ABSTRACT
We develop a low-complexity overcomplete wavelet domain method for denoising digital images corrupted with non-stationary white additive Gaussian noise. The noise level for each pixel is estimated from a local window around that pixel. We use a shrinkage function that adapts itself to the noise level and to the spatially changing statistics of the image. Experiments show that this noise model has good results for different non-stationary noise sources. Finally, we extend our method for denoising images corrupted with signal-dependent noise.

Index Terms— Image restoration, Gaussian noise, non-stationary noise, signal-dependent noise

I. INTRODUCTION
Images recorded with digital imaging devices, are often corrupted with noise. This noise originates from the sensors and signal amplification circuitries within the camera, and is device-specific. Also, after capturing the image by the sensors, often many post-processing steps [1] are performed, like color interpolation, color correction, contrast enhancement, sharpening and gamma correction, which alter the statistical properties of the noise. Local adaptive contrast enhancement [1] in bad lightening conditions, higher amounts of noise are observed in dark areas. Gamma correction introduces a non-linear transform on the noise, making the noise signal-dependent [2]. In many cases, noise removal is desirable to improve the visual quality of the image.

Multiresolution representations like wavelets have proven to be very useful in reducing noise while retaining image details. In the wavelet domain, noise is removed by shrinking (thresholding) the noisy coefficient magnitudes. In the literature, many techniques [3], [4], [5], [6] have emerged for the reduction of spatially stationary noise. Recently, a few techniques have been proposed that deal with correlated noise [5] and signal-dependent noise [7], [8].

Signal-dependent noise can be interpreted as additive non-stationary noise, with the noise levels at each spatial position depending on the unknown noise-free image [8].

In this paper, we extend a recent denoising method of [6] to cope with non-stationary additive noise. The method consists of a two step procedure: a novel local noise estimation and local adaptive shrinkage. The local adaptive shrinkage applies the Probshrink-estimator of [6] on small windows of wavelet coefficients, instead of a full subband. By letting the window slide over the image, spatially changing statistics of both the noise-free image and the noise can be captured. To limit the computational complexity, we do not take interband correlations into account.

This paper is organized as follows: in Section II, we present our technique for non-stationary noise, in Section III we explain how this algorithm can be used for signal-dependent noise. The implementation is discussed in Section IV. In Section V, we test this method on real images and compare the results.

II. REDUCTION OF NON-STATIONARY NOISE

II-A. Noise model
Let \( \mathbf{x} = [x_1, \ldots, x_N] \) denote the unknown ideal image, and let \( \mathbf{y} = [y_1, \ldots, y_N] \) be the observed noisy image, where a single index denotes the spatial position, like in raster scanning. We assume the following additive noise model:

\[
y_j = x_j + \sigma_j w_j
\]

where \( w_j \) represents independent identically distributed (i.i.d.) zero mean Gaussian noise, with unit variance and independent of \( x_j \). \( \sigma_j \) is a deterministic function, representing the noise level at spatial position \( j \). The product \( \sigma_j w_j \) models the non-stationary noise source.

Let us transform the observed image \( \mathbf{y} \) using a non-decimated wavelet transform like in [4], [6]. \( s \) denotes the scale \( (s = 1, \ldots, L) \), \( L \) is the number of wavelet decompositions and \( o \) denotes the orientation band \( (o = LH, HL, HH) \). Using the linearity of the wavelet transform,
\[ Y_j^{(s,o)} = X_j^{(s,o)} + \sum_{k \in \delta W} [Y_k^{(1,HH)}]^2 \]  
We use only information from the highest frequency sub-band, since for large noise levels, this sub-band mainly consists of noise. The choice of the window size plays an important role in this process: small windows adapt fast to changing characteristics of the noise, but contain few coefficients and yield unreliable estimates; large windows are more reliable, but less sensitive to fast noise level fluctuations. An optimal choice often depends on the noise characteristics.

II-C. Assumed prior

In the wavelet transform domain, noise-free coefficients are often modeled by generalized Laplacian priors [6] (also called generalized Gaussian priors [4]). In our approach, we model wavelet coefficients in a small window and assume that these coefficients are resulting from the same stationary process. While the window slides over the image, the image characteristics change (for instance from a smooth area to a highly textured area). These changes are captured by local estimation of the prior parameters. To limit the number of prior parameters to estimate, we use the Laplacian prior, which has only one parameter: 

\[ f(\beta|v) = \frac{1}{2v} \exp\left(-\frac{\beta}{v}\right) \]  

II-D. Adaptive Bayesian shrinkage

Consider one wavelet subband at scale \( s \) and orientation \( o \). We adapt the ProbsShrink-shrinkage rule of [6] to the small square window \( \delta_j \) with size \( W_d \) around a coefficient at position \( j \). We define a "signal of interest" as a noise-free coefficient component within this window that exceeds \( \sigma_j \).

