A minimal bimetric gravity model that fits cosmological observations

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We discuss a particularly simple example of a bimetric massive gravity model which seems to offer a valid alternative to the standard cosmological model. Just like ΛCDM, it depends on a single parameter, has an analytical background expansion law and fits well the expansion cosmological data and the linear perturbation observations. Nevertheless, it is well distinguishable from ΛCDM: its equation of state is $w(z) \approx -1.22^{+0.02}_{-0.02} - 0.64^{+0.05}_{-0.04} z/(1+z)$ for small redshifts and its growth index is approximately $\gamma = 0.47$. The predicted sensitivity of planned cosmological observations is more than sufficient to distinguish this minimal bimetric model from ΛCDM.

Introduction. The history of massive gravity dates back to 1939, when the linear model of Fierz and Pauli was published (see e.g. [1] and [2] for a review). Massive gravity requires the introduction of a second tensor field beside the metric (or some form of non-locality in the action, see [3]). The interaction of the two tensor fields creates a mixture of massless and massive gravitons that apparently avoids the appearance of ghosts [4–7].

In the model introduced in Refs. [8, 9], the second tensor field becomes dynamical, just as the standard metric, although only the latter is coupled to matter (for a generalization, see [10]). This approach, denoted bimetric gravity, keeps the theory ghosts-free and has the advantage of allowing cosmologically viable solutions. The cosmology of bimetric gravity has been studied in several papers, e.g. in Refs. [11–17].

In this paper we select among the class of bimetric models a particularly simple case, that we dub minimal bimetric model (MBM). Just like ΛCDM, this model depends on a single parameter and has an analytical background behavior that is at all times distinguishable from ΛCDM. In a previous paper we have shown that the MBM is the only one-parametric version of bimetric gravity (beside the trivial case in which only a cosmological constant is left) that is cosmologically well behaved and fits well the supernovae Hubble diagram [18] (see also [11, 12]). Here we derive its scalar cosmological perturbation equations in the sub-horizon limit and integrate them numerically, showing that the solution is stable. In this way we are able to derive the modified Poisson equation and the anisotropic stress, two functions of scale and time that completely determine the observational effects at linear level. Finally, we compare the results to a recent compilation of growth data [19]. We find that the MBM fits both supernovae and growth rate data, while remaining well distinguishable from ΛCDM. The MBM is therefore a viable, simple, testable alternative to standard cosmology.

Background equations. We start with the action of the form [8]

$$\begin{aligned} S &= -\frac{M_g^2}{2}\int d^4x\sqrt{-g}R(g) - \frac{M_f^2}{2}\int d^4x\sqrt{-f}R(f) \\ &+ m^2 M_g^2 \int d^4x\sqrt{-g}\sum_{n=0}^{4}\beta_n e_n(X) + \int d^4x\sqrt{-g}L_m \end{aligned} \quad (1)$$

where $X^\alpha_\gamma \equiv \sqrt{g^{\alpha\beta}f_{\beta\gamma}}$, e_n are suitable polynomials, β_n are arbitrary constants and $L_m = L_m(g,\psi)$ is a matter Lagrangian. Here $g_{\mu\nu}$ is the standard metric coupled to matter fields in the L_m Lagrangian, while $f_{\mu\nu}$ is an additional dynamical tensor field. In the following we express masses in units of the Planck mass M_g and the mass parameter m^2 will be absorbed into the parameters β_n. Varying the action with respect to $g_{\mu\nu}$, one obtains the following equations of motion,

$$G_{\mu\nu} + \frac{1}{2}\sum_{n=0}^{3}(-1)^n\beta_n\left[g_{\mu\lambda}Y^\lambda_{(n)\nu}(X) + g_{\nu\lambda}Y^\lambda_{(n)\mu}(X)\right] = T_{\mu\nu} \quad (2)$$

where $G_{\mu\nu}$ is Einstein's tensor, and the expressions $Y^\lambda_{(n)\nu}(X)$ are defined as

$$Y_{(0)} = I, \quad (3)$$

$$Y_{(1)} = X - I[X], \quad (4)$$

$$Y_{(2)} = X^2 - X[X] + \frac{1}{2}I\left([X]^2 - [X^2]\right) \quad (5)$$

$$\begin{aligned} Y_{(3)} &= X^3 - X^2[X] + \frac{1}{2}X\left([X]^2 - [X^2]\right)) \\ &- \frac{1}{6}I\left([X]^3 - 3[X][X^2] + 2[X^3]\right) \end{aligned} \quad (6)$$

where I is the identity matrix and [...] is the trace operator. Varying the action with respect to $f_{\mu\nu}$ we get

