

Coulomb drag between disordered two-dimensional electron gas layers

Lian Zheng and A.H. MacDonald

Department of Physics, Indiana University, Bloomington, IN 47405

We derive and evaluate expressions for the frictional Coulomb drag between disordered two-dimensional electron gas layers. Our derivation is based on the memory-function formalism and the expression for the drag reduces to previously known results in the ballistic limit. We find that Coulomb drag is appreciably enhanced by disorder at low temperatures when the mean-free-path within a layer is comparable to or shorter than the layer separation. In high mobility two-dimensional electron gas systems, where the drag has been studied experimentally, the effect of disorder on the drag is negligible at attainable temperatures. We predict that an enhancement due to disorder and a crossover in the temperature-dependence and layer-separation dependence will be observable at low temperatures in moderate and low mobility samples.

PACS numbers: 73.50.Dn

I. INTRODUCTION

Double-layer two dimensional electron gas (2DEG) systems, where electrons are confined to nearby parallel planes, are expected to exhibit many novel phenomena due to interlayer electron-electron interaction. For example, in strong magnetic fields Coulomb effects are expected to produce new incompressible ground states that exhibit the fractional quantum Hall effect [1] and to cause the collapse of certain integer quantum Hall effect gaps [2]. In zero magnetic field, it has been suggested that Wigner crystallization in double layer systems is favored by interlayer Coulomb interactions [3] and that in the case of electron-hole systems, excitonic superfluidity could result [4] from interlayer interactions. In recent experiments by Gramila et al. on electron-electron double layer systems [5,6] and in similar experiment by Sivan et al. on electron-hole systems [7], the strength of interlayer interactions was studied directly by measuring the frictional drag of one two-dimensional electron gas layer on another. In these experiments a current flowing in one layer tends to induce a current in nearby layers. If no current is allowed to flow in the nearby layer an electric field develops whose influence cancels the frictional force between the layers. The transresistance, defined as the ratio of the induced voltage in the second layer to the applied current in the first layer, directly measures the rate at which momentum is transferred from the current carrying 2DEG to its neighbor.

Drag between spatially separated electron systems due to Coulomb interactions between carriers was first considered by Pogrebinskii [8] and Price [9]. The experiments of Gramila et al. [5,6] and of Sivan et al. [7] have stimulated recent theoretical attention, especially to the case of drag between two-dimensional electron layers at low temperatures [5,6,10,11,12,13,14]. The interlayer Coulomb drag is caused by fluctuations in the density of electrons in each layer since 2D layers with charge uniformly distributed will not exert any frictional forces upon each other. In this paper we examine for the first time the possibility of enhanced frictional drag between disordered layers due to the diffusive nature of long-wavelength long-time electron density fluctuations. Disorder is known [15,16] to

enhance interaction effects and to lead to violations of Fermi liquid theory for individual two-dimensional electron gas layers. In the diffusive regime, where wavelengths are longer than the mean free path of the electrons and times are longer than the electron scattering time, the electron density-density response function possesses a diffusion pole [17] in momentum-frequency space. In perturbation theory the diffusion pole arises from dressing the electron-electron vertex [16] with corrections arising from impurity potential scattering. At shorter distances or shorter times the disorder vertex corrections are not important. We find that disorder enhances the interlayer drag at low temperatures, changing the temperature dependence of the drag from T^2 to $-T^2 \ln T$. In very high-mobility samples (on which existing experiments have been performed) or in samples with small layer separations the corrections due to disorder become important only at extremely low temperatures. For samples with lower mobility or more widely separated layers the influence on the interlayer scattering rate from disorder scattering should be easily measurable.

Most [5,10,11] previous work on interlayer friction has been based on Boltzmann transport theory which cannot capture disorder enhanced interaction effects. (An exception is the work of Vasilopoulos and co-workers [14].) In Section II we present a derivation of the expression for the frictional transresistance based on the memory function formalism [17] which is sufficiently general to treat the case of disordered 2DEG layers. Although phonon-mediated [6,14,19] interlayer interactions are not discussed explicitly in this paper the expression we derive in Section II are sufficiently general that other coupling mechanisms can be incorporated as an effective-interlayer interaction potential. Similarly particle-particle interaction vertex corrections, which are probably quantitatively important especially for electron-hole double layer systems [7], can also be incorporated into the results we derive in Section II as a contribution to the effective electron-electron interaction [18]. Readers interested only in the application to disordered double-layer systems should proceed to Section III where we discuss how disorder within the layers influences the interlayer friction. The interlayer scattering rate of typical electron-electron double layer samples is evaluated numerically in Section IV. We show that the temperatures below which disorder becomes important de-

creases very rapidly with increasing mobility. A brief summary of our findings concludes the paper in Section V.

