

A History of Network Synthesis and Filter Theory for Circuits Composed of Resistors, Inductors, and Capacitors

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Abstract—This paper is an informal history of the birth and growth of network synthesis and filter theory, as it was developed for *RLC* circuits. It includes events, experiences, and anecdotes which are not all well documented but may make interesting reading. Other papers in this issue are histories of other aspects of circuit theory; for the most part descended from the *RLC* theory.

INTRODUCTION

THIS is an invited paper on network synthesis and filters. Other historical papers by other authors, in this issue, cover active circuits, digital circuits, nonlinear circuits, etc. Therefore, in order to avoid duplications, this paper is concerned primarily with linear, time-invariant, *RLC* circuits. However, a good deal will be said about the influence of *RLC* theory on other disciplines, and vice versa.

General network synthesis as we know it now was first developed for *RLC* circuits. Vacuum tube circuits were relatively simple *circuitry* (interstage circuits in vacuum tube amplifiers, simple feedback in oscillators and “regenerative” radio receivers, etc.). There were no operational amplifiers, transistors, gyrators, impedance converters, etc. Thus network synthesis theory for *RLC* circuits is an ancestor from which more general synthesis theories are descended.

Belevitch published a very fine history of circuit theory in the May 1962, 50th Anniversary Issue of the Proceedings of the IRE [1]. He listed many key contributions and contributors, with years of publication but without specific journal references. He also noted that there have been many other contributors worthy of mention, excluded for the sake of brevity and readability.

I cannot possibly improve on Belevitch’s objective, scholarly history of circuit theory up to 1962. Since 1962, there have been important advances in *RLC theory* but they have been relatively few except for computer applications (which belong in the paper on CAD in this issue). Most of the important advances in circuit theory have concerned circuits of other kinds, such as active, switched, or digital circuits.

Also, when I try to write an impersonal, scholarly history, such as Belevitch’s, I call up a flood of personal memories, recorded in my mind’s technical database during more than half a century. I grew up expecting to be an engineer, and

chose EE (probably to be different from my father, who was an ME). I was converted from power engineering to communication by a course on transients in tuned coupled circuits taught by George Washington Pierce at Harvard (1927 or 1928), and was introduced to Campbell–Zobel filters as part of a course on transmission lines, taught by Ernie Guillemin at MIT (1929, a year or so before he became deeply involved in general network synthesis). When I joined Bell Laboratories in 1929, my first boss was Ed Norton (Norton’s theorem, and much more). My second boss (starting in the mid-1930’s) was Hendrik Bode. Thus my early formative years in communication engineering pretty much coincided with the early formative years of Network Synthesis.

So, this paper is a less scholarly, more subjective history. It includes events, experiences, and anecdotes which are not all well documented, but may make interesting reading. It also includes personal thoughts about the influence of circuit theory on those who have studied it, as well as on the evolution of other kinds of engineering.

An entire book could be written on the history of *RLC* circuits, network synthesis, and filter theory, with technical explanations, journal and patent references, and interactions with other disciplines. Its preparation would take more journal and library research than I could contemplate, more than could easily be justified.

Some readers may disagree with some of the selections of references to people and events. They are very personal choices from deserving references so numerous that they can only be sampled. Some details taken from memory may be in error in small ways, particularly as regards events of long ago. There are few surviving witnesses of the high and far off times when filter theory and network synthesis were born. Campbell, Zobel, Cauer, Guillemin, and many others died many years ago. Bode died about two years ago and Norton more like a year ago. Among the survivors, a notable example is Foster. During a recent telephone conversation, he disclosed that he was working on a long paper on topology (about 60 years after his famous “reactance theorem”) [2]. For briefer references to more people, events, and dates, not mentioned in this paper, see [1].

The term “network” is used here mostly in “network synthesis,” which is historically most familiar. Elsewhere, the synonym “circuit” has been preferred. Belevitch uses “network” specifically for idealized circuit models [1, p. 849 (footnote)]. But now we have also so many “networks” of computers, TV stations, communication channels, etc.

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The rest of the paper is divided into the following sections:

Before Network Synthesis
 Proto-Network Synthesis
 General Network Synthesis—Realization Techniques
 General Network Synthesis—Approximation Techniques
 Synthesis of Circuits with Prescribed Insertion Losses
 Loss-Phase Relations and Feedback Amplifiers
 Further Developments of Circuit Theory
 Now Versus Then
 Hereafter.

BEFORE NETWORK SYNTHESIS

“Circuit Analysis” determines characteristics of given circuits. “Network Synthesis” is the inverse. It determines circuits with given (desired) characteristics. Development of circuit analysis was a prerequisite for the development of network synthesis. First the performance of the individual components had to be formulated (“modeled”, in modern terms). The result was Ohm’s law for resistors, current derivatives for inductors, voltage derivatives for capacitors, etc. Then the operation of interconnected components had to be formulated. For linear time-invariant *RLC* circuits, Kirchhoff’s laws led to simultaneous linear differential equations in voltages and/or currents, with constant coefficients.

A mathematical theory of such differential equations was of course well known, and also applications to classical dynamics. An understanding of the differential equations as they apply to circuits included such concepts as steady state versus transient responses, the superposition theorem, Thevenin’s and Norton’s theorems, and the reciprocity theorem. The use of complex numbers made computations of steady-state responses much easier, and the frequency variable $s = j\omega$ was important in the analysis of transients and the later development of synthesis techniques.

The development of submarine cables had an important influence on the development of circuit theory. Initially, transmission lines such as submarine cables were poorly understood. Misconceptions led to serious practical difficulties. An important early contributor to an understanding of transmission lines, and other aspects of circuit theory, was Oliver Heaviside. A good biography of Heaviside was published recently by Nahin [3]. Initially, Heaviside’s work was not generally accepted and he was pretty much forgotten until the latter 1920’s. In his course on transients in tuned coupled circuits, in 1927 or 1928, G. W. Pierce did not include the Heaviside expansion theorem. In 1929, people at MIT were reading a new book on Heaviside’s operational calculus. (As I recall, it was an early edition of a book by Vanivar Bush.) But George Campbell was always interested in Heaviside [2], and was a very important, very early (born 1870) contributor to the reduction to practice of Heaviside’s mathematics [7].