$$\bar{G}_{\mu\nu} + \sum_{n=0}^{3}\frac{(-1)^n\beta_{4-n}}{2M_f^2}\left[f_{\mu\lambda}Y^\lambda_{(n)\nu}(X^{-1}) + f_{\nu\lambda}Y^\lambda_{(n)\mu}(X^{-1})\right] = 0 \quad (7)$$

where the overbar indicates $f_{\mu\nu}$ curvatures. Notice that β_0 acts as a pure cosmological constant. Finally, the rescaling $f \to M_f^{-2}f$, $\beta_n \to M_f^n\beta_n$ allows us to assume $M_f = 1$ in the following (see [17]).

We assume now a cosmological spatially flat FRW metric

$$ds^2 = a^2(t)\left(-dt^2 + dx_i dx^i\right) \quad (8)$$

where t represents the conformal time and a dot will represent the derivative with respect to it. The second metric is chosen also in a FRW form

$$ds_f^2 = -\left[\dot{b}(t)^2/\mathcal{H}^2(t)\right]dt^2 + b(t)^2 dx_i dx^i \quad (9)$$

where $\mathcal{H} \equiv \dot{a}/a$ is the conformal Hubble function. This form of the metric $f_{\mu\nu}$ ensures that the equations satisfy the Bianchi identities (see e.g. [9]).

Defining $r = b/a$, the background equations can be conveniently written as a first order system for $r, \mathcal{H}$, using $N = \log a$ as time variable and denoting d/dN with a prime ([18], see also [12]):

$$2E'E + E^2 = a^2(B_0 + B_2 r'), \tag{10}$$

$$r' = \frac{3rB_1\Omega_m}{\beta_1 - 3\beta_3 r^2 - 2\beta_4 r^3 + 3B_2 r^2} \tag{11}$$

where $\Omega_m = 1 - \frac{B_0}{B_1} r$, $E \equiv \mathcal{H}/H_0$ and the couplings β_i are measured in units of H_0^2 and finally

$$B_0 = \beta_0 + 3\beta_1 r + 3\beta_2 r^2 + \beta_3 r^3 \tag{12}$$

$$B_1 = \beta_1 + 3\beta_2 r + 3\beta_3 r^2 + \beta_4 r^3 \tag{13}$$

$$B_2 = \beta_1 + 2\beta_2 r + \beta_3 r^2 \,. \tag{14}$$

Minimal bimetric model. In Ref. [18] we identified the conditions for standard cosmological viability, i.e. for a matter epoch followed by a stable acceleration, without bounces or singularities beside the big bang. We found that among the models with a single non-vanishing parameter only two cases give a viable cosmology, namely the cases with only β_0 or only β_1. The former one is indeed the ΛCDM model, while the β_1 case is what we call the minimal bimetric model. One has then for the MBM

$$r' = \frac{3r\left(1 - 3r^2\right)}{1 + 3r^2} \,. \tag{15}$$

independency of β_1. This equation has two branches for $r > 0$, but only the one that starts at $r = 0$ and ends at $r = 1/\sqrt{3}$ is cosmologically viable. In terms of the scale factor, this solution reads [18, 20]

$$r(a) = \frac{1}{6} a^{-3} \left(-A \pm \sqrt{12a^6 + A^2}\right) , \tag{16}$$

where $A = -\beta_1 + 3/\beta_1$. These equations imply a remarkably simple and testable relation between the equation of state w and Ω_m valid at all times during matter domination:

$$w = \frac{2}{\Omega_m - 2} \,, \tag{17}$$

where the density parameter is given by

$$\Omega_m = 1 - 3r(a)^2 \,. \tag{18}$$

In [18] we found that the MBM fits well the supernovae data if $\beta_1 = 1.38 \pm 0.03$, corresponding to $\Omega_{m0} = 1 - \beta_0^2/3 = 0.37 \pm 0.02$. The equation of state turns out to be approximated at small redshifts by $w(z) \approx -1.22^{+0.02}_{-0.02} - 0.64^{+0.05}_{-0.04} z/(1+z)$. However this parametrization is not adequate at $z \geq 0.5$ and the analytic expressions (16-18) should be employed instead.