II. MEMORY FUNCTION FORMALISM DERIVATION

The memory function formalism provide a very convenient method for deriving a flexible expression for the transresistance of double layer 2DEG systems. The Kubo current-current correlation function formula for the conductivity is first converted into a force-force correlation function expression for the resistivity by making use of Mori's [20] projection operator. This force-force correlation function is then evaluated at lowest order in the screened inter-layer interaction to obtain an approximate expression for the transresistance. The advantage of using the force-force correlation function rather than the current-current correlation function is that it yields a reasonable approximation even when evaluated at lowest order [21,22]. At this level the results are physically equivalent to momentum-balance [14] approximations or to relaxation-time approximations in a Boltzmann-transport approach [23]. The derivation for the situation of present interest is sketched in the following paragraphs.

For notational simplicity we restrict ourself to the case of zero magnetic field so that the current is in the same direction as the applied electric field and the conductance, therefore, forms a 2×2 matrix with respect to the layer indices. (The final expression for the transresistance is equally valid [24] in the presence of a magnetic field.) We need to consider only the long wavelength limit of the conductance. To use the memory function formalism it is convenient to write the Kubo formula in the form

$$\sigma_{ij}(\omega) = \frac{\beta}{\nu} \int_0^\infty dt e^{i\omega t} (\hat{J}_i(t), \hat{J}_j) \quad (1)$$

where $\beta = 1/k_B T$, ν is the cross section area of the 2DEG layers, and the indices i and j are layer labels. $\sigma_{ij}(\omega)$ gives the current density induced in layer i due to an electric field in layer j . \hat{J} is the zero wavevector Fourier component of the current density operator. The inner product appearing in Eq. (1) is defined by

$$\begin{aligned}
C_{AB}(t) &\equiv (\hat{A}(t), \hat{B}) \\
&\equiv \beta^{-1} \int_0^\beta d\lambda \langle \hat{A}^\dagger(t), \hat{B}(i\hbar\lambda) \rangle
\end{aligned} \tag{2}$$

In Eq. (2) the angle brackets denote thermal averages. The following relationship can be used to change Eq. (1) into the more familiar form of the Kubo formula [22]:

$$i\beta\partial_t C_{AB}(t) = \frac{1}{\hbar} \langle [\hat{A}(t), \hat{B}] \rangle. \tag{3}$$

The projection operator method is now used to obtain an expression for the matrix inverse in layer indices of $C_{J_i J_j}$. We define a superoperator \mathcal{P} which ‘projects’ an operator \hat{O} onto the current density, and its complement \mathcal{Q} , by

$$\mathcal{P}\hat{O} \equiv \sum_k \frac{\hat{J}_k(\hat{J}_k, \hat{O})}{(\hat{J}_k, \hat{J}_k)} \tag{4}$$

$$\mathcal{P}\hat{O} \equiv \hat{O} - \mathcal{Q}\hat{O} \tag{5}$$

It is useful to define a matrix χ_{ij} :

$$\chi_{ij} = \frac{\beta}{\nu} C_{J_i J_j}(0) = (\hat{J}_i, \hat{J}_j) = \frac{n_i e^2}{m} \delta_{ij}. \tag{6}$$

Here n_i is the areal density of 2D electrons in the i -th layer. Following the usual development of the memory function formalism [17], we obtain an equation for the matrix inverse of the Fourier transform of $C_{J_i J_j}(t)$:

$$[C_{J_i J_j}(z)]^{-1} = \frac{\beta}{\nu} \chi^{-1} [-iz\mathbf{1} + M(z)] \tag{7}$$

where

$$M_{ij}(z) = \frac{\beta}{\nu} (\hat{J}_i, \frac{i}{z - \mathcal{Q}\mathcal{L}} \hat{J}_j) \chi_{jj}^{-1}. \tag{8}$$

