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Development of Linear Programming Technique for Multidimensional Analysis of Preference in Fuzzy Environment

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Abstract

Due to the nature of real world problems, the collected data usually involve some kind of uncertainty. As a matter of fact, many parts of information cannot be quantified due to their nature. Incomplete information or partial ignorance is also another cause for resorting to fuzziness. In many decision making problems, fuzzy values are approximated by crisp data. This defuzzification leads to loss of main portion of information. In this paper we develop "crisp LINMAP method" into fuzzy environment. A nonlinear programming decision model is developed based on the distance of each alternative to an unknown fuzzy positive ideal solution. Then the fuzzy positive ideal solution and the weights of attributes are estimated and ranking order of alternatives is obtained using these values. The significant features of developed model are decision maker's ideal solution considered as fuzzy variable with triangular possibility distribution and pair-wise comparisons between alternatives have degree of truth in the interval [0,1]. To illustrate the algorithm developed here, two numerical examples are presented.

Keywords: fuzzy decision making, LINMAP, fuzzy distance, linguistic variables, positive ideal solution

1 Introduction

MADM problems arise in many real world situations. A MADM problem is to find a best compromise solution from all feasible alternatives assessed on multiple attributes, both quantitative and qualitative. In these problems, decision maker have to choose one or rank alternatives $A_1, A_2,..., A_n$ based on criteria $c_1, c_2,..., c_m$. Value allocated to i^{th} alternative of j^{th} criteria is denoted with x_{ij} and w_j is the relative weight of attribute c_j where $w_j \geq 0$ (j=1,2,…,m) and $\sum_{j=1}^{m} w_j = 1$. Therefore a MADM problem can be expressed as the following decision matrix [3, 4, 5]

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_m \end{array} \\ \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{pmatrix} \end{array} \tag{1}$$

$$w = [\, w_1 \quad w_2 \quad \cdots \quad w_m \,].$$

Several methods presented to solve above MADM problem. Some of them are based on ideal alternative in the decision maker's opinion such as TOPSIS which developed by Hwang and Yoon [1] and ELECTRE. In the cases where ideal alternative and weight of criteria are not available for decision maker, aforesaid methods are not applicable. The linear programming technique for multidimensional analysis of preference (LINMAP) developed by Srinivasan and Shocker [2] is one of the methods that are more appropriate for this situation. This method uses pair-wise comparisons between alternatives to generate a weight vector and produce ideal alternative which has the least distance from that is in the mind of decision maker [3, 4, 5].

The decision maker usually assesses alternatives with linguistic variables. In addition the ideal alternative in the mind of decision maker has some vagueness. So determination of ideal alternative in the form of crisp numbers is usually illogical. Employment of the fuzzy numbers to determine these values is fairly appropriate. Sadi-Nezhad [6]

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developed LINMAP when DM's preferences are given through pair-wise comparisons with fuzzy values. He introduced two models; the first model was a mixed binary integer programming to define maximum summation of truth degree in preference relation and the second model was a fuzzy goal programming model. Li and Yang [5] have considered values of decision matrix and ideal point of decision maker opinion in terms of fuzzy numbers but with consideration crisp pair-wise comparisons between alternatives, in fact, they have finally formulated a crisp linear programming. Xia *et al.* [7] developed classical LINMAP to solve multi-attribute group decision making problems with fuzzy information on decision matrix. In particular, triangular fuzzy numbers are used in their fuzzy linear programming model to assess alternatives with respect to qualitative attributes. They considered decision matrix information which consists linguistic terms and/or fuzzy values in group decision making environments. Sadi-Nezhad and Akhtari [8] developed LINMAP method in group decision making environments and formulated the problem as a possibilistic programming with multiple objectives. Deng-Feng Li [9] extended the LINMAP method to develop a new methodology for solving multi attribute decision making problems under Atanassov's intuitionistic fuzzy (IF) environments. In this methodology, Atanassov's IF sets were used to describe fuzziness in decision information and decision making processes by means of an Atanassov's IF decision matrix.