We formulate the following two hypotheses: \( H_0^{(j)} \): "signal of interest is absent in local window" and \( H_1^{(j)} \): "signal of interest is present in local window" as:

\[ H_0^{(j)} : |X_j| \leq \sigma_j \quad \text{and} \quad H_1^{(j)} : |X_j| > \sigma_j. \]  

We consider the following shrinkage rule:

\[ \hat{X}_j = P(H_1^{(j)}|Y_j, Z_j)Y_j = \frac{\mu_j \eta_j \xi_j}{1 + \mu_j \eta_j \xi_j} Y_j \]  

where

\[ \eta_j = \frac{f_Z(Z_j|H_1^{(j)})}{f_Z(Z_j|H_0^{(j)})} \quad \text{and} \quad \mu_j = \frac{P(H_1^{(j)})}{P(H_0^{(j)})} \]  

\( Z_j \) denotes the local spatial activity indicator (LSAI), and is defined as the average magnitude of the coefficients in the local window, with exclusion of the central coefficient:

\[ Z_j = \frac{1}{M} \sum_{k \in \delta_j \backslash \{j\}} |Y_k|, \quad M = (W_d)^2 - 1 \]  

For the assumed noise model (2), the conditional probability densities of the noisy coefficients are [6]:

\[ f(Y_j|H_0^{(j)}) = \int_{-\infty}^{\infty} \phi_W(Y_j - x; \sigma_j)f(x|H_0^{(j)})dx, \]

\[ f(Y_j|H_1^{(j)}) = \int_{-\infty}^{\infty} \phi_W(Y_j - x; \sigma_j)f(x|H_1^{(j)})dx, \]  

where \( f(x|H_0^{(j)}) \) and \( f(x|H_1^{(j)}) \) denote the conditional probability densities of the noise-free coefficients and \( \phi_W(w; \sigma_j) \) is the zero mean Gaussian probability density with standard deviation \( \sigma_j \).

The conditional densities \( f(Z_j|H_0^{(j)}) \) and \( f(Z_j|H_1^{(j)}) \) are estimated by convolving the densities of the coefficient magnitudes \( |Y_k| \) over the window \( \delta_j \), with exclusion of the central coefficient magnitude [6]:

\[ f(MZ_j|H_0^{(j)}) = f(A_j|H_0^{(j)}) \ast \cdots \ast f(A_j|H_0^{(j)}), \]

\[ f(MZ_j|H_1^{(j)}) = f(A_j|H_1^{(j)}) \ast \cdots \ast f(A_j|H_1^{(j)}), \]  

where \( A_j \) denotes the coefficient magnitude \( A_j = |Y_j|, l = 0, 1 \).

II-E. Estimation of prior parameters

The variance of a Laplacian signal corrupted with additive white Gaussian noise with standard deviation \( \sigma_j \) is, using (2): \( \sigma^2_{Y,j} = \sigma^2_{X,j} + \sigma^2_W = 2\sigma^2_j + \sigma^2_j \) where \( \sigma_j \) denotes the Laplacian prior parameter at the position \( j \) and \( \sigma_j \) is defined in (3). The variance \( \sigma^2_{X,j} \) can be estimated from the local window \( \delta_j \) in each subband. We find:

\[ \hat{\sigma}_j^2 = \max(\hat{\sigma}^2_{X,j} - \hat{\sigma}^2_j, 0)/2 \]  

III. REDUCTION OF SIGNAL-DEPENDENT NOISE

Consider a point-wise signal-dependent noise model:

\[ y_j = \zeta_j(x_j, w_j) \]
where $x_j$ is the noise-free image, $y_j$ is the noisy image, $w_j$ is i.i.d. normal distributed noise. In the absence of noise, the original signal is passed ($\zeta_j(x_j, 0) = x_j$). We approximate $\zeta_j(x, w)$ using a first-order Taylor-series expansion in $w = 0$:

$$y_j \approx \zeta_j(x, 0) + \frac{\partial \zeta_j}{\partial w}(x, 0), w_j$$

(13)

We can distinguish whether a priori knowledge of $\zeta_j(x_j, w_j)$ is available.

1) If $\zeta_j(x_j, w_j)$ is unknown, we estimate $\frac{\partial \zeta_j}{\partial w}(x_j, 0)$ using local estimation of the noise variance. Substitution of $\frac{\partial \zeta_j}{\partial w}(x_j, 0) = \sigma_j$ in (13) transforms this noise model into (1).

