SEP Performance of Coherent MPSK Over Fading Channels in the Presence of Phase/Quadrature Error and I-Q Gain Mismatch

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Abstract—This letter presents a joint approach to the symbol-error probability (SEP) of coherent M-ary phase-shift keying in a situation where the phase error, quadrature error, and in-phase–quadrature (I-Q) gain mismatch problems take place all concurrently over an additive white Gaussian noise and arbitrary fading channel. A set of equations that characterizes the conditional SEP on an instantaneous fading signal-to-noise ratio is derived in the form of the Craig representation.

Index Terms—In-phase–quadrature (I-Q) gain mismatch, M-ary phase-shift keying (MPSK), phase error, quadrature error, symbol-error probability (SEP).

I. INTRODUCTION

IN A COHERENT M-ary phase-shift keying (MPSK) system, the performance of the receiver is characterized by various factors, such as the additive white Gaussian noise (AWGN), fading, phase error, quadrature error, in-phase–quadrature (I-Q) gain mismatch, and so on. The phase error comes from the phase difference between the received carrier and that generated by the local carrier-recovery circuitry. The quadrature error is generated by the phase shifts other than 90° between the I and Q paths. The I-Q gain mismatch is caused by different conversion losses in the I and Q mixers, or by different amplifier gains in the I and Q signal paths at the I-Q demodulator.

There have been a number of papers which describe the symbol-error probability (SEP) performance of MPSK in various situations. Simon and Divsalar [1] developed the analytical expressions for the SEP of MPSK over an AWGN channel in the presence of the phase error and the quadrature error separately. The Craig Q-function [2] was used in their analysis. Simon and Alouini [3], [4] investigated the SEP performance of the MPSK system over generalized fading channels. The Craig representation and the moment generating function (MGF) approach were employed in the analysis. Pawula [5] presented new expressions for the distributions of the phase angle between the two received random vectors perturbed by correlated Gaussian noises for several different special cases.

More recently, Simon [6] simplified the Craig representation of the two-dimensional (2-D) Gaussian Q-function through a clever change of variables. Chiani et al. [7] investigated new exponential bounds for the MPSK error probabilities over AWGN channels, using the results in [6]. The analytical results presented thus far, however, have been made by considering the individual situations one at a time.

In this letter, we present a joint approach to the SEP performance of the MPSK system in a more generalized framework, where the phase error, quadrature error, and I-Q gain mismatch problems take place simultaneously over an AWGN and arbitrary fading channel. A set of new expressions for the conditional SEP on an instantaneous fading signal-to-noise ratio (SNR) is derived in the form of the Craig representation. Computer simulation results are included in order to demonstrate the validity of our analysis. The expressions presented here can be conveniently applied to compute the average SEP over various fading channels by further making use of the MGF approach, as was done in [3] and [4].

II. A JOINT APPROACH TO THE SEP PERFORMANCE OF THE COHERENT MPSK

Consider the block diagram in Fig. 1 for the optimum coherent MPSK receiver, where ϕ, ψ, and (αX, αY) denote the phase error, the quadrature error, and the I-Q gain mismatch pair, respectively. The I and Q demodulation reference signals are defined as was done in [1], such that

\[ r_I(t) = \sqrt{2} \cos \omega_c t \]
\[ r_Q(t) = -\sqrt{2} \sin (\omega_c t - \psi) . \]  

(1)

Note that the phase error ϕ is not included in (1), since it does not change the nature of the joint Gaussian probability density function (pdf) in correlated Gaussian noises, and will only be used to determine the correct decision region in the I-Q signal space.

The kth received signal vector \( S_k \), for \( k = 0, 1, \cdots, M-1 \), at the outputs of the I-Q integrate-and-dumps over the AWGN channel becomes

\[ S_k = \begin{bmatrix} X \\ Y \end{bmatrix} = \sqrt{E_s} \begin{bmatrix} \alpha_X \cos \frac{2\pi k}{M} \\ \alpha_Y \sin \left( \frac{2\pi k}{M} + \psi \right) \end{bmatrix} + \begin{bmatrix} n_X \\ n_Y \end{bmatrix} \]  

(2)

where \( E_s \) denotes the instantaneous symbol energy perturbed by an arbitrary fading channel, and the two noise random variables \( n_X \) and \( n_Y \) are jointly Gaussian with zero mean and equal variance, such that \( \sigma_X^2 = \sigma_Y^2 = \sigma^2 = N_0/2 \). The correlation coefficient between \( n_X \) and \( n_Y \) is given by \( \rho_{XY} = \sin \psi \).