In this paper, fuzzy numbers are used to complete the decision matrix. Moreover, ideal point of decision maker is formulated as fuzzy variable with triangular possibility distribution and linguistic variables are employed to pair-wise comparisons between alternatives. Fuzzy numbers used in this paper are in the form of triangular fuzzy numbers. The organization of this paper is as follows. In Section 2, the basic definitions of fuzzy numbers and linguistic variables are defined as well as the fuzzy distance formula and the fuzzy decision making method. In Section 3, the developed LINMAP method in the fuzzy environment is described. For more explanation in Section 4, two numerical examples are stated. Finally in Section 5, conclusion is submitted.

2 Basic Fuzzy Concepts

2.1 Fuzzy Number

A fuzzy number $\tilde{M}$ is a convex normalized fuzzy set $\tilde{M}$ of the real line *R* such that: 1) It exists exactly one $x_0 \in R$ with $\mu_{\tilde{M}}(x_0) = 1$ (x_0 is called the mean value of $\tilde{M}$); 2) $\mu_{\tilde{M}}(x)$ is piecewise continuous.

A fuzzy number $\tilde{M}$ is of *LR*-type if there is reference function *L* (for left), *R* (for right), and scalars $\alpha > 0$, $\beta > 0$ with

$$\mu_{\tilde{N}}(x) = \begin{cases} L\left(\dfrac{m-x}{\alpha}\right) & x \le m \\ R\left(\dfrac{x-m}{\beta}\right) & x \ge m, \end{cases} \tag{2}$$

where *m*, called the mean value of $\tilde{M}$, is a real number , and α and β are called the left and right spread , respectively.

If $\tilde{M}$ is a "triangular fuzzy number", L(x) = R(x) = max (0, 1-x) is implied [10].

2.2 Distance Between Two Triangular Fuzzy Numbers

Let $\tilde{M} = (m_L, m_M, m_R)$ and $\tilde{N} = (n_L, n_M, n_R)$ be two triangular fuzzy numbers. Then the vertex method is defined to calculate the distance between them as follows [5]

$$d_{(\tilde{M},\tilde{N})} = \sqrt{\frac{1}{3}\left[(m_L - n_L)^2 + (m_M - n_M)^2 + (m_R - n_R)^2\right]}. \tag{3}$$

2.3 Linguistic Variable

A linguistic variable is a variable whose values are words, natural or artificial linguistic statements. For example the performance ratings of alternatives on qualitative attribute could be expressed using linguistic variable such as very bad, bad, medium, good and very good [11].

2.4 Fuzzy Decision Making

In 1970 Bellman and Zadeh considered classical model of a decision and suggested a model for decision making in a fuzzy environment.

Assume that we are given a fuzzy goal $\tilde{G}$ and a fuzzy constraint $\tilde{C}$ in a space of alternatives X. Then $\tilde{G}$ and $\tilde{C}$ combined to form a decision $\tilde{D}$, which is a fuzzy set resulting from intersection of $\tilde{G}$ and $\tilde{C}$. In the symbols [10]:

$$\tilde{D} = \tilde{G} \cap \tilde{C}, \mu_{\tilde{D}} = \min\left\{\mu_{\tilde{G}}, \mu_{\tilde{C}}\right\} \quad (4)$$

3 Development of LINMAP Method in Fuzzy Environment

In this method, n alternatives $A_1, A_2, ..., A_n$ and m criteria $c_1, c_2, ..., c_m$ of decision matrix $\tilde{D} = \left[\tilde{x}_{ij}\right]$: $\tilde{x}_{ij} = \left(x_{ij}{}^L, x_{ij}{}^M, x_{ij}{}^R\right)$ have been considered as n points in a m-dimensional space. The ideal alternative is named $A^* = [\tilde{x}_j^*]: \tilde{x}_j^* = \left(x_j^{*L}, x_j^{*M}, x_j^{*R}\right)$ and distance between every alternative A_i and A^* is calculated as follows

$$d_i = d_{(A_i, A^*)} = \sqrt{\frac{1}{3}\sum_{j=1}^{m} w_j \times \left[\left(x_{ij}{}^L - x_j^{*L}\right)^2 + \left(x_{ij}{}^M - x_j^{*M}\right)^2 + \left(x_{ij}{}^R - x_j^{*R}\right)^2\right]}. \quad (5)$$