Perturbation equations. We now find the perturbation equations for the MBM. For the perturbed part of the metrics we adopt the gauge defined in Fourier space as

$$ds_f^2 = 2Fb^2 \left[-\frac{\dot{b}(t)^2 \Psi_f}{b(t)^2 \mathcal{H}^2(t)} dt^2 + (\Phi_f \delta_{ij} + k_i k_j E_f) dx^i dx^j \right] ,$$

$$ds^2 = 2Fa^2 \left[-\Psi dt^2 + (\Phi \delta_{ij} + k_i k_j E) dx^i dx^j \right] \tag{19}$$

where $F = e^{i\mathbf{k}\cdot\mathbf{r}}$. After a transformation to the gauge-invariant variables [13]

$$\tilde{\Phi} = \Phi - \mathcal{H}^2 E' \tag{20}$$

$$\tilde{\Psi} = \Psi - (\mathcal{H}^2 + \mathcal{H}\mathcal{H}')E' - \mathcal{H}^2 E'' \tag{21}$$

$$\tilde{\Phi}_f = \Phi_f - \frac{r\mathcal{H}^2 E_f'}{(r' + r)} \tag{22}$$

$$\tilde{\Psi}_f = \Psi_f - \frac{\mathcal{H} r^2 (\mathcal{H} E_f')'}{(r' + r)^2} - \frac{\mathcal{H}^2 E_f' r (r^2 + 2r'^2 + 2rr' - rr'')}{(r' + r)^3} \tag{23}$$

we obtain from the Einstein equations a set of perturbation equations in $\Xi = \{\tilde{\Phi}, \tilde{\Psi}, \tilde{\Phi}_f, \tilde{\Psi}_f, E, \Delta E \equiv E - E_f\}$ and from the conservation of matter two more equations for the matter density contrast δ and the velocity divergence θ. At sub-horizon scales, i.e. $k/\mathcal{H} \gg 1$, we can assume the so-called quasi-static limit, i.e. (we skip from now on the tildas) $\Xi_i (k/\mathcal{H})^2$ much larger than any of the derivative Ξ_i', Ξ_i'' and also $\delta(k/\mathcal{H})^2, \delta'(k/\mathcal{H})^2 \gg \theta/\mathcal{H}$: then the set of differential equations becomes algebraic (except for the matter conservation equations) and we obtain the Poisson-like relations

$$\Psi = -\frac{\mathcal{H}^2 \Omega_m \delta \left(2k^2 r^3 \left(11 + 6r^2\right) + 3\beta_1 a^2 \left(1 + 7r^2 - 6r^4\right)\right)}{2k^2 \left(\beta_1 a^2 \left(1 + r^2\right)^2 \left(1 + 3r^2\right) + k^2 r^3 \left(7 + 3r^2\right)\right)}$$

$$\Phi = \frac{\mathcal{H}^2 \Omega_m \delta \left(2k^2 r^3 \left(10 + 3r^2\right) + 3\beta_1 a^2 \left(1 + 4r^2 + 3r^4\right)\right)}{2k^2 \left(\beta_1 a^2 \left(1 + r^2\right)^2 \left(1 + 3r^2\right) + k^2 r^3 \left(7 + 3r^2\right)\right)} \tag{24}$$

which reduce to the standard ones during the matter epoch, i.e. for $r \to 0$. The full set of equations and the details of the derivation will be published elsewhere [21]. We obtain then the two modified gravity parameters

$$\eta \equiv -\frac{\Phi}{\Psi} = H_2 \frac{1 + H_4 (k/\mathcal{H})^2}{1 + H_3 (k/\mathcal{H})^2} \,, \tag{25}$$

$$Y \equiv -\frac{2k^2 \Psi}{3\mathcal{H}^2 \Omega_m \delta_m} = H_1 \frac{1 + H_3 (k/\mathcal{H})^2}{1 + H_5 (k/\mathcal{H})^2} \tag{26}$$

where

$$H_1 \equiv \frac{1+7r^2-6r^4}{\left(1+r^2\right)^2\left(1+3r^2\right)} \quad (27)$$

$$H_2 \equiv \frac{1+4r^2+3r^4}{1+7r^2-6r^4} \quad (28)$$

$$H_3 \equiv \frac{2\mathcal{H}^2 r^3\left(11+6r^2\right)}{3\beta_1 a^2\left(1+7r^2-6r^4\right)} \quad (29)$$

$$H_4 \equiv \frac{2\mathcal{H}^2 r^3\left(10+3r^2\right)}{3\beta_1 a^2\left(1+4r^2+3r^4\right)} \quad (30)$$

$$H_5 \equiv \frac{\mathcal{H}^2 r^3\left(7+3r^2\right)}{\beta_1 a^2\left(1+r^2\right)^2\left(1+3r^2\right)} \,. \quad (31)$$