Let \( \gamma \) denote a random variable representing the instantaneous fading SNR, defined as \( \gamma = E_s/N_0 \). Assuming that the transmitted phase is zero (i.e., \( k = 0 \)) for simplicity, we are able to convert the received signal into a baseband pair given by

\[ X = \alpha_X \sqrt{E_s} + \alpha_X n_X \]
\[ Y = \alpha_Y \sqrt{E_s} \sin \psi + \alpha_Y n_Y . \]  

(3)
The event $C$ for the correct decision region when conditioned on $\gamma$ is then written as

$$C = \{(X, Y)| -X \tan \left(\frac{\pi}{M} - \phi\right) < Y < X \tan \left(\frac{\pi}{M} + \phi\right)\}$$  \hspace{1cm} (4)

which can be further expressed explicitly as the intersection of

$$\alpha_X n_X \tan \left(\frac{\pi}{M} - \phi\right) + \alpha_Y n_Y$$

$$> -\sqrt{E_s} \left[\alpha_X \tan \left(\frac{\pi}{M} - \phi\right) + \alpha_Y \sin \psi\right]$$  \hspace{1cm} (5)

with

$$-\alpha_X n_X \tan \left(\frac{\pi}{M} + \phi\right) + \alpha_Y n_Y$$

$$< \sqrt{E_s} \left[\alpha_X \tan \left(\frac{\pi}{M} + \phi\right) - \alpha_Y \sin \psi\right]$$  \hspace{1cm} (6)

Let $Z = \alpha_X n_X \tan(\pi/M - \phi) + \alpha_Y n_Y$ and $W = -\alpha_X n_X \tan(\pi/M + \phi) + \alpha_Y n_Y$. It is not difficult to show that the two random variables $Z$ and $W$ are also jointly Gaussian, with zero mean and variances, respectively, given by

$$\sigma_Z^2 = \sigma^2 \left[\alpha_X^2 \tan^2 \left(\frac{\pi}{M} - \phi\right) + 2\alpha_X \alpha_Y \sin \psi \tan \left(\frac{\pi}{M} - \phi\right) + \alpha_Y^2\right]$$  \hspace{1cm} (7)

and

$$\sigma_W^2 = \sigma^2 \left[\alpha_X^2 \tan^2 \left(\frac{\pi}{M} + \phi\right) - 2\alpha_X \alpha_Y \sin \psi \tan \left(\frac{\pi}{M} + \phi\right) + \alpha_Y^2\right].$$  \hspace{1cm} (8)

Furthermore, the correlation coefficient between $Z$ and $W$ becomes

$$\rho_{ZW} = \frac{\sigma_Z^2 \sigma_W^2}{\sigma_Z^2 \sigma_W} \left\{-\alpha_X^2 \tan \left(\frac{\pi}{M} - \phi\right) \tan \left(\frac{\pi}{M} + \phi\right) + \alpha_X \alpha_Y \sin \psi \left[\tan \left(\frac{\pi}{M} - \phi\right) - \tan \left(\frac{\pi}{M} + \phi\right)\right] + \alpha_Y^2 \right\}.$$

Now, from (4), the conditional probability for the correct decision region $C$ on $\gamma$ results in

$$\Pr\{C|\gamma\} = \Pr \left\{Z > -\sqrt{E_s} \left[\alpha_X \tan \left(\frac{\pi}{M} - \phi\right) + \alpha_Y \sin \psi\right], W < \sqrt{E_s} \left[\alpha_X \tan \left(\frac{\pi}{M} + \phi\right) - \alpha_Y \sin \psi\right]\right\}$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{2\pi \sqrt{1 - \rho_{ZW}^2}} \exp \left[-\frac{u^2 + v^2 - 2\rho_{ZW} uv}{2(1 - \rho_{ZW}^2)}\right] \, du \, dv$$  \hspace{1cm} (10)

where

$$m_Z = \frac{\alpha_X \tan \left(\frac{\pi}{M} - \phi\right) + \alpha_Y \sin \psi}{\sqrt{\alpha_X^2 \tan^2 \left(\frac{\pi}{M} - \phi\right) + 2\alpha_X \alpha_Y \sin \psi \tan \left(\frac{\pi}{M} - \phi\right) + \alpha_Y^2}},$$

and

$$m_W = \frac{\alpha_X \tan \left(\frac{\pi}{M} + \phi\right) - \alpha_Y \sin \psi}{\sqrt{\alpha_X^2 \tan^2 \left(\frac{\pi}{M} + \phi\right) - 2\alpha_X \alpha_Y \sin \psi \tan \left(\frac{\pi}{M} + \phi\right) + \alpha_Y^2}}.\hspace{1cm} (11)$$

Employing [8, (26.3.6)], we are able to rewrite (10) using the 2-D Gaussian $Q$-function as

$$\Pr\{C|\gamma\} = Q(-m_Z \sqrt{2\gamma}, -m_W \sqrt{2\gamma}; -\rho_{ZW})$$  \hspace{1cm} (13)

where

$$Q(x, y, \rho) = \frac{1}{2\pi \sqrt{1 - \rho^2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left[-\frac{u^2 + v^2 - 2\rho uv}{2(1 - \rho^2)}\right] \, du \, dv.$$