A dot over an operators denotes its time derivative at $t = 0$ and the Liouville superoperator is defined by $\mathcal{L}\hat{O} = [\hat{H}, \hat{O}]$, where \hat{H} is the Hamiltonian. In obtaining Eq. (7), the time reversal invariance condition $\mathcal{P}\dot{J} = 0$ has been applied. Combining Eq. (1), Eq. (7) and Eq. (8) we obtain an expression for the resistivity matrix:

$$\begin{aligned}
\rho_{ij}(z) &= \chi_{ii}^{-1} M_{ij}(z) \\
&= \chi_{ii}^{-1} \chi_{jj}^{-1} \frac{\beta}{\nu} \int_0^\infty dt e^{izt} (\dot{J}_i, e^{-i\mathcal{Q}Lt} \dot{J}_j).
\end{aligned} \tag{9}$$

ρ_{ij} relates the electric field in layer i to the current density in layer j . With the relation $\dot{J}_i = -e/mF_i$, we obtain the force-force correlation function expression for the trans-resistance,

$$\rho_{LR} = \frac{\beta}{n_L n_R e^2 \nu} \int_0^\infty dt e^{izt} (\hat{F}_L, e^{-i\mathcal{Q}Lt} \hat{F}_R). \tag{10}$$

It is easy to demonstrate that ρ_{LR} is identically zero in the absence of interlayer coupling, since the forces in left and right layers are uncorrelated. The leading contribution to the force operator from interlayer interactions can be expressed in terms of the interlayer interaction potential $U_e(q)$ and the electron density $\varrho(q)$

$$\mathbf{F}_{R(L)} = \pm \frac{i}{\nu} \sum_{\vec{q}} \vec{q} \varrho_L(\vec{q}) \varrho_R(-\vec{q}) U_e(q). \tag{11}$$

To leading order in interlayer interactions $e^{-i\mathcal{Q}Lt} \hat{F}_i$ in Eq. (10) can be replaced by $e^{-iLt} \hat{F}_i$. This replacement leads to the desired force-force correlation function expression for the trans-resistance

$$\rho_{LR}(z) = \frac{\beta}{n_L n_R e^2 \nu} \int_0^\infty dt e^{izt} (F_L(t), F_R)_0. \tag{12}$$

The subscript on the inner product in Eq. (12) indicates that it should be evaluated in the absence of interlayer interactions. Substituting the explicit expression for the interlayer forces gives

$$\rho_{LR}(z) = \frac{\beta}{2n_L n_R e^2 \nu^3} \sum_{\vec{q}} q^2 \int_0^\infty e^{izt} dt |U_e(q)|^2 (\hat{A}(t), \hat{A}^\dagger(0))_0 \tag{13}$$

with

$$\hat{A}(t) = \varrho_L(-\vec{q}, t) \varrho_R(\vec{q}, t). \tag{14}$$

The correlation function for decoupled layers appearing in Eq. (13) is related to the isolated layer density fluctuations. Using a representation of exact eigenstates that in the $z = 0$ limit it is easy to show that

$$\rho_{LR} = \frac{\pi\beta}{2n_L n_R e^2} \int \frac{d^2\vec{q}}{(2\pi)^2} q^2 |U_e(q)|^2 \int_{-\infty}^{\infty} d\omega S_L(\vec{q}, \omega) S_R(-\vec{q}, -\omega) \quad (15)$$

where $S_i(\vec{q}, \omega)$ is the dynamic structure factor for layer i :

$$S_i(\vec{q}, \omega) \equiv \frac{1}{\nu} \sum_{n,m} \exp(-\beta E_n) |\langle n | \rho_i(\vec{q}) | m \rangle|^2 \delta(\omega - (E_m - E_n)/\hbar) \quad (16)$$

It is usually more convenient to express the resistance in terms of individual layer response functions rather than the dynamic structure factor. We relate $S_i(q, \omega)$ to the retarded density-density response function for layer i , $\chi_i(\vec{q}, \omega)$ by applying the fluctuation-dissipation theorem [17]

$$S_i(q, \omega) = \frac{\hbar}{1 - e^{-\hbar\omega\beta}} \text{Im}\chi_i(\vec{q}, \omega). \quad (17)$$

This gives us the final form of our expression for the drag resistivity, which is summarized diagrammatically in Fig. (1).

$$\rho_{LR} = \frac{\hbar^2\beta}{\pi n_L n_R e^2 \nu} \sum_{\mathbf{q}} q^2 |U_e(q)|^2 \int_0^{\infty} d\omega \frac{\text{Im}\chi_R(q, \omega) \text{Im}\chi_L(q, \omega)}{e^{\beta\hbar\omega} + e^{-\beta\hbar\omega} - 2}. \quad (18)$$