2) If $\zeta_j(x, w)$ is given, we estimate $\hat{\sigma}_j$ as follows:

$$\hat{\sigma}_j = \frac{\partial \zeta_j}{\partial w} (\hat{x}_j, 0)$$

(14)

Note that (14) requires an initial estimation of $\hat{x}_j$, which one can retrieve by applying a mean filter on the observed image $y_j$, as in [8].

IV. IMPLEMENTATION

The convolutions in (9)-(10) are performed numerically, and have to be repeated for every pixel in each subband, which is a computationally expensive procedure. Analytical expressions are given in [6]. We calculate (9) off-line (which means we only have to do this once), for different choices of the parameters. We keep the noise variance $\sigma_j^2$ constant, letting the parameter $v_j$ vary, and we approximate $\log \eta_j(Y)$ and $\log \zeta_j(Y)$ (Eq. 7) to the polynomials $k_l^1 Y^2 + k_l^2 Y + k_l^3$ and $l_l^1 Y^2 + l_l^2 Y + l_l^3$ using least squares fitting. The resulting parameters $\{k_l^1, l_l^1\}$ are stored in a lookup table with a total size of 24 kilobytes. These approximations have no influence on the PSNR performance of our algorithm, and make fast implementations possible. We implemented our method in C++ for 8-bit gray-scale images using 32-bit floating point numbers. On a Pentium IV 2GHz, denoising takes 0.74 s for a 256x256 image and 4.3 s for a 512x512 image.

V. RESULTS

In this paper, results are produced using a non-decimated wavelet transform with 3 orientations and 5 decomposition levels, and the Daubechies’ wavelet of length four. The sizes of the noise estimation window $W_n$ and the signal analysis window $W_d$ are 15 and 9, respectively. We create noise artificially (see Fig. 1), and compare the PSNR-values for the Lena, Barbara and Peppers test images. Table I groups the results that correspond to the same input PSNR value for different noise sources (see Fig. 1). The denoising method of [6], implemented here with the noise estimator of [3], is used as reference method for stationary noise. For constant noise levels, the PSNR results for our method lay within approximately 0.3dB of the reference method. This is expected since our local noise estimation (from a window) is less accurate than the global one in case of the spatially stationary noise. Visually, differences are hardly noticeable. When we use other noise sources, that still produce the same input PSNR ($PSNR_{in}$), results are equally well or better. Fig. 2 shows the visual performance of our method.

We also tested our technique on signal-dependent noise and in particular on images corrupted with stationary noise before gamma correction. An obvious approach consists of inverting the gamma correction, followed by a standard filter for additive stationary noise, and afterwards gamma correction of the result (Fig. 3b, Fig. 4b). In practice, a quantization step is performed after the gamma correction, which makes the inverse gamma correction ill-conditioned, and hampers subsequent noise filtering [2]. Using a signal-dependent noise model (Fig. 4a) better denoising results are obtained. In Fig. 4b, one can see that the contrast is altered by filtering on the non-gamma corrected image. In Fig. 4c the contrast is left unchanged. Fig. 4d reveals the signal-dependent character of the noise. For Fig. 4c the gamma correction factor $\gamma$ was not available to our algorithm. If $\gamma$ is known, using (14), the PSNR further increases with +0.11dB, while visually hardly any changes can be seen. So the new method does not require prior knowledge about the gamma correction factor.

VI. CONCLUSION

In this paper, a new wavelet domain denoising method for non-stationary additive white noise is presented. This technique is also useful for denoising images corrupted with signal-dependent noise, whether a priori information of the noise model is available.

VII. REFERENCES

Fig. 1. Simulated non-stationary noise sources. Left: \( \sigma(m, n) \sim \sin(16\pi.m.n/(M.N)) \), middle: \( \sigma(m, n) \sim \sin(32\pi.m/M) \), right: linear gradient: \( \sigma(m, n) \sim m/M \)

Fig. 2. Noise reduction of non-stationary noise (a) Noisy image (PSNR=22.16 dB). (b) Proposed method (PSNR=32.82 dB) (c) Noise levels \( \sigma(m, n) \) (d) Estimated noise levels \( \hat{\sigma}(m, n) \)

Fig. 3. Two simplified denoising schemas for signal-dependent camera noise by gamma correction (a) noise reduction after inverse gamma correction (b) noise reduction of the gamma corrected signal

Fig. 4. Denoising results for signal-dependent noise with simulated gamma correction (\( \gamma=0.45, \sigma_n=35 \)). (a) Noisy image (PSNR=15.16 dB) (b) Method of [6] after inverse gamma correction (PSNR=22.45 dB), using the prior knowledge about the exact value of \( \gamma \) (c) Proposed method (PSNR=24.93 dB) without any prior knowledge about the \( \gamma \) value (d) Difference between (c) and (a).


