurements of the matter distribution and the CMB anisotropies, including the information from polarization.

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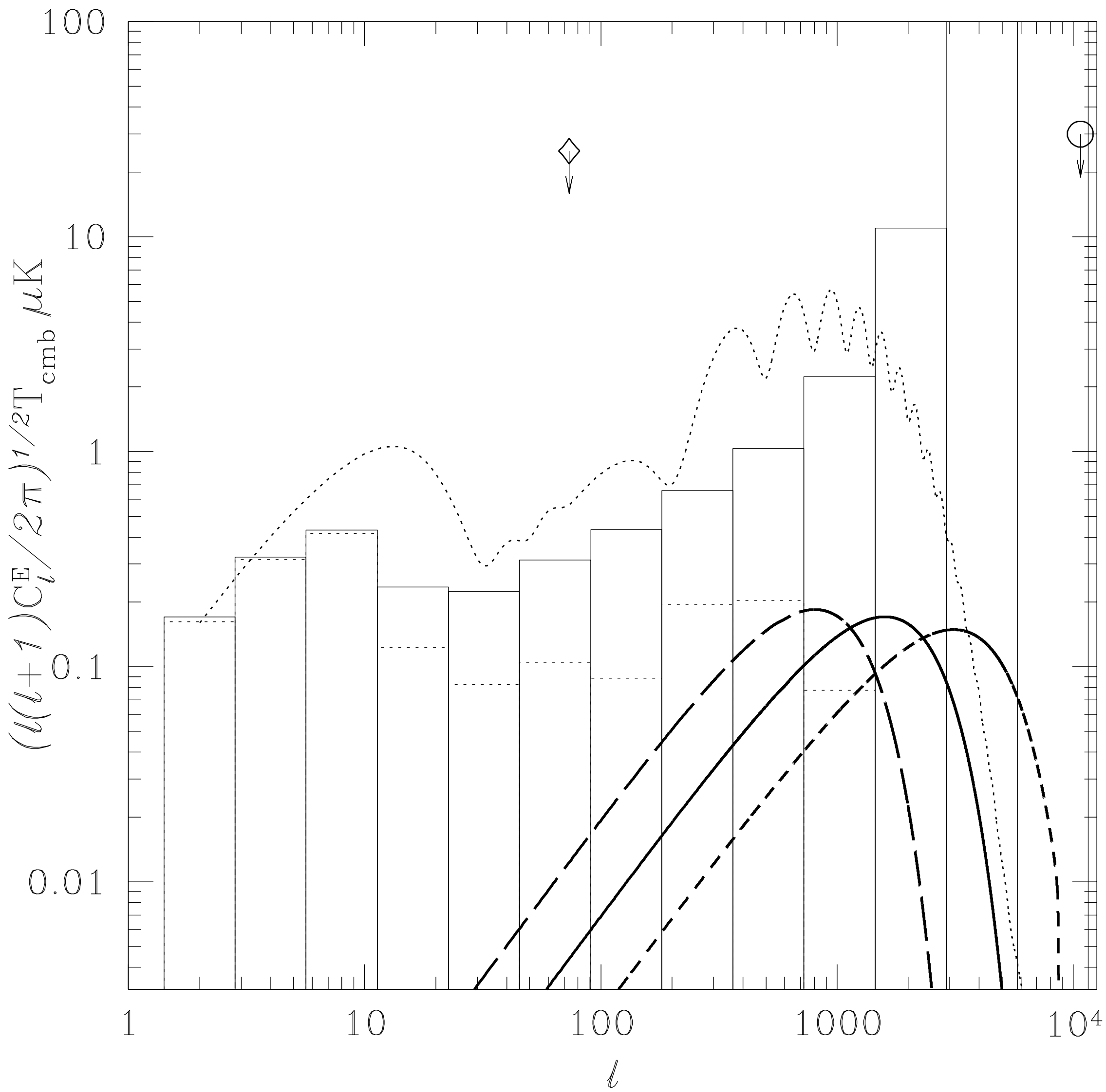

Fig. 1.— The CMB polarization anisotropy power spectrum for a model with an effective reionization time $z^* = 50$ and $\delta z^* = 20$. The dotted line is the first order contribution. The second order signal from inhomogeneous reionization is the long-dashed line for a patch size of $R = 20h^{-1}$Mpc, the solid line for $R = 10h^{-1}$Mpc and the short-dashed line for $R = 5h^{-1}$Mpc. The diamond and circled data points are the 95% confidence upper limit from the Saskatoon anisotropy experiment and the VLA 8.4 GHz CBR project, respectively. The histogram shows the logarithmically binned uncertainty limit from the polarization measurement with the Planck Surveyor. The horizontal dotted lines in the histogram correspond to the uncertainty levels due to cosmic variance only.