An understanding of transmission lines brought in such concepts as propagation constants (attenuation and phase), matched impedances, and reflections at nonmatching impedances. Lumped loading (initially proposed by Heaviside) was an ancestor of image parameter filters. Grounded versus balanced lines and “phantom” circuits brought in the concepts

of three-terminal versus balanced two-ports and longitudinal versus transverse currents.

PROTO-NETWORK SYNTHESIS

Webster’s Dictionary defines “proto” as “first in time,” or “primitive.” Here we mean circuit design techniques used before the development of general network synthesis (and for many purposes thereafter, and even until now). Successful design techniques were developed for a number of special purposes. The examples noted below are mostly from the 1910’s and 1920’s. (But loaded lines were demonstrated as early as 1900 [7].)

Filter theory evolved first from loaded lines. A line combines the effects of distributed inductance and capacitance. (Inductive loading increases the Q of the effective inductance, which is usually much lower than the Q of the capacitance.) Distributed loading increases the Q of the distributed inductance itself (e.g., by magnetic material wrapped around the conductors). Lumped loading approximates the effect by lumped inductors inserted in the line at uniformly spaced points. Heaviside proposed both distributed and lumped loading. His proposals were not generally accepted, because it was thought that the increased inductance would retard the transmission of signals [3]. Actually, of course, it does retard the transmission, but this is only an increase in delay, with a decrease in delay distortion.

Later both Campbell (of AT&T) and Pupin filed for patents on lumped loading. Since the concept and principle had been published by Heaviside, it is not clear just what they claimed—probably reduction to practice. Pupin won the patent suit and sold exclusive rights to AT&T. “Loading coils” very substantially increased the distance over which it was practical to make telephone calls before the advent of vacuum tube repeaters (but only after the development of improved inductors, entailing considerable experimentation [7]).

It is not at all surprising that continuous loading can be replaced by lumped loading if the loading coils are close enough together. Continuous loading is the limit as the size and spacing of the inductors are reduced to zero. The engineering question is: How far apart can they be placed? Transmission characteristics of loaded lines were analyzed in order to answer such questions. It turned out that a sinusoidal signal traveling along a nondissipative loaded line suffers zero attenuation if its frequency does not exceed a “cutoff frequency,” related to the spacing and size of the loading coils. At higher frequencies the signal is attenuated. Thus a loaded line is a low-pass filter. It is a simple step to reduce the distributed inductance to zero, leaving only lumped inductances. It is then reasonable to add lumped capacitive loading and to reduce the distributed capacitance to zero.

The result is a lumped *LC* low-pass filter (with all its transmission zeros at infinite frequencies). It has a “ladder” configuration: alternate “series branches” in the line and “shunt branches” across the line. Campbell also disclosed all-pass lattice sections (1920–1922) [1]. Artificial lines and attenuators were developed quite early [1] and at least some used ladder configurations [7].

Campbell and Wagner independently invented this sort of filter during World War I [1]. In Campbell's case at least, the history was more complicated, roughly as follows:¹ In 1911, AT&T filed a patent application on Campbell's filter theory, without reduction to practice. In 1913, a proposal to build and test a filter was dropped, because it appeared to be "of no great consequence physically." In 1915, Frank Jewett (Assistant Chief Engineer at AT&T and later the first President of Bell Laboratories) wrote a letter urging the importance of securing "the maximum of patent protection." The letter included the following two remarks: "The matter is highly technical and involves considerable mathematical physics relating to electrical networks, a subject few engineers even have mastered," and, as a closing remark: "You may not agree with me in this suggestion, but I feel strongly that we will some time experience the pleasures of a morning after if the matter is allowed to lapse."

The filter theories included the concept of filter sections, image impedances, and image attenuation and phase. In these terms, the filters derived from loaded lines may be described as tandem connections of identical elementary sections. Since the image impedances were matched between sections, the image attenuation and phase of the overall filter were the sums of those of the individual sections. Otto Zobel invented simple filter sections with the same image impedances and pass- and stopbands as the original iterative filters, but with different image attenuation and phase characteristics. In particular, his "*m*-derived" sections gave sharper cutoffs, and attenuation peaks at arbitrary finite frequencies. By dividing his *m*-derived sections in the middle, he obtained image impedances at the ends of filters which varied less over passband frequencies, thereby reducing terminal reflection losses.

At the same time as the development of filter theory, the general concept of two-ports was introduced in Germany and France. Corresponding impedance, admittance, and chain matrices (2×2 matrices of frequency functions) were introduced and used to compute corresponding matrices of series, parallel, and cascade connected two-ports [1].

There was now a scientific basis for developing a technique for filter design. Additional filter sections were devised, not only for low-pass filters but also for high-pass, bandpass, and bandstop filters. For a while, a circuit theorist might seek to raise his visibility by inventing still another filter section. Over the years, a large body of art was added to the science, the result was a large assortment of practical filters meeting many practical requirements.

In the 1930's a more systematic approach to image parameter filter theory was introduced by Bode (1934) and Piloty (1937–1938) and by Cauer. Here one looks at the relation between the open- and short-circuit impedances of the whole filter and the image impedances and attenuation and phase functions. The image impedances are identified with the square root of the product of the corresponding open- and short-circuit impedances: $Z_I = \sqrt{Z_0 Z_s}$. The image impedances are real over theoretical passbands and imaginary over stopbands. This requires zeros and poles of Z_0 or Z_s to be canceled by poles

or zeros of Z_s or Z_0 inside passbands, and zeros and poles of Z_0 and Z_s to be identical when they occur inside stopbands. The arbitrary locations of the matched singularities determine the way in which the image impedances, as well as the image attenuation and phase, vary with frequency.

Tuned circuits are also a simple kind of (bandpass) filter. They were essential to the development of radio receivers. Tuned coupled circuits can give fairly flat passbands, and cascades of such circuits are useful when cutoffs need not be very sharp and requirements on passband flatness are not very severe. In radio receivers, passbands were positioned on the frequency scale by ganged variable capacitors, and, later, by tuning the heterodyne oscillators in Armstrong's superheterodyne receivers.

