Relativistic signatures in laser-assisted scattering at high field intensities

C. Szymanowski and A. Maquet

Laboratoire de Chimie Physique — Matière et Rayonnement ,
11 rue Pierre et Marie Curie , Université P. et M. Curie Paris VI ,
F - 75 231 Paris Cedex 05 , France

szymanow@ccr.jussieu.fr , alm@ccr.jussieu.fr

Abstract: The complete Dirac-Volkov relativistic treatment of the first Born limit of laser-assisted potential scattering of electrons within a circularly polarized laser field has been compared to the nonrelativistic Bunkin-Fedorov approach. The dependence of the quiver energy on the electron four-momentum in an ultrastrong laser field leads to different energy transfer cross sections depending on the scattering geometry with respect to the laser propagation direction. Visible differences between the relativistic and non-relativistic differential cross sections for small-angle scattering occur already for $10^{16}\,\mathrm{W/cm^2}$ intensity of near infrared wavelength and moderate electron initial energies.

OCIS codes: (020.4180) Multiphoton processes; (020.5580) Quantum electrodynamics; (020.2070) Effects of collisions

References

1. see, for instance, "High-field interactions and short-wavelength generation", edited by H. Milchberg and R. Freeman, part 1 of special issue of *J. Opt. Soc. Am.* **B13**, 51-222 (1996).
2. see, for instance, "High-field interactions and short-wavelength generarion", edited by H. Milchberg and R. Freeman, part 2 of special issue of *J. Opt. Soc. Am.* **B13**, 314-468 (1996).
3. D. L. Burke, R. C. Field, G. Horton-Smith, J. E. Spencer, D. Walz, S. C. Berrigde, W. M. Bugg, K. Shmakov, A. W. Weidemann, C. Bula, K. T. McDonald, E. J. Prebys, C. Bamber, S. J. Boege, T. Koffas, T. Kotseroglou, A. C. Melissinos, D. D. Meyerhofer, D.A. Reis, and W. Ragg, "Positron Production in Multiphoton Light-by-Light Scattering", *Phys. Rev. Lett.* **79**, 1626-1629 (1997).
4. C. Bula, K. T. McDonald, E. J. Prebys, C. Bamber, S. Boege, T. Kotseroglou, A. C. Melissinos, D. D. Meyerhofer, W. Ragg, D. L. Burke, R. C. Field, G. Horton-Smith, A. C. Odian, J. E. Spencer, D. Walz, S. C. Berrigde, W. M. Bugg, K. Shmakov, and A. W. Weidemann, "Observation of Nonlinear Effects in Compton Scattering", *Phys. Rev. Lett.* **76**, 3116-3119 (1996).
5. P. A. Norreys, M. Zepf, S. Moustaizis, A. P. Fews, J. Zhang, P. Lee, M. Bakarezos, C. N. Danson, A. Dyson, P. Gibbon, P. Loukakos, D. Neely, F. N. Walsh, J. S. Wark, and A. E. Dangor, "Efficient Extreme UV Harmonics Generated from Picosecond Laser Pulse Interactions with Solid Targets", *Phys. Rev. Lett.* **76**, 1832-1835 (1996).
6. P. Monot, T. Auguste, P. Gibbon, F. Jakober, G. Mainfray, A. Dulieu, M. Louis-Jacquet, G. Malka, and J. L. Miquel, "Experimental Demonstration of Relativistic Self-Channeling of a Multiterawatt Laser Pulse in an Underdense Plasma", *Phys. Rev. Lett.* **74**, 2953-2956 (1995).
7. C. I. Moore, J. P. Knauer, and D. D. Meyerhofer, "Observation of the Transition from Thomson to Compton Scattering in Multiphoton Interactions with Low-Energy Electrons", *Phys. Rev. Lett.* **74**, 2439-2442 (1995).
8. L. S. Brown and T. W. B. Kibble, "Interaction of Intense Laser Beams with Electrons", *Phys. Rev.* **133**, A705 (1964).
9. H. R. Reiss, "Absorption of Light by Light", *J. Math. Phys.* **3**, 59-67 (1962).