III. DISORDER AND SCREENING

The electron density-density response function $\chi(q, \omega)$ in Eq. (18) can be obtained by applying many-body perturbation theory methods to a 2DEG whose Hamiltonian contains disorder and (or) interaction terms. For a non-interacting disorder-free 2DEG the response function can be evaluated analytically [25]. Disorder leads to an enhancement in $\text{Im}\chi(q, \omega)$ at low frequencies and long wavelengths and, as we discuss below, can enhance the interlayer friction. The enhancement reflects the increased spatial correlation of states with nearby energies in disordered systems. The effect of disorder on the friction can be described without making a specific model of disorder by invoking the Einstein relation between the conductivity and the diffusive density-density response at long wavelengths and low frequencies [26]. We introduce a phenomenological intralayer electron (transport) scattering time τ . τ is related to the mobility by $\mu = e\tau/m$ and at low temperatures is related to the mean-free-path

by $l = \tau \hbar k_f / m$. For $ql > 1$ or $\omega\tau > 1$ we assume that disorder is unimportant and approximate the density-density response function by the non-interacting electron result [25]. At zero temperature and zero disorder

$$\begin{aligned} \chi_i(q, \omega) &\equiv \chi_i^B(q, \omega) \\ &= \frac{dn}{d\mu} \frac{m}{q^2} \left\{ \frac{q^2}{m} - C_+ |(k_f q / m)^2 - (\omega + \varepsilon_q)^2 / \hbar^4|^{1/2} - C_- |(k_f q / m)^2 - (\omega - \varepsilon_q)^2 / \hbar^4|^{1/2} \right\} \end{aligned} \quad (19)$$

where $\varepsilon_q = \hbar q^2 / 2m$, μ is the chemical potential and the C_{\pm} are:

$$\begin{aligned} C_{\pm} &= \text{sign}(\varepsilon_q \pm \omega) & \text{if } (k_f q / m)^2 - (\omega \pm \varepsilon_q)^2 / \hbar^2 < 0, \\ C_{\pm} &= \pm i & \text{if } (k_f q / m)^2 - (\omega \pm \varepsilon_q)^2 / \hbar^2 > 0 \end{aligned}$$

However for $ql < 1$ and $\omega\tau < 1$ disorder becomes important. In this diffusive regime the electron density-density response function is completely characterized by the diffusion constant [16]:

$$\begin{aligned} \chi_i(q, \omega) &\equiv \chi_i^D(q, \omega) \\ &= \frac{dn}{d\mu} \frac{Dq^2}{Dq^2 - i\omega} \end{aligned} \quad (20)$$

$$q < 1/l \quad \omega < 1/\tau$$

where $D = l^2 / 2\tau$ is the diffusion constant.

The derivation of the expression for the transresistivity in the previous section is valid up to second order in the interlayer interaction. For the system of physical interest the interlayer interaction is Coulombic and it is essential to include screening in order to get qualitatively correct results. In this paper we adopt the usual expediency of employing the second-order expression with the interlayer interaction replaced by a screened interlayer interaction and argue that this includes the most important higher-order effects. In the disorder free limit our expression for the transresistivity then becomes identical to those derived using other approaches in earlier work [5,10,11,12,14]. The random-phase-approximation (RPA) screened interlayer interaction is

$$U_e(q, \omega) = \frac{V_e(q)}{[1 + V_a(q)\chi_L(q, \omega)][1 + V_a(q)\chi_R(q, \omega)] - V_e^2(q)\chi_L(q, \omega)\chi_R(q, \omega)} \quad (21)$$

where the bare intra- and inter-layer electron-electron interaction potentials are $V_a(q) = 2\pi e^2/q$ and $V_e(q) = V_a(q)e^{-qd}$ where d is the separation between the layers. In the above expression either the ballistic or the diffusive form for $\chi_{L(R)}$ should be used as appropriate. Note that the interlayer interaction is cut-off by the factor e^{-qd} for $q > 1/d$. The layer separation dependence of the friction in both diffusive and ballistic limits results from this cutoff. Physically the cutoff reflects the fact that charge fluctuations in one layer with a wavelength shorter than the layer separation get averaged out when viewed from the other layer.