For $\beta_1 \to 0$ the only consistent background solution is $r \to 0$: in this limit the model reduces to pure CDM and consequently $H_{1,2} = 1$ and $H_{3,4,5} = 0$. The expressions (25,26) have the same structure as the Horndeski Lagrangian [22–24] since both Lagrangians produce second-order equations of motion. The matter evolution equations can now be written as a single equation:

$$\delta_m'' + \delta_m'\left(1+\frac{\mathcal{H}'}{\mathcal{H}}\right) - \frac{3}{2}Y(k)\Omega_m\delta_m = 0 \,. \quad (32)$$

Integrating numerically this equation along the background solution (16) we find that near $k = 0.1h/\mathrm{Mpc}$ and $\beta_1 = 1.38$ we can approximate $f \equiv \delta'/\delta \approx \Omega_m^\gamma$ [25] with $\gamma \approx 0.47$ in the range $z \in (0,5)$ (see Fig. 1). Near $\beta_1 = 1.38$ the dependence on β_1 at $k = 0.1h/\mathrm{Mpc}$ can be linearly approximated as $\gamma = 0.26 + 0.15\beta_1$, while the weak dependence on k is approximately

$$\gamma(k) = 0.47 + 0.0035\left(\frac{k}{0.1h/\mathrm{Mpc}}\right)^{-1/2} \,. \quad (33)$$

Future experiments, like the Euclid satellite [26], plan to measure γ to within 0.02: this will amply allow to distinguish MBM from ΛCDM and standard quintessence, that predict $\gamma \approx 0.54$.

Let us remark, however, that the growth rate is significantly larger than 1 for redshifts $z \gtrsim 1$ and can not be well approximated with the standard Ω_m^γ fit. We find that an additional correction

$$f \approx \Omega_m^{\gamma_0}\left(1+\frac{\gamma_1}{z+1}\right) \quad (34)$$

with $\gamma_0 = 0.58$ and $\gamma_1 = 0.07$ reproduces much better our numerical result.

Comparing to growth rate. Our perturbation theory results can be compared to measurements of $f(z)\sigma_8(z)$ where $\sigma_8(z) = \sigma_8 G(z)$, $G(z)$ being the growth rate normalized to unity today . The likelihood is given by

$$\chi^2_{f\sigma_8} = \sum_{ij}\left(d_i - \sigma_8 t_i\right)C_{ij}^{-1}\left(d_j - \sigma_8 t_j\right) \quad (35)$$

in which d_i and t_i are vectors containing the measured and theoretically expected data, respectively, and C_{ij} denotes the covariance matrix. Since the current constraints on σ_8 depend on the theory of gravity, for generality we marginalize analytically the likelihood over σ_8 (see e.g. [27]). Since current data are not binned in k-space, we choose an average value $k = 0.1h/\mathrm{Mpc}$ in (32).

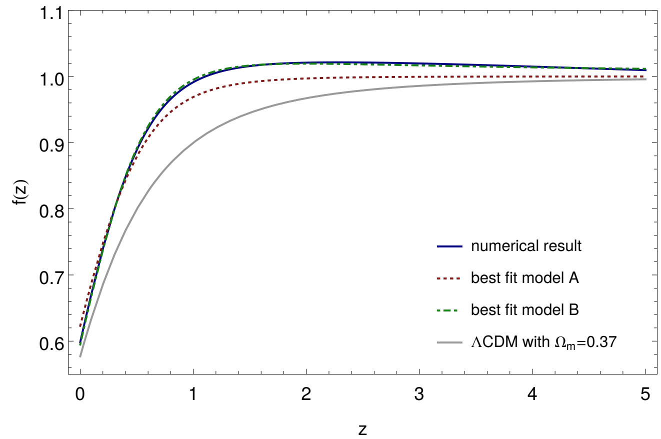

Figure 1: Growth rate $f = \delta'/\delta$ in the quasi-static limit for $\beta_1 = 1.38$ and $k = 0.1h/\mathrm{Mpc}$. The numerical result (in blue) is approximated by the fitting model $f = \Omega_m^\gamma$ (model A, red dotted curve) and $f = \Omega_m^{\gamma_0}(1+\frac{\gamma_1}{z+1})$ (model B, green dashed-dotted curve). For a comparison we plotted the ΛCDM result (gray dashed line) while using $\Omega_{m0} = 0.37$ which corresponds to the present matter density in our analyzed MBM.