Circuits other than filters were also developed. Equalizers came into use for various purposes. Sally Pero (later Sally Pero Mead) designed a very early special-purpose equalizer. As I recall, it was a simple one-port across the line in the receiver for a submarine telegraph cable, designed to permit transmission of telegraph signals at faster rates. I do not know whether it did more to correct phase or amplitude distortion. Telegraph signals were pulses, transmitted at quite low rates. On land lines, pulses were regenerated by electromechanical relays. Under an ocean, repeaters could not be used and distortion limited the feasible number of pulses per second.

Otto Zobel's "constant resistance equalizer sections" displaced other more rudimentary equalizers. Since their image impedances are independent of frequency, constant resistance sections can be connected in cascade and terminated in resistors, with no reflections. Constant resistance all-pass lattice sections permit phase equalization and approximations to constant delays (but are balanced four-terminal, not grounded three-terminal circuits).

One of the early uses of artificial delay lines was in a hydrophone system developed by George Washington Pierce. (He taught a course in hydrophone engineering at Harvard, reputed to be the only such course in the world.) Pierce's system was an electrical improvement on the purely acoustical hydrophone used in World War I. In modern terms it was a passive sonar using a two-element array receiver. Direction measurements required a (permissibly piecewise) variable delay. Pierce used an iteration of simple filterlike three-terminal sections. Each section comprised a pair of coupled series inductors and a capacitor from their common point to ground. This is a low-pass filter section with an "*m*" > 1, which puts the transmission zero at a real *s* (imaginary *f*). A suitable coefficient of coupling gives a more linear phase than uncoupled inductors (*m* = 1). As I recall, Pierce's patent was bought, or licensed by the Bell System.

Telephone engineers encountered an increasing need for multiport transformers, or combinations of transformers. Examples are antisidetone hybrid coils, separation of phantom and ordinary balanced lines (longitudinal versus transverse currents), and connections between two-way and pairs of one-way transmission systems. In 1920, Campbell and Foster published a paper on optimum energy relations in resistance terminated nondissipative four-ports [7]. Belevitch [1] describes this as probably the first publication on network

¹According to (possibly imperfect) copies of two internal AT&T letters given to Foster by Campbell many years ago.

synthesis in its true sense—including biconjugacy of the networks, enumeration of all realizations, and circuits composed explicitly of ideal transformers. This all used, of course, the concept of equivalent circuits.

Foster once recounted the following story concerning the enumeration of the four-port combination of ideal transformers: There could be various numbers of transformers with various numbers of windings, connected to each other and the various four-ports in various ways. At first, Campbell and Foster found a number of circuits which looked useful and asked their patent department to patent them. The patent attorneys replied that there was no use patenting some of the circuits if there were other, equivalent circuits (which could be used to circumvent the patent). They should patent either all or none. So Campbell and Foster set out to enumerate all the equivalent circuits. They arrived at such a large number that it was deemed not feasible to patent all (83, 539 distinct circuits [7]). So it was decided to publish the enumeration, thereby excluding the possibility that somebody else might patent one or more specific embodiments which they might want to use.

Ideal transformers are not only useful in theory. They are actually realized in many bandpass filter designs, by means of the circuit equivalence illustrated in Fig. 1. The left-hand circuit in the Fig. is a three-terminal two-port “L” comprising a series branch with impedance Z_1 followed by a shunt branch with impedance Z_2 . The ratio Z_1/Z_2 must be constant (independent of frequency), but Z_1 and Z_2 may individually be proportional to a function of frequency. The circuit is exactly equivalent to a shunt branch with impedance Z'_2 followed by a series branch with impedance Z'_1 followed by an ideal transformer. Suppose Z'_1 and Z'_2 are part of a desired filter and it is desired to add the ideal transformer. The same effect can be realized by replacing the combination by branches Z_1 and Z_2 . The transformer can be moved to the other end of the corresponding “L” if Z'_1 and Z'_2 are each divided by r^2 . For transmission from left to right in the circuits as shown in the figure, it is always a step-down transformer. For a step-up transformer turn each of the two circuits around. When Z'_1/Z'_2 is larger than required for a desired transformer ratio, the series impedance and/or the shunt admittance can be divided into two parts, only one of which is used in the circuit of Fig. 1.

Realizing ideal transformers by “turning around L’s” is a common expedient in a bandpass filter design. I learned about it first from Norton, but do not know whether he was the first to discover it. Many other circuit equivalences are, of course, also used in filter design, such as equivalent “T’s” and “II’s” and “bridged T” equivalents of some lattices.

GENERAL NETWORK SYNTHESIS—REALIZATION TECHNIQUES

General network synthesis theories come in two parts: realization techniques and approximation techniques. The first evolved in the 1920’s and early 1930’s, to describe more general approaches to circuit design than the more specialized techniques described in the previous section.

Different realization techniques are needed for different kinds of circuits, but in general they differ only in the details of a common set of principal parts. The first “complete” realization technique concerned LC one-ports. All possible LC

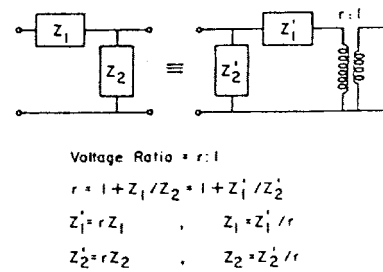


Fig. 1. Realization of an ideal transformer.

one-ports may be regarded as an abstract class of circuits. The various impedances produced by the circuits in the circuit class may be regarded as an abstract class of functions (of frequency). Foster’s reactance theorem (1924) established necessary and sufficient conditions which define the function class in mathematical terms.

