Asymptotic Detection Performance of Energy Detector in Fixed-gain Cooperative Relay Networks

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Abstract—Energy detectors are preferred choice for spectrum sensing in cognitive radio networks due to their low implementation complexity. Recently, there is a lot of work being done to analyze the energy detectors in both diversity and cooperative relay networks. In this work, detection performance of energy detector is analyzed in a fixed-gain cooperative relay network with maximum-ratio combiner (MRC) at the destination. Upper and lower bounds on instantaneous signal-to-noise ratio (SNR) of the relay path are used in this work to mathematically tract the analyses. Probability density function (PDF) is derived for these asymptotic instantaneous SNR bounds to obtain upper and lower bound expressions of average detection probability \( P_d \) for the relay path transmission. Similarly, PDF is derived for the combined signal from direct and relay path at destination with MRC, which is then used to derive asymptotic bounds on average \( P_d \) of the cooperative system. The derived upper and lower bounds are tight approximations of the exact average \( P_d \), which is also validated through simulations.

Keywords—Cognitive radio, energy detection, maximum-ratio combining.

I. INTRODUCTION

Cognitive radio technology attained much popularity during the last decade when Federal Communications Commission (FCC) of the U.S. reported that some frequency bands are largely vacant at particular times and in particular geographic locations [1]. This opened up new avenues for the wireless communications researchers to exploit these under-utilized frequency bands for communication by the unlicensed users. Cognitive radio is a technology that allows these unlicensed users to use the idle licensed bands [2]. Cognitive radio is (but not always) built upon software defined radio (SDR) with spectrum intelligence capabilities [3] and allows cognitive users (unlicensed users) to access licensed frequency band when it is vacant.

Spectrum intelligence essentially means that a cognitive user requires to perform spectrum sensing of its surrounding environment. There are different methods to achieve this task such as matched filter detection, cyclostationary detection and energy detection [4]. Among these, energy detector offers the least implementation complexity and it was studied under different fading environments in [5]. However, energy detectors perform poorly at low signal-to-noise ratio (SNR) and therefore they were analyzed with diversity combining techniques such as equal gain combining (EGC), selection combining (SC), switch-and-stay combining (SSC), square-law combining (SLC) and square-law selection (SLS) in [6], [7]. Recently, an extensive study on the detection performance of energy detector under different fading conditions and with different diversity combiners, such as, maximum-ratio combiner (MRC), EGC and SC, was performed in [8].

Cooperative relay networks have also shown improved system performance in terms of outage behavior and error rates [9], [10]. It was apparent that detection performance of energy detector will also benefit from cooperative relays. Hence, authors in [11], [12] demonstrated improved detection performance of energy detectors in cooperative relay networks. Since then, there have been many works mathematically analyzing the energy detectors in cooperative relay networks [13]–[17]. Briefly outlining, [13] derived upper-bound average detection probability \( P_d \) expressions for fixed-gain relays and SLC at the destination, while [14], [15] obtained upper-bound average \( P_d \) expressions for variable-gain relay with MRC and SC at the destination. Authors in [16] evaluated the exact average \( P_d \) expressions with fixed-gain relay and SC at the destination while [17] addressed the same problem in multihop (hop \( \geq 2 \)) variable-gain relay networks with MRC and SC at the destination.

A concise look into above works and references therein, reveals that fixed-gain cooperative network with MRC at the destination has not been exploited so far for the problem of energy detection. Therefore, in this paper are obtained, asymptotic expressions of average \( P_d \) in a fixed-gain cooperative relay network with MRC at the destination. Upper and lower bounds on relay path SNR are considered for mathematical tractability. Then, probability density function (PDF) of these approximate SNR bounds is evaluated to compute average \( P_d \) expressions for relay path signal reception. Similarly, PDF is evaluated for the cooperative system with MRC to combine the relay path and direct path signals. This PDF is then used to compute average \( P_d \) expressions for the considered cooperative system. Since the derived analytical expressions were in the form of infinite summation series, their respective truncation error bounds are also obtained to compute the required number of terms necessary to achieve a given accuracy.

II. SYSTEM MODEL

An orthogonal system in time, such as, time division multiple access (TDMA) based wireless system is considered in this work. The system consists of an information source...
A fixed-gain relay (R) and a destination node (D) as shown in Fig. 1. This cooperative system is assumed to be operating over independent and identically distributed (IID), slowly varying and flat, Rayleigh fading channels, where $h_{xy}$ is the channel gain for the $X \rightarrow Y$ link. Therefore the instantaneous SNR, denoted as $\gamma$, is distributed according to exponential distribution whose CDF and PDF can be expressed respectively as,

$$F_{\gamma_{xy}}(\gamma) = 1 - e^{-\frac{\gamma}{\gamma_{xy}}}$$  \hspace{1cm} (1)

$$f_{\gamma_{xy}}(\gamma) = \frac{1}{\gamma_{xy}} e^{-\frac{\gamma}{\gamma_{xy}}}$$  \hspace{1cm} (2)

where $\gamma_{xy}$ is the average SNR over the $X \rightarrow Y$ link.

