Joint Power Allocation and Relay Selection in Cooperative Networks

Khoa T. Phan†, Duy H. N. Nguyen‡, and Tho Le-Ngoc‡
†Electrical Engineering Department, University of California, Los Angeles (UCLA), Los Angeles, USA
‡Department of Electrical and Computer Engineering, McGill University, Montreal, CANADA
Email: kphan@ee.ucla.edu, duy.h.nguyen@mail.mcgill.ca, tho.le-ngoc@mcgill.ca

Abstract—In this paper, we study the joint power allocation and relay selection problem for multi-user amplify-and-forward (AF) cooperative networks. To increase the system’s spectral efficiency under the orthogonal transmission assumption, each source-destination pair is constrained to be assisted by a small subset of a set of available relays. The aim of this work is to establish a framework that determines which relays to help which users and with how much power. In particular, we propose the joint schemes under two design criteria: i) maximization of user rates, and ii) minimization of the total transmit power at the relays. As the original problem formulations are shown to be nonconvex integer optimization problems, and thus, are combinatorially hard, we also propose an efficient convex relaxation approach to solve the problems with low complexity. Numerical results demonstrate the effectiveness of the proposed approaches.

I. INTRODUCTION

Recently, cooperative communication via relays has been introduced to enhance the performance of wireless networks [1]. Additionally, efficient radio resource management also provides performance enhancement of wireless networks [2]. Numerous works on radio resource allocation for relay networks (for example, see [3], [4] and references therein) focus on single user scenarios. In this paper, we consider a network model, in which multiple source-destination pairs share radio resources from a set of relays [5]. A typical example of such scenarios is the deployment of a few relays in a cellular network to assist mobile users located in poor coverage areas of a cell for both uplink and downlink transmissions. Unfortunately, resource allocation schemes for single-user relay networks cannot be directly applied to multi-user cases.

In cooperative systems, to avoid the interference among different transmission links, orthogonal channels are often considered [1]. Hence, the diversity advantage results in a reduction in spectral efficiency. One possible way to improve the spectral efficiency is to select only a subset of the available relays to assist each source-destination link. In [6], the authors showed that full diversity can still be achieved with systems where only the “best” relay is selected. A distributed game theoretical framework was proposed in [7] for multi-user cooperative communication systems to achieve optimal relay selection and power allocation. In [8], algorithms were proposed to pick a set of cooperating relays to minimize the total transmission time of a fixed amount of data. The authors in [9] proposed and analyzed two relay selection schemes. The first scheme selects a fixed $M$ out of $K$ relays while the second scheme adaptively minimizes the number of selected relays given an achieved outage probability. However, a general framework for joint power allocation and relay selection has been rarely studied.

This paper considers an optimization-based framework to the mentioned problem for multi-user amplify-and-forward (AF) relay networks. Specifically, we seek the best way(s) to select a subset of available relays to assist the transmissions of each source-destination pair, together with corresponding power allocation. Different schemes are proposed for two criteria: i) maximization of user rates, including minimum user rates and weighted-sum user rates, and ii) minimization of the total transmit power at the relays. We show that the original formulations are nonconvex integer programming problems, and thus, are combinatorially hard to solve optimally. Therefore, a low-complexity albeit sub-optimal solution is desirable. Consequently, we propose a simple but efficient algorithm to solve the joint power allocation and relay selection problem. Simulation results show the effectiveness of the proposed approach.

II. SYSTEM MODEL AND ASSUMPTIONS

Consider a multi-user relay network with $M$ source nodes $s_i$, transmitting data to their corresponding destination nodes $d_i$, $i \in \{1, \ldots, M\}$, and $L$ relay nodes $r_j$, $j \in \{1, \ldots, L\}$. Orthogonal transmissions are used for simultaneous transmissions among different users by using different channels, (e.g., different frequency bands) and time division multiplexing is employed by AF cooperative diversity for each user [1].

Let $P_{s_i}$ denote the power transmitted by source node $s_i$. Let $x_{s_ir_j}$ be the relay selection variable, i.e., $x_{s_ir_j} = 1$ if $r_j$ relays data for source $s_i$, otherwise $x_{s_ir_j} = 0$. In the former case, the power transmitted by relay $r_j$ for assisting source $s_i$ is denoted by $P_{s_ir_j} > 0$, otherwise $P_{s_ir_j} = 0$. Denote the channel gains for links $s_ir_j$ and $r_jd_i$ as $a_{s_ir_j}$ and $a_{r_jd_i}$, respectively, where the channel gains may include the effects of path loss, shadowing, and fading. Also let $N_{r_j}$ and $N_{d_i}$ be the variances of additive circularly symmetric white Gaussian noise (AWGN) at the relay $r_j$ and at destination node $d_i$, respectively. In the first time interval, source $s_i$ broadcasts the
is a concave increasing function of $P_d$ as at destination node $d$, where