Corresponding to each function in the function class (except the simplest), there are a number of “equivalent” LC one-ports with identical impedances but different configurations. One or another kind of configuration can be chosen to define a “canonical” subclass of the circuit class such that it can be used to realize any impedance function in the function class. Four well-known canonical LC one-ports are: parallel-connected series resonances, series-connected parallel resonances (Foster), “Ladders” with series L’s and shunt C’s, and vice versa (Cauer). For any function in the function class, the component sizes for any one of the four canonical circuits can be found by straightforward computations. Belevitch [1] notes that Foster’s reactance theorem (1924) was “a transition from analytical dynamics to modern network synthesis,” but that “the first paper dealing explicitly with the realization of a one-port whose impedance is a prescribed function of frequency is Cauer’s 1926 contribution.” But Foster recalls correspondence with Cauer between 1924 and 1926, concerning Cauer’s dissertation, which was his 1926 paper [2].

Realization techniques for many other kinds of circuits follow similar patterns. Table I lists the principal parts in general terms. (See also Foster’s paper on Theoretical Aspects of Circuit Theory [4].)

Brune published his realization for RLC one-ports in 1931 [1]. For item 3 in Table I he introduced the concept of positive-real functions, as important in itself as his canonical configuration, which frequently uses many mutual inductances. Later, configurations were found which avoid mutual inductances but at a cost of extra components (Bott and Duffin, 1949).

Foster’s LC one-port theorem was soon extended to LC two-ports. The first papers assumed symmetrical two-ports, but in 1931 Cauer solved the general problem for all LC two-ports and even for LC n-ports [1]. Cauer’s canonical circuits included many mutual inductances, and other undesirable features. More useful equivalent circuits were found later.

Since then, as practical needs and available components have proliferated, many other realization techniques have been derived. Reference [5] is a survey of realization techniques, published in 1955. It includes a table of 21 complete realization

TABLE I

1.	A class of circuits (component types, number of ports, sometimes restrictions on configurations).
2.	A corresponding class of functions of frequency or time (immittances, or matrices of immittances, or impulse responses, etc.)
3.	Necessary and sufficient conditions which define the function class in mathematical terms.
4.	A canonical subclass of the circuit class, which covers the function class.
5.	A straight forward procedure for computing the component sizes for the canonical circuit corresponding to any given function in the function class.

techniques, as described above. They differ as to the circuit class, defined by kinds of components and kinds of circuit configurations (one-ports, two-ports, n -ports, three-terminal or balanced circuits, mutual inductances permitted or not). They also differ as to the characteristics represented by the function class (driving-point immittances, complete immittance matrices for two-ports or n -ports, transfer immittances, insertion loss only, impulse response time functions, etc.).

The table is by no means an exhaustive list of the realization techniques known in 1955, and of course more have been devised since that time.

There are other realization techniques which are more or less incomplete (as defined above). An example is three-terminal ladder filters free from mutual inductances. The function class includes most practical filter functions. Some sufficient conditions for no mutual inductances are known, and some conditions under which they are more likely to be needed. But complete necessary and sufficient conditions defining the function class are either not known or, at least, not in a form that is easily applied.

GENERAL NETWORK

SYNTHESIS—APPROXIMATION TECHNIQUES

In most circuit design problems there are ideal characteristics which can only be approximated. In terms of Table I, no functions in the function class match exactly the ideal external characteristics. Then the problem is to find a function in the function class which best approximates the ideal, according to one or another criterion. The best known approximation techniques are those applied to filter design, assuming that ideal filters have perfectly flat passbands and infinite attenuations over stopbands.

In 1930, Butterworth used the maximally flat approximation in the design of multistage amplifiers [1]. A maximally flat approximation makes a maximum number of derivatives zero at a particularly important frequency, mid-band for tuned coupled circuits, zero for low-pass filters. At about the same time, or perhaps somewhat earlier, Norton applied the maximally flat concept to the design of electromechanical and mechanical-acoustical devices.

Norton's work had to do with electrically powered recorders for cutting wax master phonograph records and improved mechanical-acoustical record players. (Bell Laboratories developed these for the Victor Talking Machine Co.) Norton designed the amplifier-to-cutting stylus and the needle-to-exponential horn mechanisms in terms of equivalent electrical circuits. (Masses are modeled by inductors, compliances by capacitors, and damping means by resistors.) The electrical circuit analogs had the configuration of low-pass filters, terminated at one end only. Norton designed the filters for maximally flat response. I believe the "Orthophonic Victrolas" were marketed before Butterworth's paper, but from a remark of Norton's, which I barely recall, his maximally flat designs may have been for a later model.

In one way at least, Norton's work was symbolic of the coming of age of circuit theory. Early circuit analysis drew heavily on the previous analytical dynamics of vibrating mechanical systems. Norton may have been the first to turn the analog around, to design mechanical systems in terms of the more advanced circuit theory.

In 1931, Cauer introduced into circuit theory the use of "approximations in the Chebyshev sense," for which "minimax approximations" is a simpler modern term (the maximum error is minimized) [1]. He derived realizable Chebyshev approximations to constant image impedances over arbitrary portions of theoretical passbands. He also derived Chebyshev approximations to infinite attenuation (minimum attenuation maximized) over arbitrary portions of theoretical stopbands (the same mathematical problem except for a change of variable). In each instant, they assumed an arbitrary number of arbitrary parameters, determined by (or determining) the complexity of the filter.

At Bell Laboratories, a number of us first learned about some of Cauer's canonical circuits and his Chebyshev approximations at a conference on Cauer's proposed sale of some of his patents. It was an important event in my professional life.

Cauer's patents stated his Chebyshev formulas without proof. Sergei Schelkunoff soon provided us with a proof. At the same time, he conceived of a new general theorem (not needed for that specific problem), which applies to all Chebyshev problems with "equal ripple" solutions. I am still seeking more new applications.

SYNTHESIS OF CIRCUITS WITH PRESCRIBED INSERTION LOSSES

Image parameter filters are somewhat restricted by the required cancellation or coincidence of most of the zeros and poles of the open- and short-circuit impedances, inherent in the method. The actual insertion loss is not chosen directly, but by correcting the chosen line-type attenuation for reflection and interaction (multiple reflections) at the terminations.