Suppose $\Omega = \{(k,l)\}$ is a set of pairs (A_k, A_l) that A_k is preferred to A_l. This preference is described in linguistic variable "better ($\succ$)" by decision maker. For example " A_i is better than A_j ". Supposing that decision maker has stated alternative k better than $l\,(k \succ l)$, this implies that $d_l \geq d_k$, but this prediction may be false, so this error is stated as d^-. The amount of errors in pair-wise comparisons of alternatives is stated as B

$$d_{k,l}^- = \begin{cases} d_k - d_l & \mathrm{d}_k > \mathrm{d}_l \\ 0 & \mathrm{d}_k \leq d_l \end{cases} \Rightarrow d_{k,l}^- = \max\{0, d_k - d_l\}, \quad (6)$$

$$\mathrm{B} = \sum_{(\mathrm{k,l}) \in \Omega} d_{k,l}^- . \quad (7)$$

In the opposition of B, a new value called credibility judgment degree is defined between two alternatives k and l, which is denoted by G:

$$d_{k,l}^+ = \begin{cases} d_l - d_k & \mathrm{d}_l > \mathrm{d}_k \\ 0 & \mathrm{d}_l \leq d_k \end{cases} \Rightarrow d_{k,l}^+ = \max\{0, d_l - d_k\}, \quad (8)$$

$$\mathrm{G} = \sum_{(\mathrm{k,l}) \in \Omega} d_{k,l}^+ . \quad (9)$$

Regarding this fact that judgment between two alternatives is described as linguistic statement, therefore membership function (10) is considered to determine credibility degree of decision maker's judgment in relation to preference of alternative k to l (membership function of "better" statement), and membership function (11) is considered to determine incredibility degree of decision maker's judgment (membership function of "worse" statement). These two membership functions are depicted in Figure 1.

$$\mu_{\succ}(d^+) = \begin{cases} 0 & \mathrm{d}_k > \mathrm{d}_l \\ \dfrac{1}{1 + \dfrac{1}{d^{+^2}}} & \mathrm{d}_l \geq \mathrm{d}_k, \end{cases} \quad (10)$$

$$\mu_{\prec}(d^-) = \begin{cases} 0 & \mathrm{d}_l > \mathrm{d}_k \\ \dfrac{1}{1 + \dfrac{1}{d^{-^2}}} & \mathrm{d}_k \geq \mathrm{d}_l. \end{cases} \quad (11)$$

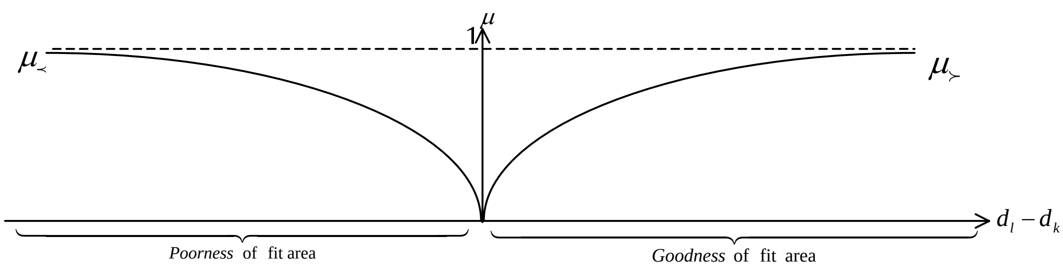

Figure 1: Membership function of goodness of fit and poorness of fit.