The expression for the trans-resistance of Eq. (18) can be split into contributions from the ballistic and diffusive regimes. With $\rho_{LR}^{-1} \equiv n_R e^2 \tau_{LR} / m$, we have

$$\tau_{RL}^{-1} = \tau_B^{-1} + \tau_{\Delta}^{-1} \quad (22)$$

where

$$\tau_B^{-1} = \frac{\hbar^2 \beta}{2\pi^2 m n_L} \int_0^\infty dq q^3 \int_0^\infty d\omega |U_e|^2 \frac{\text{Im}[\chi_L^B(q, \omega)] \text{Im}[\chi_R^B(q, \omega)]}{e^{\beta\hbar\omega} + e^{-\beta\hbar\omega} - 2} \quad (23)$$

and

$$\tau_{\Delta}^{-1} = \tau_D^{-1} - \frac{\hbar^2 \beta}{2\pi^2 m n_L} \int_0^{\frac{1}{d}} dq q^3 \int_0^{\frac{1}{d}} d\omega |U_e|^2 \frac{\text{Im}[\chi_L^B(q, \omega)] \text{Im}[\chi_R^B(q, \omega)]}{e^{\beta\hbar\omega} + e^{-\beta\hbar\omega} - 2} \quad (24)$$

with

$$\tau_D^{-1} = \frac{\hbar^2 \beta}{2\pi^2 m n_L} \int_0^{\frac{1}{d}} dq q^3 \int_0^{\frac{1}{d}} d\omega |U_e|^2 \frac{\text{Im}[\chi_L^D(q, \omega)] \text{Im}[\chi_R^D(q, \omega)]}{e^{\beta\hbar\omega} + e^{-\beta\hbar\omega} - 2} \quad (25)$$

τ_B^{-1} is the result for a disorder-free 2DEGs and τ_{Δ}^{-1} is the correction due to the the enhanced fluctuations at long wavelengths and low frequencies in disordered systems. In the next section we discuss the evaluation of these expressions.

IV. NUMERICAL RESULTS AND DISCUSSION

The dependence of the interlayer scattering rate on temperature and on layer separation depends on whether the interlayer scattering is dominated by τ_B^{-1} or τ_D^{-1} . In Fig. (2) and

Fig. (3) we show numerical results for τ_B and τ_Δ as a functions of temperature calculated for two different values of layer separation for a high mobility two-dimensional electron gas sample. The data in these figures are obtained from numerical evaluation of Eq. (23) and Eq. (25) with the input parameters taken from the experiment of ref. [5]. In Fig. (2) we see that $\tau_B^{-1} \sim T^2/d^4$ at low temperatures, as pointed out in Ref. [5]. (The d^{-4} dependence can be recognized by noticing that the scattering rate decreases by a factor of approximately four when the layer separation increases by a factor of $\sqrt{2}$.) From Fig. (3) one can see that τ_Δ^{-1} falls off more slowly with both temperature and layer separation as $T \rightarrow 0$. The inset to Fig. (3) establishes that for $T \ll T_\tau$, $\tau_D^{-1} \sim -T^2 \ln T/d^2$. The dimensionless temperature scale for Fig. (2) is the Fermi temperature ($T_F \equiv E_F/k_B$) which is about 60K for this sample, while the dimensionless temperature scale for Fig. (3) is the disorder temperature ($T_\tau \equiv \hbar/k_B\tau$) which is $\sim 56\text{mK}$ for this sample. Comparing Fig. (2) and Fig. (3) we see that for this high-mobility sample the disorder correction is smaller than one part in 10^5 at temperatures above $\sim 10\text{mK}$.

The origin of the temperature and layer dependence seen in Fig. (2) and Fig. (3) can be understood by looking at the limit of large layer separations where the interlayer scattering rates can be evaluated analytically. The evaluation of τ_{LR}^{-1} at large layer separations and low temperatures for the disorder-free limit, where the relation $\tau_B \propto T^2/d^4$ holds, has been carried out previously by several authors [5,10,11]. (The results quoted in Ref. ([5]) and Ref. ([10]) are in error by a factor of two.) We rederive those results here to allow a comparison with the disordered case. For $T \ll T_F$ and $d \gg k_F^{-1}$ only the low frequency and long wavelength limit of $\text{Im}\chi$ contributes importantly to Eq. (18). (k_F is the Fermi wavelength.) From Eq. (19) it follows that in this limit $\text{Re}\chi(q, \omega) = dn/d\mu$, and $\text{Im}\chi(q, \omega) = dn/d\mu(2\hbar\omega/E_F)(k_F/q)$. For $T \ll T_F$, we can ignore the contribution of $\text{Im}\chi$ to screening the interlayer interaction and it follows from Eq. (21) that we can replace the interlayer interaction by