An alternative is to specify the insertion-loss function directly, and to determine a circuit therefrom. The general insertion-loss theory was preceded by Norton's constant-resistance filter pairs (published in 1937, but invented in the very early 1930's). Norton's pairs of filters were connected in parallel (or series) at one end. Together, they offered a constant resistance at their common end. Norton showed that each filter

acted like a single filter terminated by a resistance at one end only (and by an open- or short-circuit at the other). He chose insertion-loss functions suitable for his constant resistance pairs of filters, found therefrom the open- or short-circuit impedances of the filters at their terminated ends, and derived a step-by-step procedure for calculating components in ladder configurations from the open- or short-circuit impedances.

More general insertion-loss theories were developed independently by Cocci (1938–1940), Darlington (1939), Cauer (1939–1941), and Piloty (1931–1941) [1]. The general theory is a sequence of several major parts. It starts with the following realization technique, in the sense of Table I: The circuit class is the class of all LC two-ports inserted between input and output resistors. The frequency function of interest is the “power ratio” $|V_2/V_m|^2$, where V_2 is the voltage across the output resistor and V_m is the maximum possible V_2 (corresponding to an impedance-matching transformer between the two resistors). The function class is the class of all such functions corresponding to circuits in the circuit class. Necessary and sufficient conditions on the functions require $|V_2/V_m|^2$ to be a nonnegative rational function of the square of the frequency, nowhere > 1 .

Open- and short-circuit impedances of a corresponding LC two-port can be found from any such function. Finding relations for this was part of the development of the general theory. It can be shown that the impedances meet Cauer’s conditions on LC two-ports. Then Cauer’s canonical LC two-port becomes one canonical circuit for the general insertion-loss theory.

Cauer’s canonical LC two-port was convenient for providing the necessary and sufficient conditions on the insertion-loss functions, but it is unsatisfactory for most practical applications. A second major part of the insertion-loss theory determined an alternative canonical LC two-port consisting of cascade connected simple “sections.” For a general LC two-port, sections of four kinds may be required. Series one-ports, shunt one-ports, a “ T ” of a pair of coupled series inductors separated by a shunt capacitor, and a section with twice as many components but required only for transmission zeros at complex frequencies. (For complete generality one must also include an ideal transformer at one end of the two-port, not needed in the insertion-loss theory if the ratio of the terminating resistances is subject to designer’s choice). Most practical filters can be arranged in a ladder configuration of alternating series and shunt one-ports, with no mutual inductances.

A new canonical RLC one-port was found as a corollary to the realization technique. It has only one resistor, but it generally requires some redundant reactive components and some coupled inductors. It has been of some use for frequency-dependent impedance matching.

The next major part of the insertion-loss theory concerns approximation techniques for insertion-loss filters. For efficient use of components, frequencies of zero loss and infinite loss are usually chosen to be real. Then, the power ratio $|V_2/V_m|^2$ can be written $1/(1 + JF^2)$ in which F is either an odd or an even function of frequency. The frequencies of zero loss are the zeros of F and the transmission zeros are the poles

of F . Increasing (positive) scale factor J increases stopband losses at a cost of increased loss variations over passbands. The zeros and poles of F and scale factor J can be chosen arbitrarily. Then it is a simple matter to compute the insertion loss at any frequencies.

Incidentally, when F is odd corresponding filters are (externally) symmetric; when F is even they are antimetric.

Chebyshev or maxflat F ’s correspond to Chebyshev or maxflat insertion losses. For a Chebyshev or maxflat passband F is a Chebyshev or maxflat approximation to zero over the passband frequencies; for a Chebyshev or maxflat stopband $1/F$ is a Chebyshev or maxflat approximation to zero over the stopband frequencies. Chebyshev pass- and stopbands can both be realized in the same filter. Design techniques were found for low-pass and high-pass filters and others derived therefrom. They use Jacobian elliptic functions different from, but similar to those used by Cauer for image parameter filter functions.

Design techniques for other combinations of pass- and stopband characteristics were found, including maxflat pass- and stopband, Chebyshev passband and maxflat stopband, maxflat passband and Chebyshev stopband, and Chebyshev passband and arbitrary stopband transmission zeros.

Procedures were also developed for computing component sizes from power ratio functions. First the poles of $|V_2/V_m|^2$ are found, and from them the complex frequency function V_2/V_m and the complex reflection coefficients at the terminating resistors. From these, open- and short-circuit impedance functions of the LC two-port are easily formulated. Finally, the filter components are computed from one or more of the impedances, using basically Norton’s step-by-step procedure. (It was later refined and improved by various circuit theorists, especially to reduce sensitivities to computational roundoff errors, which can be high.)

My version of the insertion-loss theory included a fourth part. Bode discovered a simple transformation on the frequency variable whereby one can compute the effect of replacing all nondissipative L ’s and C ’s by dissipative L ’s and C ’s all of the same Q . It turned Bode’s transformation around, to get a “predistorted” voltage ratio for nondissipative L ’s and C ’s such that the desired voltage ratio is obtained (except for a flat loss) when the L ’s and C ’s are replaced by dissipative components all of the same (specified) Q .

As with image parameter filters, the scientific skeleton had to be fleshed out by a good deal of art for practical design. The insertion-loss filter design method was not generally accepted until some years later. It had to compete with established image parameter methods, used by experienced, highly skilled filter designers. The filters obtained by the insertion-loss method were generally better, but not very much better than image parameter counterparts. The lengthy computations required for insertion-loss designs were a serious drawback.

The advent of computers and of non- LC filters changed the situation. For example, the design of cascade connected biquadratic RC -op-amp filters does not require the lengthy computation of ladder components for insertion-loss filters and does require the computation of the poles of resistance terminated image parameter filters.

The dates with the references cited above were publication dates. Considering the extensive nature of the insertion-loss theory, each of the publications was probably preceded by studies carried out over a fairly long interval. Mine started in the early 1930's. My initial inspirations were Norton's constant resistance filters; Foster, Cauer, and Brune's realization techniques; and Cauer's Chebyshev approximations for image parameter filters. By 1933 I knew how to compute symmetric and antisymmetric Chebyshev filters, but (I believe) I had not yet proved the canonical nature of LC two-ports between terminating resistors. In 1936, I wrote an extensive set of notes for internal Bell Laboratories use (more than 400 pages of typing). It included almost, but not quite all, the 1939 publication (and a lot more details). The manuscript of the publication was received by the editors of the *Journal of Mathematics and Physics* in May 1938, but was not published until there was room for it in the September 1939 issue.