Considering $B = \sum_{(k,l)\in\Omega} d_{k,l}^{-}$, we can calculate $\mu_{\prec}(B)$ by the following formula

$$\mu_{\prec}(B) = \mu_{\prec}\Big(\sum_{(k,l)\in\Omega} d_{k,l}^{-}\Big)\,. \tag{12}$$

So we must minimize $\mu_{\prec}(B)$ as far as possible. On the other side, goodness of fit must be greater than poorness of fit ($G \underset{\sim}{\succ} B$), amount of this difference that is called *h*, should be determined by decision maker. In other words, $G - B \underset{\sim}{\geq} h$. Due to the fact that this inequality is in the term of fuzzy one, so membership function of $G - B \underset{\sim}{\geq} h$ is considered as equation (13)

$$\mu_{(G-B)} = \frac{(G-B)-(h-P)}{P} = \frac{\sum_{(k,l)\in\Omega} (\mathrm{d}_l - \mathrm{d}_\mathrm{k}) - (h-P)}{P}\,. \tag{13}$$

The value of *p* should be presented by decision maker. So we must maximize $\mu_{(G-B)}$ in addition to minimizing $\mu_{\prec}(B)$. Instead, we can minimize $1-\mu_{(G-B)}$. So λ is defined in equation (14)

$$\lambda = \max\{\mu_{\prec}(B), 1-\mu_{G-B}\}\,. \tag{14}$$

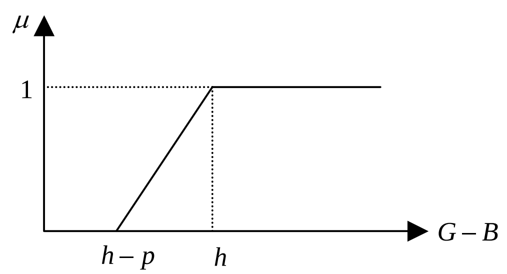

Figure 2: Membership function of *G-B*

Now in order to determine weight vector (w), and ideal alternative (A^*), a fuzzy nonlinear programming (FNLP) model is formulated as below:

$$\min \lambda$$

s.t.

$$\lambda \geq 1\text{-}\mu_{\text{G-B}} \tag{15}$$

$$\lambda \geq \mu_{\prec}(B) \tag{16}$$

$$\sum_{j=1}^{m} w_j = 1 \tag{17}$$

$$x_j^{*L} \leq x_j^{*M} \qquad \forall \mathrm{j} \in \{1,...,\mathrm{m}\} \tag{18}$$

$$x_j^{*M} \leq x_j^{*R} \qquad \forall \mathrm{j} \in \{1,...,\mathrm{m}\} \tag{19}$$

$$B = \sum_{(k,l)\in\Omega} d_{k,l}^{-} \tag{20}$$

$$d_{k,l}^{-} \geq 0 \qquad \forall(\text{k,l}) \in \Omega \tag{21}$$

$$d_{k,l}^{-} \geq d_k - d_l \qquad \forall(\text{k,l}) \in \Omega \tag{22}$$

Equations (15) and (16) satisfy equation (14). Equation (17) asserts weights being normalized. Equation (18) and (19) are considered in order to fulfill format of triangular fuzzy number. Equation (20) determines the value of poorness of fit (B). Equations (21) and (22) satisfy equation (6). In order to more illustration of the proposed algorithm, two numerical examples are offered in the next section.

4 Numerical Examples

Example 1. In order to validate the proposed algorithm in this paper, following example has been chosen from [4]. We have converted crisp numbers of the decision matrix in this example to the form of the triangular fuzzy numbers, and then we have compared the results of the two different approaches. Because of this fact that we are solving a crisp problem with two approaches (crisp and fuzzy LINMAP), it is expected that answers be close to each other.