#2709 - $15.00 US
Received September 25, 1997; Revised March 13, 1998

10. I. I. Gol'dman, "Intensity Effects in Compton Scattering", *Zh. Éksp. Teor. Fiz.* **46**, 1412-1417 (1964) [*Sov. Phys. JETP* **19**, 954-957 (1964)].

11. A. I. Nikishov and V. I. Ritus, "Quantum Processes in the field of a plane electromagnetic wave and in a constant field I", *Zh. Éksp. Teor. Fiz.* **46**, 776-796 (1964) [*Sov. Phys. JETP* **19**, 529-541 (1964)].

12. A. I. Nikishov and V. I. Ritus, "Quantum Processes in the field of a plane electromagnetic wave and in a constant field II" *Zh. Éksp. Teor. Fiz.* **46**, 1768-1781 (1964) [*Sov. Phys. JETP* **19**, 1191-1199 (1964)].

13. J. H. Eberly, "Proposed Experiment for Observation of Nonlinear Compton Wavelength Shift", *Phys. Rev. Lett.* **15**, 91-93 (1965).

14. T. W. B. Kibble, "Mutual Refraction of Electrons and Photons", *Phys. Rev.* **150**, 1060 - 1069 (1966).

15. J. H. Eberly and H. R. Reiss, "Electron Self-Energy in Intense Plane-Wave Field", *Phys. Rev.* **145**, 1035 (1966).

16. H. R. Reiss and J. H. Eberly, "Green's function in intense field electrodynamics", *Phys. Rev.* **151**, 1058-1066 (1966).

17. J. H. Eberly, "Interaction of very intense light with free electrons", *Prog. Opt.* **7**, 361-415, edited by E. Wolf, (Noth-Holland, Amsterdam, 1969).

18. F. V. Bunkin and M. V. Fedorov, " Bremsstrahlung in a strong radiation field", *Zh. Éksp. Teor. Fiz.* **49**, 1215-1221 (1965) [*Sov. Phys. JETP* **22**, 844-847 (1966)].

19. M. M. Denisov and M. V. Fedorov, "Bremsstrahlung effect on relativistic electrons in a strong radiation field", *Zh. Éksp. Teor. Fiz.* **53**, 1340-1348 (1967) [*Sov. Phys. JETP* **26** 779-783 (1968)].

20. C. Szymanowski, V. Véniard, R. Taïeb, A. Maquet, and C. H. Keitel, "Mott scattering in strong laser fields", *Phys. Rev. A* **56** 3846-3859 (1997).

21. V. P. Krainov and S. P. Roshupkin, "Relativistic effects in the angular distribution of ejected electrons in tunneling ionization of atoms by strong electromagnetic fields", *J. Opt. Soc. Am.* **B9**, 1231-1233 (1992).

22. V. B. Berestetzkiĭ, E. M. Lifshitz, and L. P. Pitaevskiĭ, *Quantum Electrodynamics*, 2nd Edition (Pergamon Press, Oxford, 1982).

23. D. M. Volkov, "Über eine Klasse von Lösungen der Diracschen Gleichung", *Z. f. Phys.* **94**, 250-260 (1935).

24. S. P. Goreslavskiĭ, "The BSI model and relativistic ponderomotive scattering", *Laser Phys.* **6**, 74-78 (1996).

25. N. B. Narozhnyi and M. S. Fofanov, "Creation of a pair by a photon colliding with a short focused laser pulse", *Laser Phys.* **7**, 141-149 (1997).

26. C. H. Keitel, C. Szymanowski, P. L. Knight, and A. Maquet, "Radiative reaction in ultra-intense laser-atom interaction", *J. Phys. B* **31**, L75-L83 (1998).

27. F. V. Hartemann and A. K. Kerman, "Classical Theory of Nonlinear Compton Scattering", *Phys. Rev. Lett.* **76**, 624-627 (1996).

28. C. Szymanowski, R. Taïeb, and A. Maquet, "Laser-assisted scattering of polarized electrons at high field intensities", *Laser Phys.* **8**, 102-106 (1998).