We compute the likelihood from the dataset compiled by [19] which contains measured growth histories from the 6dFGS [28], LRG_{200}, LRG_{60} [29], BOSS [30], WiggleZ [31] and VIPERS [32] surveys. Our results are shown in Fig. 2. The growth data constraints appear much broader than, but consistent with, the SN Ia data. The combined result SN+growth data is $\beta_1 = 1.38 \pm 0.03$, practically identical to the best fit from SN Ia alone. We also plot in Fig. 2 the likelihood from CMB and BAO measurements where we used the results from the first peak angular size WMAP 7.2 data [33] and the SDSS DR7 sample including the LRG and 2dFGRS data set [34]. The combined result from all data, SN + CMB + BAO + growth turns out to be $\beta_1 = 1.43 \pm 0.02$. However, one should keep in mind that the CMB data analysis assumes a pure ΛCDM so it is not obvious that it can be applied here without corrections. Note that including the CMB/BAO data does not change the best fit parameters for $w(z)$ and γ significantly.

Finally, in Fig. 3 we compare the growth history corresponding to the most likely MBM with the measured growth data and the ΛCDM expectation.

Conclusions. We have shown that a minimal bimetric model exists which closely reproduces the success and the simplicity of ΛCDM at background and linear level. We fix its single parameter, β_1, to percent accuracy by

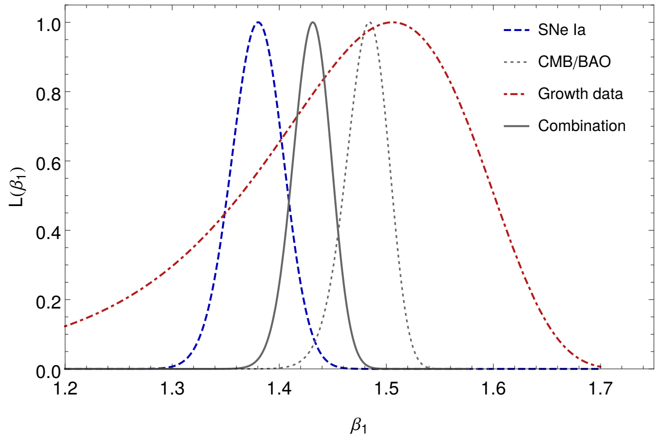

Figure 2: Likelihood for β_1 obtained from observed SNe Ia (blue dashed), measured growth data (red dot-dashed) and the combination of CMB and BAO measurements (dotted gray). The full combined likelihood is indicated by a gray solid line. All likelihoods are rescaled to unity at their maximum. For the most likely values we obtain $\beta_1 = 1.38^{+0.03}_{-0.03}$ ($\chi^2_{\rm min} = 578.3$) and $\beta_1 = 1.51^{+0.09}_{-0.13}$ ($\chi^2_{\rm min} = 4.93$) for the comparison with SNe Ia and growth data, respectively. Due to the broad width of the growth likelihood, its combination with the other probes does not changes sensibly the results.

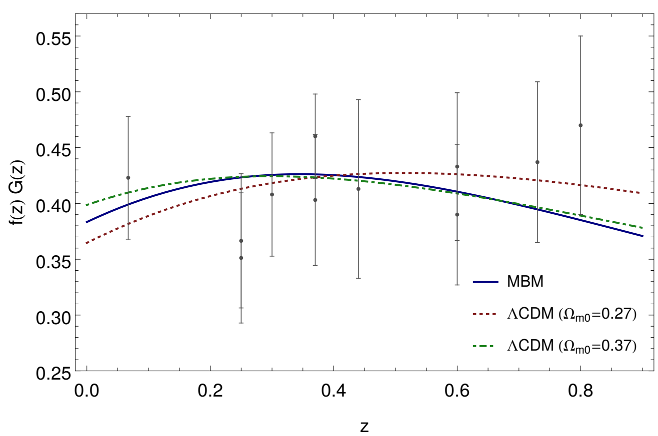

Figure 3: Comparison of growth histories in MBM with $\beta_1 = 1.38$ (blue) and ΛCDM (dotted red: $\Omega_{m0} = 0.27$, dot-dashed green: $\Omega_{m0} = 0.37$ which corresponds to the present matter density in the best fit MBM). The data points are taken from [19]. Note that the normalization of the curves is immaterial due to the marginalization over σ_8.

fitting to supernovae and to the growth data. The MBM has several unique signatures, like the $w - \Omega_m$ relation (17), the phantom equation of state, the k-dependence of the growth factor (33), the values above unity of f (34), that will make it easily distinguishable from ΛCDM with future experiments.

Acknowledgments

We acknowledge support from DFG through the project TRR33 "The Dark Universe". We thank Mariele Motta for discussions and cross-checks on the perturbation equations and Valerio Marra for help with the supernovae catalog, and Y. Akrami, T. Koivisto, and A. Solomon for discussions on the perturbation equations.

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