$$U_e(q) = \frac{\pi e^2 q}{k_{TF}^2 \sinh qd} \quad (26)$$

where $k_{TF} \equiv 2\pi e^2 dn/d\mu$ is the single layer Thomas-Fermi screening wavevector. Note that for $q \rightarrow 0$ the effective screening wavevector is $2k_{TF}^2 d$, which is proportional to the layer separation. (For GaAs $k_{TF} = 0.2\text{nm}^{-1}$ independent of electron density.) With these approximations the integral over frequency and wavevector are known and we obtain

$$\tau_B^{-1} = \frac{-\pi\zeta(3)(k_B T)^2}{16\hbar\varepsilon_f(k_{TF}d)^2(k_F d)^2}. \quad (27)$$

In this result two powers of d^{-1} may be associated with the enhanced screening of the interlayer interaction at large separations and two powers of d^{-1} with the combination of phase space considerations which cause the integrand to vary as q^1 for small q . For layer separations larger than the mean free path the electron response is diffusive over the entire range of wavevectors contributing importantly to Eq. (18). For low temperatures it follows from Eq. (20) that we may use $\text{Re}\chi(q, \omega) = dn/d\mu$ and $\text{Im}\chi(q, \omega) = dn/d\mu[\omega Dq^2/(\omega^2 + (Dq^2)^2)]$. Again we may ignore the contribution to screening from $\text{Im}\chi$ so that the relevant limit of the screened interlayer interaction is unchanged. At small ω , $\text{Im}\chi \propto q^{-2}$ compared to the q^{-1} of the ballistic case. The integrand of the wavevector integral thus goes as q^{-1} at small wavevector. This logarithmically divergent wavevector integral is cutoff at $q \sim \omega/D^{1/2}$. The remaining frequency integral is elementary and we obtain

$$\tau_D^{-1} = \frac{-\pi(k_B T)^2 \ln(T/T_\tau)}{12\hbar\varepsilon_f(q_{TF}d)^2(k_F l)^2}. \quad (28)$$

The change in the layer separation from d^{-4} in the ballistic case to d^{-2} in the diffusive case can be traced directly to the change in the wavevector dependence of $\text{Im}\chi$ from q^{-1} to q^{-2} .

For $d \ll l$ Eq. (28) correctly gives the contribution to the drag from $q \ll l^{-1}$ and Eq. (27) gives the contribution from $l^{-1} < q < d^{-1}$. Because of the different temperature dependence it is still true that the contribution from the diffusive regime will dominate at sufficiently low temperatures. Comparing Eq. (28) and Eq. (27) we can estimate the crossover temperature:

$$T_c \sim T_\tau \exp[-3(l/d)^2/4\zeta(3)] \quad (29)$$

For high-mobility samples the diffusive enhancement of the drag will be observable only at extremely low temperatures. For example, in GaAs T_τ is about 0.2K and $l \sim 10\mu$ for a

sample with a mobility of $\sim 10^6 \text{cm}^2/\text{sV}$ and a typical density. For a layer separation of $\sim 500\text{\AA}$ this implies that $T_c \sim 10^{-100} \text{K}$. For the samples in the experiment of ref. [5], which are of extremely high mobility and small layer separations, the value of T_τ is 56mK and the d/l is about 10^{-3} . τ_Δ^{-1} is smaller than τ_B^{-1} by a factor of $10^{6\sim 7}$ at $T \sim T_\tau$. The correction term, τ_Δ^{-1} , will be difficult to observe for accessible temperatures. In samples with lower mobility and/or thicker barriers between the layers, τ_Δ^{-1} and τ_B^{-1} have comparable amplitudes for $T \sim T_\tau$ and the contribution from the diffusive regime dominates at lower temperatures. In Fig. (4) we plot the relative contribution to the drag from the diffusive regime vs. mobility for a layer separation of 50nm and $n = 1.5 \times 10^{11} \text{cm}^{-2}$. These results show that the effect of disorder will become easily observable at typical low temperatures for samples with mobilities below $\sim 10^5 \text{cm}^2 \text{V}^{-1} \text{s}^{-1}$.