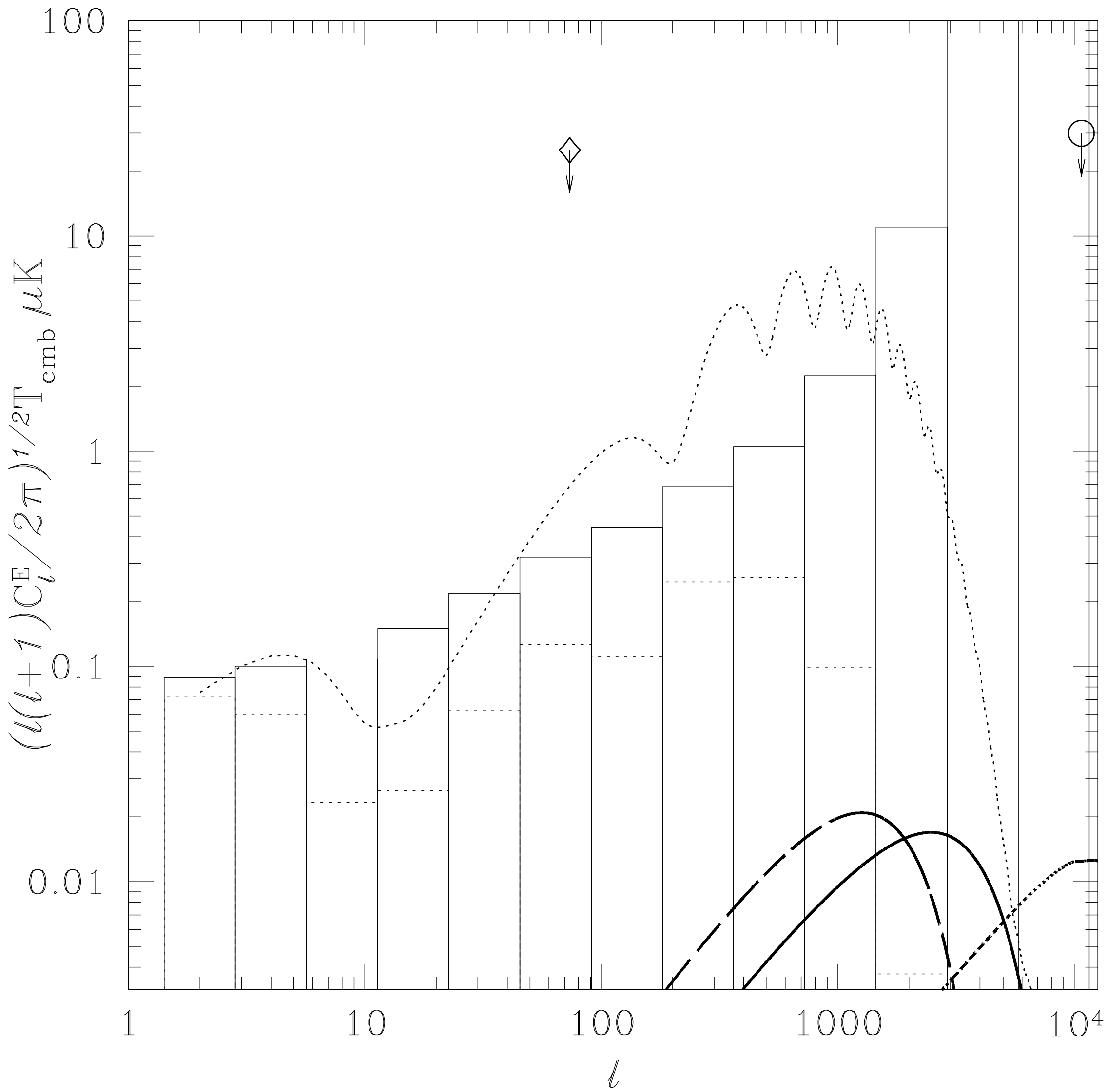

Fig. 2.— The same plot as figure 1 but for an effective reionization at $z^* = 10$ and $\delta z^* = 5$. The patch sizes are $R = 10h^{-1}$Mpc for the long-dashed line, $R = 5h^{-1}$Mpc for the solid line and $R = 1h^{-1}$Mpc for the short-dashed line.