Destination receives two copies of the source transmitted signals at the end of two time slots as follows:

$$x_{pd}(t) = h_{pd}s(t) + n_{pd}(t)$$

$$x_{rel}(t) = G_r h_{rd} (h_{pr}s(t) + n_{pr}(t)) + n_{rd}(t) \hspace{1cm} (3)$$

where $h_{xy}$ is the fading channel for $X \rightarrow Y$ link, $s(t)$ is the transmitted signal, $n_{xy}$ is the circularly symmetric and complex additive white gaussian noise (AWGN) for the $X \rightarrow Y$ link, having zero mean and variance $N_0$, and $G_r = \sqrt{\frac{1}{E[|n_{pr}|^2]+N_0}}$ is the relay gain with $E[\cdot]$ being the expectation operator. It can be shown that the received instantaneous SNR for the $P \rightarrow R \rightarrow D$ link is expressed as [18]

$$\gamma_{rel} = \frac{\gamma_{pr} \gamma_{rd}}{C + \gamma_{rd}} \hspace{1cm} (4)$$

where $C = E_{rd}/(G_r^2 N_0)$ is the constant for fixed gain. Finally, the destination node combines the direct path and the relay path signals using MRC. Then the whole received SNR at the output of the MRC can be expressed as,

$$\gamma_{MRC} = \gamma_{pd} + \gamma_{rel}. \hspace{1cm} (5)$$

Although, the notion of using MRC in colligation with energy detector seems contradictory, the objective of this assumption is to obtain the theoretically optimum achievable detection performance of energy detector in a fixed-gain cooperative relay network. Many recent works assume the use to pre-detection combiners in conjunction with energy detector, e.g., [6], [8], [19]. Note that all pre-detection combiners require full or partial CSI of all branches at the destination which can be available at the cognitive receiver via a control channel or a broadcast channel through an access point in a cognitive radio network [20], [21]. Fig. 2 depicts a hypothetical block diagram of the assumed destination (or receiver) terminal [17]. As shown, destination employs an energy detector preceded by a dual branch MRC, where, $w_{pd}$ and $w_{rel}$ are the weight factors on each branch. It is gratuitous to say that the best option for MRC is to choose the weights to be the fading on each branch.

III. DETECTION PROBABILITY ANALYSIS

In this section, we will outline the formulation for energy detection in non-fading environment followed by the analytical derivations for the average detection probability in fading environment for our proposed system model.

A. Energy detection in non-fading environment

Spectrum sensing problem is a decision problem between the following two hypothesis [6],

$$x(t) = \begin{cases} n(t) & H_0 \\ h_s(t) + n(t) & H_1 \end{cases} \hspace{1cm} (6)$$

where $H_0$ is the hypothesis for the absence of signal while $H_1$ is the hypothesis for the presence of the signal.

In a non-fading environment, the probabilities of false alarm ($P_f$) and detection ($P_d$) are expressed as [7]

$$P_f = \frac{\Gamma(u, \lambda/2)}{\Gamma(u)} \hspace{1cm} (7)$$

$$P_d = \frac{Q_u(\sqrt{2\gamma}, \sqrt{\lambda})}{\sqrt{\lambda}} \hspace{1cm} (8)$$

where $\Gamma(\cdot, \cdot)$ is the upper incomplete gamma function [22, eq. (8.350.2)], $u$ is the time-bandwidth product of the observed signal which signifies the number of observed samples, $\lambda$ is the detection threshold of the energy detector, $\Gamma(\cdot)$ is the complete gamma function and $Q_u(\cdot, \cdot)$ is the $u$th order generalized marcum-Q function [23]. The generalized marcum-Q function
can be expressed by an infinite series representation [15] as follows:

\[
Q_u(\sqrt{2}\gamma, \sqrt{\lambda}) = \sum_{n=0}^{\infty} e^{-\gamma n} \frac{\Gamma(n + u, \lambda/2)}{n!(n + u - 1)!}.
\]  

(9)

In the subsequent analysis, we will use this form to simplify the involved mathematical derivations.

The average \( P_d \) does not depend on the statistics of the wireless channel and can easily be obtained from (7) given \( u \) and \( \lambda \). Therefore, we focus on the detection probability in the sequel.

**B. Average \( P_d \) over fading channels with blind relay**

The average detection probability \( (P_d) \) can be obtained by averaging (8) over the PDF statistics of the instantaneous received SNR, as follows

\[
P_d = \int_{\gamma} Q_u(\sqrt{2}\gamma, \sqrt{\lambda}) f_{\gamma}(\gamma) d\gamma.
\]

(10)

Using the PDF expression of \( \gamma_{rel} \) obtained in [18], authors derived an exact average \( P_d \) expression for the relay-path in dual-hop, fixed-gain relay system in [16]. For reasons that will become lucid in the next subsection, we obtain a tight upper-bound and a lower-bound on the average \( P_d \) for the relay path signals in the following.