$$x = \sum_{r_j \in \mathcal{R}(s_i)} \alpha_{r_j} P_{r_j} x_{r_j} + \beta_{r_j}$$

and $\mathcal{R}(s_i)$ denotes the set of relays that assist source $s_i$. It can be shown that the rate function of user $i$ defined as

$$\tilde{r}_i = \log_2(1 + \gamma_i) = \log_2 \left(1 + \sum_{r_j \in \mathcal{R}(s_i)} \frac{P_{r_j} x_{r_j}}{\alpha_{r_j} P_{r_j} x_{r_j} + \beta_{r_j}} \right) \text{(bits/Hz/s)}$$

is a concave increasing function of $P_{r_j}$ for some fixed relay assignments $x_{r_j}$ [10].

In the following sections, we shall consider efficient joint power allocation and relay selection schemes to either maximize the minimum user rates $\tilde{r}_i$ or the weighted-sum of user rates with relay power constraints or minimize the sum relay power with user rate constraints. It should be noted that the proposed schemes are based on a centralized approach with assumed complete knowledge of channel gains. This assumption involves some timely and accurate channel estimation and feedback techniques, which are beyond the scope of this paper.

### III. Joint Power Allocation and Relay Selection: Problem Formulations

This paper considers resource allocation problems for multiuser relay networks under two different scenarios: with and without QoS requirements. In the former case, we try to maximize the minimum rate amongst all users. For the latter case, the total relay power consumption will be minimized.

#### A. Rate Maximization Based Power Allocation

In general, resource allocation in wireless networks should take into account the fairness among users. It is well-known that the max-min fairness is the fairest allocation criterion. The joint power allocation and relay selection problem under max-min rate fairness can be mathematically formulated as

$$\text{maximize} \quad \min_{s_i} \tilde{r}_i \quad \text{(2a)}$$

subject to:

$$\begin{align*}
\sum_{r_j} P_{r_j} x_{r_j} &\leq P_{r_j}^{\max}, \quad j = 1, \ldots, L \\
N_{s_i} x_{r_j} &\geq 0, \quad x_{r_j} \in \{0, 1\}, \forall i, j \\
P_{r_j}^{s_i} &\geq 0, \quad P_{r_j}^{s_i} \leq \tilde{r}_j \text{, \quad } \forall i, j
\end{align*} \quad \text{(2b, 2c, 2d)}$$

where $P_{r_j}^{\max}$ is the maximum power available to relay $r_j$. Constraints (2c) state that each source $s_i$ is assisted by $x_{r_j}^{s_i}$ relays while the constraints in (2d) require that the transmit power $P_{r_j}^{s_i}$ and selection variables $x_{r_j}^{s_i}$ must be nonnegative and binary, respectively.

The optimization problem (2a)–(2d) appears to be intractable since it is a nonconvex integer programming. Fortunately, the problem can be reformulated as follows

$$\text{maximize} \quad \min_{s_i} \tilde{r}_i \quad \text{(3a)}$$

subject to:

$$\begin{align*}
\sum_{r_j} P_{r_j} x_{r_j} &\leq P_{r_j}^{\max}, \quad j = 1, \ldots, L \\
0 &\leq P_{r_j}^{s_i} \leq x_{r_j}^{s_i} P_{r_j}^{\max}, \quad \forall i, j \\
0 &\leq P_{r_j}^{s_i} \leq x_{r_j}^{s_i} P_{r_j}^{\max}, \quad \forall i, j
\end{align*} \quad \text{(3b, 3c)}$$

where $P_{r_j}^{\max}$ is the maximum power available to relay $r_j$. Constraints (3c) relate the transmit power variable $P_{r_j}^{s_i}$ to the relay selection variable $r_{r_j}^{s_i}$. Specifically, when $x_{r_j}^{s_i} = 1$, i.e., the relay $r_j$ assists source $s_i$, $P_{r_j}^{s_i}$ is less than $P_{r_j}^{\max}$. On the other hand, if relay $r_j$ does not help source $s_i$, i.e., $x_{r_j}^{s_i} = 0$, the constraint in (3c) automatically makes $P_{r_j}^{s_i} = 0$. Thus, it can be seen that the optimization problems (3a)–(3c) and (2a)–(2d) are indeed equivalent.