The availability of very intense ultrashort laser pulses of near-infrared radiation in a number of research laboratories [1,2] enables now investigations of laser-induced processes in ultrastrong fields well beyond the atomic field strength intensity in the range of $10^{16}\,\mathrm{W/cm}^2 - 10^{19}\,\mathrm{W/cm}^2$. Recent exciting experiments give clear evidence of several relativistic effects [3,4,5,6,7], and there is considerable interest in theoretical predictions. It has already been noticed in early studies in the 1960s [8,9,10,11,12,13,15,16,17], that in this regime the averaged quiver energy becomes relativistic and as well dependent on the four-momentum of the electron. The first Born nonrelativistic treatment of laser-assisted collisions by Bunkin and Fedorov [18] has formally been extended to the relativistic domain by Denisov and Fedorov [19]. Recently, the interplay of spin-orbit and spin-laser interaction in Coulomb scattering in strong laser fields has been discussed in detail, especially for large-angle diffusion, demonstrating the importance of the full Dirac-Volkov treatment [20]. The purpose of the present contribution is to show that the modifications of the differential cross section for small-angle scattering in a Yukawa potential, chosen for account of screening effects in a plasma environment, in the presence of an ultraintense field provide a remarkable signature of relativistic effects in electron-laser interaction. Let us mention that also in strong-field ionization $1/c$-corrections at relatively low laser intensity have been realized [21].

The principal steps of the relativistic calculation are briefly outlined before proceeding to some selected results of our investigations. The solutions to the Dirac equation for an electron with four-momentum p^μ (outside the field and genuine quantum number) inside a circularly polarized plane wave with vector potential A^μ and wavevector k^μ propagating along the $\hat{e}_z$ direction read in Feynman slash notation [22,23]:

$$\psi_q = \langle \vec{x}|q\rangle = \left[1 + \frac{\not k \not A}{2c(kp)}\right] \times \frac{u}{\sqrt{2QV}} \times \exp\left[-i\,(qx) - i\int_0^{kx} \frac{(pA)}{c(kp)}\, d\phi\right], \qquad (1)$$

where u represents a free bispinor normalized by $\bar{u}u = u^*\gamma^0 u = 2c^2$ in the volume V. The physical significance of $q^\mu = (Q/c, \vec{q}\,)$ is the averaged four-momentum (dressed momentum) of the particle inside the laser field: $q^\mu = p^\mu - k^\mu\,[A^2/2(kp)c^2]$. $\vec{p}$ and $\vec{q}$ have different longitudinal but equal transverse momentum projections with respect to $\vec{k}$. This implies that angles might change due to field envelope effects [24,25]. For scattering off a Yukawa-type potential the matrix element for the transition $|q\rangle \to |q'\rangle$ is

$$T_{\mathrm{q'}\leftarrow \mathrm{q}} = \frac{iZ}{c}\int d^4x\,\bar{\psi}_{q'}\,\frac{\gamma^0 \exp\big(-\alpha|\vec{x}\,|\big)}{|\vec{x}|}\,\psi_q\,. \qquad (2)$$

The first Born approximation is certainly valid for the high energies of the incoming projectiles in ultrastrong fields. The breakdown of the validity of the model is expected at intensities so high that pair creation (and also radiative reaction [26,27]) becomes significant.

The essential steps in the explicit calculation of the cross section are: Using trigonometric relations and the generating function of ordinary Bessel functions J_n a sum over Fourier components whose order are associated to the net number of exchanged photons is obtained which leads to well-known Fourier transform integrals of the central potential. The appropriate expression for the square of the δ-function is given by the usual procedure [22]. This leads to the expression for the differential cross section into the solid angle o:

$$\frac{d\,\sigma}{do} = \sum_n \frac{d\,\sigma^{(n)}}{do}\Big|_{Q'=Q+n\omega}, \qquad (3)$$

where $|\vec{q'}|$ is fixed through the condition $q^\mu q_\mu = q'^\mu q'_\mu$. The differential cross section $d\,\sigma^{(n)}/do$ for each net n-photon process for unpolarized projectiles is [20]:

$$\begin{aligned}\frac{d\,\sigma^{(n)}}{do} \;=\; & \frac{Z^2|\vec{q'}|}{c^2|\vec{q}|} \times \frac{1}{|(\vec{q}-\vec{q'}+n\vec{k})^2+\alpha^2|^2} \times \Bigg(\\ & 2J_n(\zeta)^2\Bigg\{\left[c^2 + \frac{QQ'}{c^2} + \vec{q}\vec{q'} + \frac{A^2}{2c^2}\left(\frac{(k\tilde{q}')}{(kq)} + \frac{(k\tilde{q})}{(kq')}\right) + \frac{(A^2)^2\omega^2}{2c^6(kq)(kq')}\right] \times \\ & \left(1 - \frac{A^2\omega^2}{c^4(kq)(kq')}\right) - \frac{A^2}{2c^2}\left[1 - \frac{(k\tilde{q}')(k\tilde{q})}{(kq)(kq')}\right] + \frac{(A^2)^2\omega^2}{c^6(kq)(kq')} \\ & + \frac{(A^2)^2\omega^2}{2c^6(kq)^2(kq')^2}\left[(kq')(k\tilde{q}') + (kq)(k\tilde{q}) + \frac{A^2\omega^2}{c^4}\right]\Bigg\}\end{aligned}$$

#2709 - $15.00 US
Received September 25, 1997; Revised March 13, 1998

$$
+\Big[J_{n+1}(\zeta)^2+J_{n-1}(\zeta)^2\Big]\Bigg\{\frac{-A^2}{2c^2}\Big[\frac{(k\tilde{q}')}{(kq)}+\frac{(k\tilde{q})}{(kq')}+1+\frac{(k\tilde{q})(k\tilde{q}')-2\omega^2}{(kq)(kq')}
$$

$$
+\frac{2A^2\omega^2}{c^4(kq)(kq')}+\frac{A^2\omega^2}{c^4(kq)(kq')}\Big((kq')(k\tilde{q}')+(kq)(k\tilde{q})+\frac{A^2\omega^2}{c^4}\Big)
$$

$$
-\frac{2\omega^2}{c^2(kq)(kq')}\Big(\frac{QQ'}{c^2}+\vec{q}\vec{q}'+\frac{A^2}{2c^2}\Big(\frac{(k\tilde{q}')}{(kq)}+\frac{(k\tilde{q})}{(kq')}\Big)+\frac{(A^2)^2\omega^2}{2c^6(kq)(kq')}\Big)\Big]
$$

$$
+\frac{2A^2\omega^2}{c^4(kq')(kq)}\Big[q'_xq_x+q'_yq_y\Big]\Bigg\}+J_{n+1}(\zeta)J_{n-1}(\zeta)\cos(2\phi_0)\frac{2A^2\omega^2}{c^4(kq')(kq)}
$$

$$
\times\Big[q'_xq_x-q'_yq_y\Big]+J_n(\zeta)\Big[J_{n+1}(\zeta)+J_{n-1}(\zeta)\Big]\Bigg\{-\frac{2A^2\omega^2}{c^4(kq)(kq')}\Big[\frac{(\mathring{A}q)}{c}
$$

$$
+\frac{(\mathring{A}q')}{c}\Big]+\frac{(\mathring{A}q')(k\tilde{q})}{c(kq')}+\frac{(\mathring{A}q)}{c}+\frac{(\mathring{A}q')}{c}+\frac{(\mathring{A}q)(k\tilde{q}')}{c(kq)}\Bigg\}\Bigg)\Bigg|_{Q'=Q+n\omega}. \quad (4)
$$

Here the following abbreviations are used: $\tilde{v}^\mu$ denotes $(v^0,-\vec{v^i})$, $\zeta = A\Big\{\big[(p_x/c(kp)-p'_x/c(kp')\big]^2+\big[p_y/c(kp)-p'_y/c(kp')\big]^2\Big\}^{1/2}$, $\phi_0=\arccos\{A[p_x/(kp)-p'_x/(kp')]/c\zeta\}$ and $\mathring{A}\equiv A(0,\cos\phi_0,\sin\phi_0,0)$. The influence of the spin-laser interaction on the degree of polarization of initially polarized electrons has also been studied [28].