The Jacobian elliptic functions needed for simultaneously Chebyshev pass- and stopbands were described in various 19th century texts. In the New York City, NY, library one could consult Jacobi's original paper published in Latin in 1829. Remarkably, there were foldouts which tabulated elliptic function transformations, including transformations directly applicable to both image parameter and insertion-loss Chebyshev approximations.

LOSS-PHASE RELATIONS AND FEEDBACK AMPLIFIERS

A quite different important part of network synthesis concerns feedback amplifiers (and analogous automatic control systems). Harold Black invented negative feedback amplifiers with net gain around the feedback loop. Nyquist provided his criterion, which shows whether a given amplifier will be stable. But that is circuit analysis. Design of a stable feedback amplifier remained a network synthesis problem.

The key to the problem lay in the now famous loss-phase relations. They permit the phase to be computed from a given loss, or vice versa, by certain integrals, assuming a stable circuit. In particular, they can be used to relate loss and phase around an opened feedback loop, a preliminary to application of the Nyquist criterion. Belevitch cites various early publications on the mathematics of the loss-phase relations. Others might be included.

In the middle 1930's, efficient feedback amplifiers had to be developed for the repeaters in the first coaxial cable telephone transmission system. We had a paper on the loss-phase relations published by Wiener and Lee in the *Journal of Mathematics and Physics* (I don't recall the date). We recognized the probable importance of the relations, but as published they had two difficulties. The mathematics indicated multiple solutions, but without physical interpretation. The relations were in the form of integrals which did not give an insight into how the loss affects the phase. Bode removed both difficulties. He identified and defined "minimum phase" and showed that all other permitted phases differed from it by "all-pass" phases. He transformed the classical loss-phase integrals into equivalent integrals which gave the needed insight. This entailed a change in the variable of integration from frequency to log frequency, followed by an integration by parts. The

result was the now famous decibels per octave rule. The minimum phase at a given frequency is a weighted average of the derivative of the loss with respect to log frequency over all frequencies.

Starting with the decibels per octave rule, Bode developed an extensive synthesis theory and art for designing feedback amplifiers. It included ideal loop gain characteristics, restrictions on feedback and/or bandwidth due to forward circuit asymptotic behavior, integrals defining "resistance efficiency," etc., and variable equalizers needed for coaxial cable systems. Some of this work used classical function theory generally unfamiliar to circuit engineers of that time.

Efficient amplifiers were designed by Bode's method well before World War II. His classical book on feedback, published in 1945, is little different from notes he wrote for a Bell Laboratories course which he taught before the war.

FURTHER DEVELOPMENTS OF CIRCUIT THEORY

Further developments of circuit theory in general, and network synthesis in particular became increasingly numerous, with exceedingly complicated interrelations. As a measure of circuit theory activity Belevitch [1] uses the number of circuit theory papers published per year: < 1 before 1910, 5 to 25 in 1920 to 1940, > 100 after 1954. In a 1973 paper on "leap frog" active filters, Szentirmai notes: "In the last two decades, the question of how to realize a prescribed rational function [of frequency]... by RC active structures was the subject of more than a thousand learned papers." [6]

New developments in circuit theory exploited new developments in other disciplines: new components, new communication needs, new computer needs, etc. Conversely, new developments in circuit theory and practice sparked new developments in other disciplines. They are too numerous to describe in detail in this paper. Anyway, it is more interesting to look at the interactions in more general terms.

Table II compares the growth of various disciplines in four columns. It does not include all possible items. It merely illustrates the complexity of the growth. In the table, time increases downward, not necessarily linearly. The vertical position of an item is supposed to represent when it first became important to circuit theory (on a not necessarily linear time scale). But such times are extremely fuzzy. Many items grew to importance over a long period, from a small start.

It is tempting to model the growth of circuit theory in terms of other systems [8]. One model is a tapestry. Looking at it from the front, different viewers perceive different patterns. From the back, one sees only a tangle of knotted threads.² A more dynamic model is a river representing circuit theory, fed by tributaries representing other technologies. The tributaries have raised circuit theory from a trickle to a wide river with many interacting currents. But can computers be described as a mere tributary to circuit theory?

Other possibilities are more homocentric. One is genealogical. Present-day circuit theory may be regarded as a descendent from many ancestors of which previous generations of circuit theory are only the male line. Other ancestors included compo-

²With apology to Thornton Wilder. In his novel *The Eighth Day*, he compares interrelations among people and events to a tapestry, in similar terms.

TABLE II
(THE ITEMS ABOVE THE DASHED LINE ARE EARLIER DEVELOPMENTS WHICH CONTRIBUTED TO THE BIRTH OF MODERN CIRCUIT THEORY)

Physics Needed for Component Development	Lines (poorly understood)		Mathematics
	Loaded Lines (proposed)		Computational Skills
<u>Components</u>	<u>Linear Circuit Theory</u>	<u>Filters</u>	<u>Digital Computers</u>
L, C, R	Circuit Analysis	Iterative	
Vacuum Tube		Image Param. Sections	Mechanical (Desk Comps.)
Electromechanical	Realization Techs.		
	Optimization Techs.	General Image Param.	
	Topological	Crystal Filters	
	Insertion Loss Th.	Insertion Loss	
Vacuum Tube Op. Amp.	Feedback		
	More General Circuits (Active, etc)	More General Kinds of Frequency Functions	Electromechanical (Relays, etc.)
Transistor		Active Analog	Vacuum Tube
Transistor Op. Amp.	State Space		Software
Solid State Devices	Computer Aided Design	Computer Aided Design	
	Digital Circuits	Digital	Transistor
Integrated (Logic and Linear)	A to D & D to A	AD-Digital-DA	
		Integrated	Integrated
L.S.I.		L.S.I.	L.S.I.
V.L.S.I.		V.L.S.I.	V.L.S.I.
	Switched Capacitor	Switched Capacitor	Small Comps. & Calcs. Microprocessors

ment technologies, computers, etc. But the genealogical model poses difficult decisions as to who begat whom. (Can A be a parent of B at the same time that B is a parent of A?)