The optimization problem (3a)–(3c) is now a convex integer programming. However, it is still combinatorially hard with worst case exponential complexity. In fact, optimal solution using exhaustive search requires solving the max-min optimization problem for each of the $L^{\sum_{i=1}^{M} x_{r_j}^{s_i}}$ possible combinations. This approach is clearly not viable for practical implementation. Consequently, an efficient joint power allocation and relay selection algorithm with a low complexity is highly desirable.

We shall investigate such an algorithm in Section IV. A remark about the optimization problem (3a)–(3c) is that it is always feasible. Moreover, since the objective function $\min_{s_i} r_i$ increases with the allocated powers, the inequality constraints (2b) should be met with equality at optimality.

It is well-known that although max-min fairness based allocation helps to improve performance of the worst users, it
results in a loss in the network throughput (also confirmed by numerical results). Therefore, this criterion is applicable for the network where all users are (almost) equally important. This could be the case, for example, when all wireless users have similar QoS requirements. On the other hand, the maximization of the weighted-sum of rates can potentially achieve higher throughput, which can be mathematically posed as

$$\text{maximize} \quad \sum_{s_i} w_i r_i$$

subject to: Constraints (2b), (2c), (2d), (3c).

where $$w_i$$ denote the allocated weight for user $$i$$. To prevent unfairness among users, larger weights can be allocated to users in unfavorable condition. Moreover, this objective also captures the scenarios in which one needs to perform QoS differentiation for each user. Then, the users of higher service priority can be allocated larger weights.

B. Power Minimization Based Allocation

In wireless networks, power allocation can help to achieve the minimum QoS and low power consumption for devices. Subject to the rate requirement for each user, the resulting formulation can be posed as

$$\text{minimize} \quad \sum_{r_j, s_i} P_{s_i}$$

subject to: $r_i \geq r_{i,\min}$, $i = 1, \ldots, M$ (5b)

Constraints (2c), (2d), (3c) (5c)

where $$r_{i,\min}$$ is the minimum rate for the $$i$$th user and we assume that the threshold $$r_{i,\min}$$ is achievable. In addition, a weighted sum of powers may be also considered to cover the general case of non-homogeneous relays. There are applications where the maximum transmit power of the relays is of concern. In such applications, minimizing the maximum transmit power can be a more appropriate objective.

It can be observed that at optimality, the inequality constraints in (5b) must be met with equality. This is because $$r_i$$ is an increasing function of $$P_{s_i}$$. In order to minimize the objective function, $$r_i$$ must attain their minimum values. Note that we have implicitly assumed in (5a)-(5c) that none of the relays needs to transmit more than $$P_{r_{j,k}}$$ at optimality or the constraints (2b) can be included if necessary.

Some remarks:

1. In the aforementioned formulations, when $$x_{s_i} = L$$, $$i = 1, \ldots, M$$, this effectively means that each source-destination pair can be assisted by all the available relays, i.e., no relay selection at all. These problems become straight power allocation problems as studied in [10].

2. If $$x_{s_i} = 1$$, $$i = 1, \ldots, M$$, the problems are equivalent to selecting the “best” relay for each source-destination link [6]. This scheme achieves the largest spectral efficiency since each link requires only two orthogonal channels.

3. As being considered in this work, $$x_{s_i}$$, $$i = 1, \ldots, M$$ can take any number between 1 and L.

IV. CONVEX RELAXATION TO JOINT POWER ALLOCATION AND RELAY SELECTION

A. The relaxed relay selection problem

In order to avoid the intractability of the convex integer program in (2a)–(2d), this section considers a simple, yet efficient relaxation approach to the problem. By replacing the nonconvex constraints $$x_{s_i}^r \in \{0,1\}$$ with the convex constraints $$x_{s_i}^r \in [0,1]$$, we obtain the convex relaxation of the relay selection problem (3a)–(3c) as follows

$$\text{maximize} \quad \min_{s_i} r_i$$

subject to: Constraints (2b), (2c), (3c)

$$0 \leq x_{s_i}^r \leq 1, \forall i, j.$$ (6c)

This problem, unlike the original problem (3a)–(3c), is convex. Thus, the efficient interior-point algorithm can be applied to solve it optimally. Note that this relaxation approach to the 0–1 programming problems has been commonly considered in the literature, for example see [11], [12] and references therein.

B. Approximate relay selection solution

The relaxed joint power allocation and relay selection problem (6a)–(6c) is obviously not equivalent to its original version (3a)–(3c); in particular, the optimal $$x_{s_i}^r$$ can be fractional. However, we can take advantage of the relation between the two problems to find an approximate solution to the original problem. Note that the optimal objective value of the problem (6a)–(6c), denoted $$r^+$$, is an upper bound on $$r^*$$, the optimal objective value of (3a)–(3c) since the feasible set of the former contains that of the latter. A straightforward way to generate the suboptimal solution $$x_{s_i}^{r+}$$ for (3a)–(3c) from the optimal solution $$x_{s_i}^{r*}$$ of (6a)–(6c) is as follows.