The magnitude of the cross section of each n-photon energy transfer depends crucially on the weight given by the squares and products of Bessel functions. It is shown below that the main changes occurring in the final energy spectra are caused by the differences between the argument ζ and its nonrelativistic limit. The physical meaning of ζ is the maximal attainable transient energy difference per photon energy during a collision [20]. It describes the "overlap" of the energy dressing of the Volkov states weighted by Bessel functions for each n-photon contribution. Simulations have been performed in a wide range of parameters of intensities and for $\omega = 0.043\,\text{a.u.}$, corresponding to the Neodymium laser angular frequency (1.17 eV photon energy). For illustrating the main features of our results we will concentrate here on the case of small-angle scattering, which dominates the total cross section.

Consider Fig. 1: Differential cross sections for a deflection angle of $\angle(\vec{q},\vec{q}')=0.6\,\text{mrad}$ are shown for an intensity of $3.5\times10^{16}\,\text{W/cm}^2$ and a moderate initial electron energy of $W_{\text{kin}}\simeq 2.7\,\text{keV}$. The red curve gives the envelope of the result according to Eq. (4) for an incoming electron with momentum parallel to the laser propagation direction $\vec{k}$, while the yellow curve denotes the case of antiparallel propagation. The black line sketches the nonrelativistic result virtually independent from the direction of $\vec{k}$. A significant dependence on geometry of the amount of photon energy transfer is noticed.

Received September 25, 1997; Revised March 13, 1998

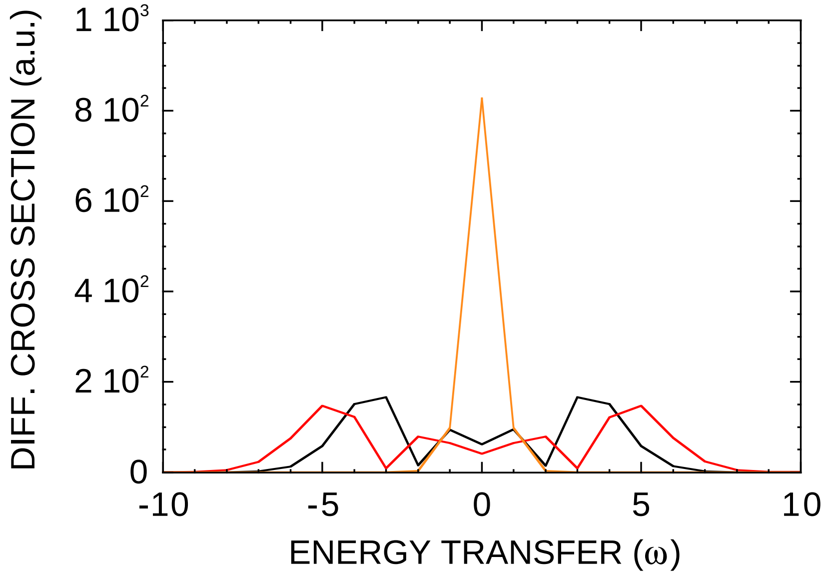

Fig. 1. Envelope of differential cross section $d\sigma/do$ in atomic units as a function of energy transfer ΔQ scaled in units of the laser photon energy ω for an electrical field strength of $E = 1\,\mathrm{a.u.}$ or vector potential $A = Ec/\omega = 3186\,\mathrm{a.u.}$. The Yukawa parameter is $\alpha = 0.25$, the deflection angle equals $\angle(\vec{q}, \vec{q'}) = 0.0002\pi$ and the initial kinetic energy is $W_{\mathrm{kin}} = 100\,\mathrm{a.u.}$. The different curves are explained in the text.

The obvious discrepancies in the weights of each n-photon process can be traced back to the sensitivity of the Bessel functions to variations of their arguments ζ, meaning to the change in the energy dressing difference. The simulations clearly show that the nonrelativistic limit $1/\omega$ of the four-product $1/(kp)$ is not any longer a good approximation in this laser intensity regime. In fact, for a kinetic energy of $W_{\mathrm{kin}} = E - c^2 \simeq 2.7\mathrm{keV}$ the differences amount to about $+11\%$ in propagation direction parallel to $\vec{k}$ and about -9% in direction antiparallel to $\vec{k}$. In the full Dirac-Volkov treatment, four-vector products of this form come additionally into play due to the Volkov prefactor $(\propto A)$ containing γ-matrices. However, these terms of the cross section describing the spin-laser interaction do not contribute significantly at intensities considered here.