The best homocentric model may be sociological (or anthropological): a number of tribes whose cultures are enriched by frequent intermarriages. For example, there is the tribe which worships circuit theory and the tribe which worships computers. Many members of each tribe are unrelated to the other. (And many of the real people working in each tribe know and care little about the growth of the other.) But all the tribes have benefitted enormously by the numerous intermarriages. And there has certainly been a population explosion.

The following example may be of interest: Everybody knows that computers have had a tremendous effect on circuit theory. An early cross fertilization in the other direction is less well known. In the late 1930's, George Stibitz invented an electrical computer out of telephone switching components. It had a "terminal" in the form of a teletypewriter. Numbers and instructions were typed in and the computer typed out the answers. In 1940 it was demonstrated with the "terminal" at Hanover, NJ, and the "CPU" in a closet at Bell Laboratories in New York City. The machine performed only the simple arithmetic of complex numbers, but its design introduced several novel ideas and concepts used ever after in larger computers.³

³For this invention, Stibitz was recently inducted into the "National Inventor's Hall of Fame."

Stibitz' design of a computer specifically for complex number arithmetic was no accident. At Bell Laboratories, where Stibitz was working, circuit development departments had a well-established need for better means to do complex number arithmetic. They used a great deal of routine complex number arithmetic in circuit analysis, for checking the responses of new circuit designs. The complex number arithmetic was performed by sequences of positive real arithmetic operations on desk computers. The work was tedious, and difficult to perform without accidental errors. (It was done by women called "computresses"; those were sexist days.)

It is obvious that a number of items in Table II deserve more detailed individual histories. Some are covered by other papers in this issue (active circuits, digital circuits, etc.). A few others are mentioned briefly below.

World War II inspired a rapid development of special-purpose signal processing circuits and optimization theory. A small sampling is: *LC* pulse generators for radar transmitters; optimum pulse shapes (theory) for noise reduction in radar receivers ("North filters," etc.); range measuring and sweep circuits for radar displays; invention of operational amplifiers (Och and Swartzel); *LC*-op-amp smoothing and prediction circuits for analog antiaircraft fire control computers. The last item inspired Norbert Wiener's famous World War II paper on smoothing and extrapolation of Gaussian signals. Rudy Kalman's smoothing and prediction in state-space terms came later and finds many applications. Most of the wartime development of high-frequency electronic skills was, of course,

a great boon to the post-war development of television. A patent by Dietzold cited *RC*-single op-amp circuits as a means for realizing general transfer functions.

Kalman also sparked a widespread interest in state-space applications to classical circuit analysis and synthesis. Actually, state-space concepts had been around for a long time. Early in the history of circuit theory one talked about “degrees of freedom” and “natural modes.” Heaviside’s expansion theorem was, in fact, a state-space application to the computation of *RLC* circuit impulse responses. Mathematicians had long converted scalar differential equations of order n into vector differential equations of order 1. Different state-space applications use different state spaces (linearly related in the case of linear circuits). Modern interest in state-space circuit theory began with a space in which there is a one-to-one correspondence between dimensions in the space and circuit components. This usually starts with a singular matrix, but Bashkow showed how it could be reduced to his nonsingular “*A* matrix.”

The *concept* of transversal filters dates well before the time suggested in Table II. In the early (or possibly mid-1930’s). Wiener and Lee patented a circuit for approximating an arbitrary function of frequency. (See also the reference to Fourier series in [1].) It comprised a cascade of identical all-pass sections, with the signal input at one end. Outputs were taken off between the sections, multiplied by arbitrary constant factors, and added. The output was described as a truncated Fourier series in a transformed frequency variable (determined by the frequency response of the individual sections). However, transversal filters did not receive much attention until wide-band *time* functions became important, as in radar, television, and high-speed data transmission.

A substantial interest in topological aspects of circuit theory started around the mid-1930’s. Foster was an early enthusiast in the field, and his interest has continued even to now [2]. The topology usually concerns circuits with no transformers or mutual inductances, frequently n -ports. For further history, see the paper on topology in this issue.

Scattering parameters were developed initially for distributed components such as lines, but they were applied effectively also to lumped component circuits, including insertion-loss filters as well as impedance matching.

The effect of circuit theory training on people may also have been significant. Many one-time circuit theorists have gone on to system theory. Ideally, tables like Table I can also be an objective in a great many other engineering design fields. But usually, practical complications or theoretical difficulties leave the ideals much farther from reality. Circuit theory seems to have been particularly satisfying, perhaps because the theoretical models are quite close to reality and the mathematical problems are solvable in not too difficult ways. It can be a valuable illustration of an important ideal (particularly when taught by great teachers like Ernie Guillemin).

On the other hand, I can remember over-enthusiastic circuit theorists who seemed to think that a system organization like Table I was new to engineering. Here is a very elderly counter example. Consider a spacecraft in orbit around a planet (device class). Sir Isaac Newton found differential equations

for the motions of celestial objects due to gravitational forces (our function class is the special case of the two-body problem with one body much smaller than the other). For two bodies, he showed that the orbits must be either elliptical, parabolic, or hyperbolic and found formulas for dimensions versus velocities, etc. (necessary and sufficient conditions defining our function class in mathematical terms). Spacecraft of different weights describe the same orbits as long as their weights are small compared to that of the planet (any one can be a canonical subclass). Finally, centuries later came the “Hohmann transfer ellipse.” the most efficient orbit for transferring a spacecraft from one elliptical orbit to another.⁴

NOW VERSUS THEN

In 1939, at MIT, electrical communication was Department VIC, as opposed to ordinary electrical engineering, Department VI. Filter theory happened to be taught that year—at the end of a one semester course on communication transmission lines. Now, the electronics of communication, computers, and control overshadow power engineering. It is no longer feasible to teach thoroughly all significant aspects of electronics in a four-year engineering curriculum.