It is possible to fabricate double-layer systems in which one layer is much more disordered than the other. In particular one may have $l \gg d$ for one layer and $l \leq d$ for the other layer. Following the same steps leading to Eq. (28), it is possible to derive an expression for the low-temperature transresistance in such a system by using one diffusive response function and one free electron response function:

$$\tau_{LR}^{-1} = \frac{\pi^3 (k_B T)^2}{72 \hbar \varepsilon_f (k_{TF} d)^2 (k_F d) (k_F l)}. \quad (30)$$

The dependences on temperature and layer separation, $\tau_{LR}^{-1} \sim T^2/d^3$, are easily understood by comparing it to Eq. (27) and Eq. (28) and noticing that the integrand in this case approaches a constant at small wavevector transfers. In Fig. (4), results are shown for the relative correction due to disorder enhancement for the case where one layer consists of free electrons while the other layer has a finite mobility. The relative correction is essentially independent of temperatures at low temperatures and it is weaker than in the case where both layers are disordered.

V. SUMMARY

Using the memory-function method, we have derived an expression for the transresistance of double layer systems which is sufficiently general to treat the case of disordered layers. The expression has been evaluated as a function of temperature, layer separation (d) and in-plane mobility μ . Both the case where only one layer is disordered and the case where both layers are disordered have been considered. We find that the drag varies as d^{-4} , d^{-3} and d^{-2} at low temperatures for clean, single-layer disorder, and double-layer disorder cases respectively. In the case of double-layer disorder the transresistance varies as $-T^2 \ln(T)$ at low temperatures, otherwise the transresistance is proportional to T^2 . The low-temperature drag is proportional to μ^{-2} and μ^{-1} for double-layer disorder and single-layer disorder respectively. Except for the case of extremely low temperatures the crossover from clean to disordered regimes occurs when the mean-free-path within a layer becomes smaller than the layer separation. In very high mobility two-dimensional electron gas systems, where the transresistance has been studied experimentally up to the present, the effect of disorder on the drag is negligible at available temperatures. We predict that the enhancement due to disorder and the associated crossovers in the temperature-dependence and layer-separation dependence will be observable at low temperatures in moderate and low mobility samples.

ACKNOWLEDGMENTS

The authors are grateful for stimulating interactions with T.J. Gramila and J.P. Eisenstein and with Jun Hu and Anthony Chan. Informative discussions with B.I. Altshuler during the initial stages of this work and helpful communications with A.P. Jauho, A.G. Rojo and G.D. Mahan are also acknowledged. This work was supported by the National Science Foundation under Grant No. DMR-9113911.

REFERENCES

- [1] For a brief review of theoretical work see A. H. MacDonald, Surf. Sci. **229**, 1(1990) and references therein. For recent experiments see Y.W. Suen, L.W. Engel, M.B. Santos, M. Shayegan, and D.C. Tsui, Phys. Rev. Lett. **68**, 1379 (1992); J.P. Eisenstein, G.S. Boebinger, L.N. Pfeiffer, K.W. West, and Song He, Phys. Rev. Lett. **68**, 1383 (1992).
- [2] A. H. MacDonald, P. M. Platzman and G. S. Boebinger, Phys. Rev. Lett **65**, 775(1990); L. Brey, Phys. Rev. Lett. **65**, 903 (1990). See also H.A. Fertig, Phys. Rev. B **40**, 1087 (1989).
- [3] L. Swierkowski and D. Neilson, Phys. Rev. Lett **67**, 240(1991).
- [4] T. Fukuzawa, E.E. Mendez, and J.M. Hong, Phys. Rev. Lett. **64**, 3066 (1990) and references therein.
- [5] T. J. Gramila, J. P. Eisenstein, A. H. MacDonald, L. N. Pfeiffer and K. W. West, Phys. Rev. Lett **66**, 1216(1991).
- [6] T.J. Gramila, J.P. Eisenstein, A.H. MacDonald, L.N. Pfeiffer, and K.W. West, Surface Science **263**, 446 (1992); T.J. Gramila, J.P. Eisenstein, A.H. MacDonald, L.N. Pfeiffer, and K.W. West, Phys. Rev. B, to appear (1993).
- [7] U. Sivan, P. M. Solomon and H. Shtrikman, Phys. Rev. Lett **68**, 1196 (1992). U. Sivan, P.M. Solomon and H. Shtrikman, preprint (1993).
- [8] M. B. Pogrebinskii, Fiz. Tekh, Poluprovodn. **11**, 637 (1977) [Sov. Phys. Semicond. **11**, 372(1977)].
- [9] P. J. Price, Physica **117/118 B+C**, 750(1983).
- [10] P. M. Solomon and B. Laikhtman, Superlattices and Microstructure **10**, 89 (1991).
- [11] A. P. Jauho and Henrik Smith, Phys. Rev. B, to appear (1993).
- [12] J. D. Mahan, private communication, 1991.