For each $$s_i$$, let $$x_{s_i}^{r+,1}, x_{s_i}^{r+,2}, \ldots, x_{s_i}^{r+,K_i}$$ denote the elements of $$x_{s_i}^{r+}$$ re-arranged in descending order. Then, the relays with indexes $$k_1, \ldots, k_{K_i}$$, i.e., the indices corresponding to the largest elements of $$x_{s_i}^{r+}$$, are assigned to assist source $$s_i$$. This “rounding technique” is commonly employed to obtain sub-optimal solution from relaxed optimal solution [11]. After determining the relay assignment for each source, we can solve the following convex optimization to find the resulting power allocation

$$\text{maximize} \quad \min_{s_i} r_i$$

subject to: $\sum_{s_i \in S(r_j)} P_{s_i} \leq P_{r_{j,k}}$, $j = 1, \ldots, L$

where $$S(r_j)$$ denotes the set of sources which are assisted by relay $$r_j$$. The optimal objective value $$r^+$$ of (7a)–(7c) is a lower bound on the optimal value of (3a)–(3c). In general, we have the inequality

$$r^+ \leq r^* \leq r^+.$$ (8)

The difference between $$r^+$$ and $$r^+$$ is called the relaxation gap which is always nonnegative. If it happens to be zero, the
optimal value of the relaxation problem will be also optimal for the original problem. Since the approximate solution to the relay selection is based on finding the indices associated with the largest \( x_{\text{max}} \) values of the optimal solution instead of the exact values of the optimal solution, the relaxed problem (6a)–(6c) might not be solved with high accuracy. A similar approach can be employed to find suboptimal solutions to the joint power allocation and relay selection problems (4a)–(4b) and (5a)–(5c). That is, we first find the relay assignment for each source by relaxing the binary variables, carrying out the rounding process; then, the power allocation is done using the resulting relay assignment found.

We should stress here that more efficient algorithms in terms of performance and/or complexity are also possible. However, as simulation results reveal, the simple proposed algorithm performs quite well. An investigation for “better” algorithms is an interesting research direction but beyond the scope of this paper.

V. SIMULATION RESULTS

Consider a wireless relay network as shown in Fig. 1 with 10 users and 5 relays distributed in a 2-D region of a size 20m×20m. The relays are fixed at coordinates (10,5), (10,7.5), (10,10), (10,12.5), (10,15). The source and destination nodes are initially deployed randomly and then fixed in the area inside the box areas \([(0,0), (5, 20)]\) and \([(15, 0), (20, 20)]\), respectively. In our simulation, each user is assisted by two relays. The channel gain for each transmission link is affected by the path loss and Rayleigh fading. The path loss component is \( \alpha = [1/d]^2 \) where \( d \) being the Euclidean distance between two transmission ends, while the variance of the Rayleigh fading is set at unity. The results are averaged over 1000 channel instances. The noise power is normalized to 1. All users are assumed to have the same minimum rate \( r_{\text{min}} \) when necessary and unit transmit power. Relays are assumed to have identical transmit power \( P_{\text{max}} \). Due to the high complexity of the optimal solution which requires searching over \( 10^{10} \) possible combinations, only the lower bound and upper bound of the optimal value for each investigated problem as mentioned above are plotted. We also study the “closest relays” selection scheme where the two closest relays to source \( s_i \) will assist data transmission of \( s_i \). The performance of this scheme provides another lower bound on the optimal power allocation and relay selection scheme. We have used software package [13] for solving convex programs in our simulations.

![Fig. 1. System model.](image1)

![Fig. 2. Minimum achievable rate amongst users. Solid lines: the relaxation scheme; dashed lines: the rounding scheme; and dashed-dot lines: the “closest relays” scheme.](image2)

![Fig. 3. Achievable network throughput. Solid lines: the relaxation scheme; dashed lines: the rounding scheme; and dashed-dot lines: the “closest-relays” scheme.](image3)
In this paper, several joint power allocation and relay selection schemes have been proposed for multi-user relay networks based on amplify-and-forward cooperative diversity. Various design criteria have been considered: minimum-rate maximization, weighted-sum rate maximization, and power minimization. Since the original problems are convex integer programming, a convex relaxation approach with low complexity has also been proposed. Simulation results have demonstrated the effectiveness of the proposed schemes.

**References**


**VI. Conclusion**

We are grateful to Dr. Long Le from Massachusetts Institute of Technology (MIT) and Prof. Mai Vu from McGill University for helpful discussions on the subject.