In the 1920’s and 1930’s it was not too difficult to keep track of the publications in circuit theory. Now, a single IEEE Circuits and Systems Society’s “International Symposium” can generate a symposium “Proceedings” of more than a thousand pages.

In the 1920’s one was lucky to find a single kind of circuit meeting a design need (for filters: resistance-terminated *LC* ladders). Now one has to choose between a number of quite different circuit kinds (for filters: *RLC*, electromechanical, *RC*-op-amp (in any of numerous configurations), digital, switched capacitor, transversal, microwave, etc.). The best design for a given application depends on a balance between numerous practical considerations.

On the other hand, a filter of a particular sort may combine design theories developed for several others. Here is an example. Start with an *RLC* ladder configuration. Find the component sizes per the Chebyshev (elliptic function) approximation to an ideal filter. Replace the *L*’s by *RC*-op-amp equivalents. Replace all *R*’s by switched capacitors. The switched-capacitor filter samples the signal. The switches can be so arranged that the “bilinear *Z* transform” of digital filter theory applies. In the original *RLC* design, use the bilinear *Z* transfer to adjust the filter cutoffs. Finally, integrate the entire switched-capacitor filter on a single chip.

Three different approximation techniques are available for obtaining filters with (more or less) flat passbands and high-loss stopbands: image impedance filters, insertion-loss filters, and tuned coupled circuits. Insertion-loss filters generally make somewhat more efficient use of the components. For *RLC* designs, they require substantially more complicated computations, but computers are generally available. But for

⁴I learned about the Hohmann transfer ellipse through association with some space projects. It was probably included in a German book by Hohmann in 1925, mentioned in C. C. Adams’ book *Space Flight*, New York, McGraw-Hill, 1958, p. 18.

some popular RC -op-amp configurations computations are about the same for insertion-loss and image parameter filters.

Pairs of tuned coupled circuits have been known, of course, for a very long time. A generalization is a tandem connection of more than two tuned circuits with coupling between each tuned circuit and the next. The coupling can be either inductive or capacitive. I am told, on good authority, that useful filters are frequently obtained by “tweaking” all the capacitors in a capacitively coupled cascade of more than two tuned circuits. The configuration comprises shunt inductors separated by “T’s” of capacitors (equivalent to “II’s” of capacitors). All the capacitors are adjusted experimentally to obtain a satisfactory (not necessarily optimum) filter characteristic. The filters are appropriate for quite narrow passbands which do not have to be very flat, and stopbands which do not have to give very high losses too close to the passbands. Equally satisfactory designs using fewer components are likely to be possible, but may not be worth the greater design and adjustment effort. The difference in performance between “tweaked” and “ideal” designs probably increases very rapidly as the number of tune circuits increases.

Insertion-loss filter theory gives some insight into tweaked filter possibilities. Suppose the passband is quite narrow. All the inductors except those at the ends of the filter can have any values, within certain limits, with no change in the external filter characteristics. This is because the capacitors can be tweaked to produce the equivalent of ideal impedance transformers (per Norton). An insertion-loss filter (of nondissipative L 's and C 's) with the maximum number of real frequencies of zero loss requires equal inductors at the ends of the filter (for equal terminating resistors). If both end inductors differ from the design by equal, small amounts, the filter can be adjusted for a slightly different bandwidth. Corresponding to slightly unequal end inductors there will be insertion-loss designs which include an added constant loss. The problem is not the existence of capacitor adjustments for the insertion-loss characteristic, but how to find the adjustments.

Applications of analog filter theory to digital filters depend on the nature of the digital filters. The analog approximation theory can be applied directly to infinite impulse digital filters (rational functions of “ Z ”) by means of the bilinear Z transform. But the bilinear Z transform is usually not appropriate for finite impulse response filters (polynomials in Z). The *concepts* of maxflat, Chebyshev pass- or stopbands, and elliptic filters still apply but applications are more difficult. Also, for finite impulse response (FIR) filters there are competing concepts and techniques, notably Fourier series and “windows.”

HEREAFTER

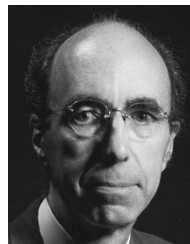
In our increasingly digital future, an increasingly large portion of our filters will undoubtedly be digital. For many analog filters it may become increasingly economical to use digital

filters between aD, DA converters, integrated on single chips. Perhaps a few programmable filters of this sort will replace many individually designed filters, the way microprocessors have dominated other applications. But analog filters will surely still be needed for *some* purposes, depending on the frequency range, filter requirements, nature of the signals, number of units to be manufactured, etc.

Anyway, the birth and growth of network synthesis and filter theory, during the last half century and more, has been a rewarding experience for many participants and has been important to the birth and growth of electronics as we know it now. It was my great good fortune to choose communication engineering, more or less by chance, at a time when nobody could possibly foresee its evolution, via electronics, into the present technical and social revolution.

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In 1929 he joined the Technical Staff of Bell Telephone Laboratories. He retired from Bell Laboratories in 1971 but stayed on as a part time Consultant for the next three years. At the time of his retirement, he was a Department Head in the Mathematics Research Center. Since 1971, he has been an Adjunct Professor of Electrical Engineering at the University of New Hampshire, Durham, NH, where he has occasionally taught a graduate course on filters. He was a Visiting Lecturer at the University of California, Berkeley, during the 1960’s, for periods of 1 to 6 weeks, and was a Visiting Lecturer at the University of California, Los Angeles, during November, 1978. His interests have included circuit theory, communication systems, analog computers, data smoothing, radars, rocket guidance, and design of flight paths for spacecraft. He has published numerous papers and been awarded 35 patents.

Dr. Darlington is a past Chairman of the IRE Circuit Theory Group. He received the IEEE Edison Medal in 1975 and the IEEE Medal of Honor in 1981. He was elected to the National Academy of Engineering in 1975 and to the National Academy of Sciences in 1978.