- [13] A. G. Rojo and G. D. Mahan, Phys. Rev. Lett **68**, 2074(1992).
- [14] H.C. Tso and P. Vasilopoulos, Phys. Rev. B **45**, 1333 (1992); H.C. Tso, P. Vasilopoulos, and F.M. Peeters, Phys. Rev. Lett. **68**, 2516 (1992); I.I. Boiko, P. Vasilopoulos, and Yu. M. Sirenko, Phys. Rev B **45**, 13538 (1992); Yu. M. Sirenko and P. Vasilopoulos, Phys. Rev. B **46**, 1611 (1992); H.C. Tso, P. Vasilopoulos, and F.M. Peeters, preprint 1993.
- [15] B.L. Altshuler and A.G. Aronov, Solid State Commun. **39**, 115 (1979).
- [16] Patrick A. Lee and T.V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985) and work cited therein.
- [17] See for example D. Forster, in *Hydrodynamic Fluctuations, Broken Symmetry, and Correlation Functions* (Addison-Wesley,1990).
- [18] Lian Zheng and A. H. MacDonald, unpublished.
- [19] A.H. MacDonald, T.J. Gramila, and J.P. Eisenstein, preprint (1993).
- [20] H. Mori, Progr. Theort. Phys. (Kyoto) **34**, 423 (1965).
- [21] Y. Shiwa and A. Isihara, J. Phys. C, **16**, 4853(1983).
- [22] ICda Cunha Lima and S. C. Ying, J. Phys. C, **18**, 2887(1983).
- [23] See for example the related discussion in G.D. Mahan *Many-Particle Systems*, (Plenum, New York, 1990).
- [24] Lian Zheng and A.H. MacDonald, unpublished.
- [25] F. Stern, Phys. Rev. Lett. **18**, 546 (1967); P.F. Maldague, Surface Science **73**, 296 (1978).
- [26] For detailed discussions of this point see M.T. Béal-Monod, Phys. Rev. B **47**, 3495 (1993); R.N. Bhatt, P. Wölfle, and T.V. Ramakrishnan, Phys. Rev. B **32**, 569 (1985).

FIGURES

FIG. 1. Schematic diagrammatic rendering of the memory function expression for the transresistivity to lowest order in interlayer interactions. (Eq. (12))

FIG. 2. The interlayer scattering rate for a disorder-free 2DEG samples as a function of temperature. $t \equiv T/T_F$ where T_F is the Fermi temperature. The solid curve was obtained for $d = 315\text{\AA}$ and the dashed curve was obtained for $d = \sqrt{2} \times 315\text{\AA}$. These curves were calculated for typical and equal densities in the two layers: $n_L = n_R = 1.5 \times 10^{11} \text{cm}^{-2}\text{V}$ so that $T_F \sim 60\text{K}$.

FIG. 3. The correction to the interlayer scattering rate due to disorder-enhanced interactions at long wavelengths and low temperatures. Here $t = T/T_\tau$. The insert shows that $\tau_D^{-1} \propto T^2 \ln T / d^2$ at low temperatures. The solid curve is for $d = 315\text{\AA}$ and the dashed curve is for $d = \sqrt{2} \times 315\text{\AA}$. Other parameters used are taken from the experiments of Ref. ([5]): $n_L = n_R = 1.5 \times 10^{11} \text{cm}^{-2}$, and $\mu = 3.4 \times 10^6 \text{cm}^2/\text{sV}$. The corresponding $T_\tau = 56\text{mK}$ so that disorder corrections would not be observable at available temperatures.

FIG. 4. Relative correction to the interlayer scattering rate due to disorder-enhanced interactions for several temperatures as a function of sample mobility. These results were obtained for a typical layer separation: $d = 500\text{\AA}$. The dashed line is for $T = 1.0\text{K}$, the dotted line for $T = 0.1\text{K}$ and the solid line for $T = 0.01\text{K}$. The density was taken from the experiments of Ref. ([5]). For these temperatures the diffusive contribution become important for mobilities below about $10^5 \text{cm}^2\text{s}^{-1}\text{V}^{-1}$. The marked lines show the result for the case where one layer has a finite mobility while the other layer has an infinitively large mobility.