Suppose following decision matrix is

$$D = \begin{matrix} & C_1 & C_2 \\ A_1 & 0 & 5 \\ A_2 & 5 & 4 \\ A_3 & 0 & 2 \\ A_4 & 1 & 3 \\ A_5 & 4 & 1 \end{matrix} \tag{23}$$

and below pair-wise comparisons between alternatives have been submitted by decision maker:

$$\Omega = \{(1,2),(3,1),(4,1),(5,1),(2,3),(2,4),(2,5),(4,3),(3,5),(4,5)\}, \text{ h=1}. \tag{24}$$

Solution of this problem with crisp LINMAP has lead to below answers:

$$W = \{\frac{1}{3},\frac{2}{3}\}, A^* = \{3,3.5\}. \tag{25}$$

Now, in order to solve the above problem with proposed algorithm, we have converted the decision matrix's data to the form of the triangular fuzzy numbers:

$$\tilde{D} = \begin{matrix} & C_1 & C_2 \\ A_1 & (0,0,0) & (5,5,5) \\ A_2 & (5,5,5) & (4,4,4) \\ A_3 & (0,0,0) & (2,2,2) \\ A_4 & (1,1,1) & (3,3,3) \\ A_5 & (4,4,4) & (1,1,1) \end{matrix}. \tag{26}$$

With consideration of available pair-wise comparisons that presented by decision maker, like (24) and $h= 1$, and with supposing $p= 0.5$, we have solved this problem with the proposed fuzzy nonlinear programming algorithm and below answers have been produced

$$W = \{\frac{1}{3},\frac{2}{3}\}, A^* = \{(3,3,3),(0,0,10.5)\}. \tag{27}$$

As observed, if we defuzzify A^* with existing methods, we exactly achieve the answers in (25).

Example 2. *Y* is one of *X* company's products which is in it's descend age of its life cycle. This company intends to substitute this article with another product. Manager of company aims to select one of products *T*, *S*, *A*, *Z* as a substitution for *Y*. Selection criteria are three attributes: uncovered market demand (C_1), margin of benefit (C_2) and change of technology which used to produce *Y* in order to produce new good (C_3). Decision matrix, *h, P* and Ω have been presented in Table 1.

Obviously, the values of the third parameter are qualitative. Triangular fuzzy numbers for linguistic variables are depicted in Table 2.

Table 1: Decision making data

Alternative \ Criteria	$C_1(\times 10^5)$	C_2	C_3
Z	10	2	low
A	7	2.3	high
S	14	1.9	Too high
T	15	1.5	medium
H=0.5		P=0.1	
$\Omega = \{(Z,A),(Z,S),(Z,T),(A,S),(A,T),(S,T)\}$			

Table2: Fuzzy numbers for linguistic variables [11]

Linguistic variable	Triangular fuzzy number
too low	(0,0,0.3)
low	(0,0.3,0.5)
medium	(0.3,0.5,0.7)
high	(0.5,0.7,1)
too high	(0.7,1,1)

Using the proposed model the ranking order of alternatives obtained as $A>Z>S>T$. Weight vector (w) and fuzzy positive ideal solution (A^*) are resulted as follows:

$$w = (0.178, 0, 0.822), A^* = \{(699998, 699999, 699999), (0.547, 0.782, 0.980), (0.34, 0.35, 1)\}.$$

5 Conclusion

Criteria in a MADM problem may be qualitative and so fuzzy concept could be used to assess the alternatives with regard to attributes. Employments of the fuzzy numbers or the linguistic variables to judge such problems are highly appropriate and increase the stability of answers. In this paper we have tried to develop crisp LINMAP method in the fuzzy environment by utilization of fuzzy concept in decision maker's judgments and employment of the linguistic variables to achieve answers closer to reality. In the conditions where decision maker cannot definitely state his opinion about preference of an alternative to another one and although in conditions where definite assessment of alternatives based on criteria is impossible, this technique can be used to determine weight of criteria and ideal alternative of the decision maker opinion.

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