The net photon energy transfer is even higher for more intense laser fields as is shown in Fig. 2. Here, differential cross sections for the same deflection angle of $\angle(\vec{q}, \vec{q'}) = 0.6\,\mathrm{mrad}$ and for a relativistically high intensity of about $10^{18}\,\mathrm{W/cm^2}$ are plotted. The incoming electron energy is again about $2.7\,\mathrm{keV}$, and the incoming electron momentum is oriented parallely to the laser propagation direction. The red curve denotes the full Dirac-Volkov result according to equation 4. The black line corresponds to a simplified treatment for spinless relativistic particles, while the yellow line sketches the outcome of the nonrelativistic calculation. The discrepancy between the relativistic and nonrelativistic treatments amount now to a factor 2 in the occurence of the cutoff. The maximal energy transfer is fixed by the condition $n \simeq \zeta$ when the Bessel functions reach asymptotically the Airy function Ai. One can verify in classical terms that ζ corresponds to the maximal attainable transient energy difference per photon energy

Received September 25, 1997; Revised March 13, 1998

of the electron travelling in the laser field. Nevertheless in comparison to the spinless particle case the complete Dirac-Volkov calculation predicts a higher cross section due to the spin-laser interaction pushing the electron in the laser propagation direction. This effect is mediated by the additional terms in Eq. 4 proportional to powers of A. In comparison, the blue line denotes the contribution arising only from the first part of the Volkov prefactor ($\propto 1$) which is largely dominating in the nonrelativistic and moderately relativistic intensity regime.

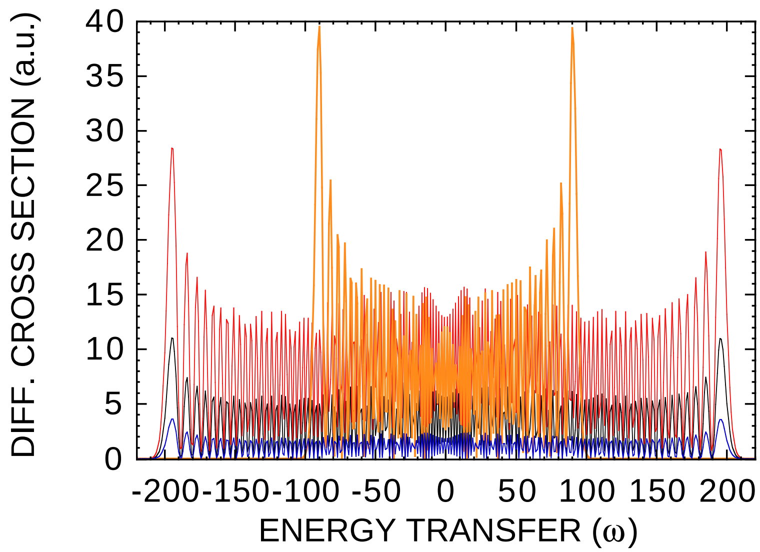

Fig. 2. Envelope of differential cross section $d\sigma/do$ scaled in atomic units as a function of energy transfer ΔQ scaled in units of the laser photon energy ω for an electrical field strength of $E = \omega c = 5.89$ a.u. or vector potential $A = c^2 = 18769$ a.u.. The parameter α, the deflection angle and the initial kinetic energy are the same as for Fig. 1. The different curves are explained in the text.

In brief, our calculations show that at laser intensities currently available in the near-infrared significant relativistic effects in small-angle laser-assisted scattering occur already for moderate electron energies. The influence of relativity essentially is the dependence of the differential cross section for fixed deflection angle on the orientation of the scattering geometry of the scattering with respect to the laser propagation direction. A larger net photon energy transfer is predicted in comparison to the nonrelativistic calculation in this direction.

C.S. acknowledges a DAAD-fellowship HSP III financed by the German Ministry of Education, Science, Research and Technology. This work is supported as well in part by the European Union through contract CRB CHRX CT 940470. The Laboratoire de Chimie Physique — Matière et Rayonnement is UMR 7614 du CNRS. Valuable discussions with Drs. R. Taïeb, V. Véniard and C. H. Keitel and with Profs. H. R. Reiss and M. V. Fedorov at the LPHYS'97 workshop in Prague are gratefully acknowledged.