The conditional SEP of MPSK on $\gamma$ is now readily evaluated using (13) as

$$\Pr\{E|\gamma\} = 1 - \Pr\{C|\gamma\}.\hspace{1cm} (15)$$
Writing (15) in the form of the Craig representation [6], [9], we obtain (16)–(19), as shown at the bottom of the page, where

$$\tan^{-1}\left(\frac{x}{y}\right) = \frac{\pi}{2} \left[1 - \text{sgn}(y)\right] + \text{sgn}(y) \tan^{-1}\left(\frac{x}{|y|}\right). \quad (20)$$

The expressions in (16)–(19) can be conveniently used to obtain closed-form results for the average SEP over a variety of fading channels by further making use of the MGF approach, as was done in [3] and [4].

III. NUMERICAL RESULTS

In order to demonstrate the validity of our analytical results, we compute the numerical values of the conditional 8-PSK SEP using (16)–(19), and compare them with those obtained from the past work in [1]. Fig. 2 illustrates the numerical results, where only the following three special cases are considered: 1) $\phi = 0$ and $\psi = 0$; 2) $\phi = \pi/50$ and $\psi = 0$; and 3) $\phi = 0$ and $\psi = -\pi/100$. The I-Q gain pair is chosen to be $(\alpha_X, \alpha_Y) = (1, 1)$ for each of the three cases. We observe in Fig. 2 that our results agree very well with those in [1] for all these special cases.

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$$\Pr\{E|\gamma\} = \frac{1}{2\pi} \tan^{-1}\left(\frac{\sqrt{1 - \frac{m_Z^2}{m_W^2}}}{1 + \frac{m_Z^2}{m_W^2}}\right) \exp\left(-\frac{\gamma m_Z^2}{\sin^2 \theta}\right) d\theta + \frac{1}{2\pi} \tan^{-1}\left(\frac{\sqrt{1 - \frac{m_Z^2}{m_W^2}}}{1 + \frac{m_Z^2}{m_W^2}}\right) \exp\left(-\frac{\gamma m_W^2}{\sin^2 \theta}\right) d\theta,$$

for $m_Z > 0$ and $m_W > 0$ \hspace{1cm} (16)

$$\Pr\{E|\gamma\} = 1 - \frac{1}{2\pi} \tan^{-1}\left(\frac{\sqrt{1 - \frac{m_Z^2}{m_W^2}}}{1 + \frac{m_Z^2}{m_W^2}}\right) \exp\left(-\frac{\gamma m_Z^2}{\sin^2 \theta}\right) d\theta + \frac{1}{2\pi} \tan^{-1}\left(\frac{\sqrt{1 - \frac{m_Z^2}{m_W^2}}}{1 + \frac{m_Z^2}{m_W^2}}\right) \exp\left(-\frac{\gamma m_W^2}{\sin^2 \theta}\right) d\theta,$$

for $m_Z \leq 0$ and $m_W > 0$ \hspace{1cm} (17)

$$\Pr\{E|\gamma\} = 1 + \frac{1}{2\pi} \int_{0}^{\pi} \exp\left(-\frac{\gamma m_Z^2}{\sin^2 \theta}\right) d\theta - \frac{1}{2\pi} \tan^{-1}\left(\frac{\sqrt{1 - \frac{m_Z^2}{m_W^2}}}{1 + \frac{m_Z^2}{m_W^2}}\right) \exp\left(-\frac{\gamma m_Z^2}{\sin^2 \theta}\right) d\theta,$$

for $m_Z > 0$ and $m_W \leq 0$ \hspace{1cm} (18)

$$\Pr\{E|\gamma\} = 1 - \frac{1}{2\pi} \int_{0}^{\pi} \exp\left(-\frac{\gamma m_W^2}{\sin^2 \theta}\right) d\theta - \frac{1}{2\pi} \tan^{-1}\left(\frac{\sqrt{1 - \frac{m_Z^2}{m_W^2}}}{1 + \frac{m_Z^2}{m_W^2}}\right) \exp\left(-\frac{\gamma m_W^2}{\sin^2 \theta}\right) d\theta,$$

for $m_Z \leq 0$ and $m_W \leq 0$ \hspace{1cm} (19)
IV. CONCLUSION

In this letter, we have described a joint approach to the SEP performance of the MPSK system in the simultaneous presence of phase error, quadrature error, and I-Q gain mismatch over an AWGN and arbitrary fading channel. A set of equations for the conditional SEP on the fading channel has been derived. It was shown that our results numerically agree with those from past work. The expressions for the conditional SEP presented in this letter can be conveniently applied to compute the average SEP over various fading channels by further making use of the MGF approach.

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