Upper and lower bounds on \( \gamma_{rel} \) are defined as [10]

\[
\gamma_{rel} < \gamma_{rel}^{UB} = \min(\gamma_{pr}, \gamma_{rd})
\]

and

\[
\gamma_{rel} \geq \gamma_{rel}^{LB} = \frac{1}{2}\min(\gamma_{pr}, \gamma_{rd}).
\]

(11)

From probability theory, we know that minimum \( \gamma_{rel}^{UB} \) of two exponential random variables with parameters \( 1/\gamma_{pr} \) and \( 1/\gamma_{rd} \) is also exponentially distributed. Hence, the CDFs of \( \gamma_{rel}^{UB} \) and \( \gamma_{rel}^{LB} \) are respectively expressed as

\[
F_{\gamma_{rel}^{UB}}(y) = 1 - e^{-\left(\frac{1}{\gamma_{pr}} + \frac{1}{\gamma_{rd}}\right)y}
\]

and

\[
F_{\gamma_{rel}^{LB}}(y) = F_{\gamma_{rel}^{UB}}(2y).
\]

(12)

Taking first order derivative of the above with respect to \( y \), we get the respective PDFs of \( \gamma_{rel}^{UB} \) and \( \gamma_{rel}^{LB} \) as follows:

\[
f_{\gamma_{rel}^{UB}}(y) = \left(\frac{1}{\gamma_{pr}} + \frac{1}{\gamma_{rd}}\right) e^{-\left(\frac{1}{\gamma_{pr}} + \frac{1}{\gamma_{rd}}\right)y}
\]

and

\[
f_{\gamma_{rel}^{LB}}(y) = 2 \left(\frac{1}{\gamma_{pr}} + \frac{1}{\gamma_{rd}}\right) e^{-\left(\frac{1}{\gamma_{pr}} + \frac{1}{\gamma_{rd}}\right)y}.
\]

(12)

Now it is possible to substitute (12) and (9) in (10) to obtain average \( P_d \) over the relay path can be respectively expressed as

\[
P_{d,rel}^{UB} = \left(\frac{1}{\gamma_{pr}} + \frac{1}{\gamma_{rd}}\right) \sum_{n=0}^{\infty} \left[ \frac{1}{(1 + \frac{1}{\gamma_{pr}} + \frac{1}{\gamma_{rd}})^{n+1}} \right] \frac{\Gamma(n+u, \gamma_{pr})}{\Gamma(n+u)}
\]

(13)

Having obtained the asymptotic expressions for average \( P_d \) of the fixed-gain relay path in the form of infinite summation series, it is necessary to truncate this infinite summation series to finite terms. For fixed values of \( \gamma_{pr}, \gamma_{rd}, u, \) and \( \lambda \), following the method as adopted in [15], i.e., taking a finite sum up to \( N \) terms and subtracting it from the infinite sum, the error bound in truncating the infinite summation series of (13) and (14) to finite terms in this case can be calculated respectively as

\[
|E_{rel}^{UB}| = \left[ \left( F_{\gamma_{rel}^{UB}}(x) - \sum_{n=0}^{N} \frac{1}{(1+\gamma_{pr})^n} \right) \frac{\gamma_{pr}}{\Gamma(n+u, \gamma_{pr})} \right] \frac{\Gamma(n+u, \gamma_{pr})}{\Gamma(n+u)}
\]

(15)

\[
|E_{rel}^{LB}| = \left[ \left( F_{\gamma_{rel}^{LB}}(x) - \sum_{n=0}^{N} \frac{1}{(1+\gamma_{pr})^n} \right) \frac{\gamma_{pr}}{\Gamma(n+u, \gamma_{pr})} \right] \frac{\Gamma(n+u, \gamma_{pr})}{\Gamma(n+u)}
\]

(16)

where the function \( F_{\gamma_{rel}^{UB}}(x) = \sum_{n=0}^{N} \frac{1}{(1+\gamma_{pr})^n} x^n \) is a special case of generalized hypergeometric series [22, eq.(9.14.1)], \((a)_n\) denotes the Pochhammer symbol such that \((a)_n = \frac{\Gamma(a+n)}{\Gamma(a)}\) and \(\gamma_{pr} = \frac{1}{\gamma_{pr}} + \frac{1}{\gamma_{rd}}\). The finite number of terms \(N\) required to achieve a given figure of accuracy can easily be obtained from this expression.

**C. Average \( P_d \) over fading channels for proposed cooperative system**

When the relay path and direct path signals are combined as per (5), it is required to compute the convolution of the PDFs of \( \gamma_{rel} \) and \( \gamma_{pd} \) in order to evaluate the PDF of the combined signal SNR \( \gamma_{MRC} \). It seems impossible to evaluate this convolution integral due to the presence of modified bessel functions \( K_0(\cdot) \) and \( K_1(\cdot) \) in the exact PDF expression of \( \gamma_{rel} \) which was utilized in [16]. Hence, most of the works involving fixed-gain relays and MRC directly compute the moment generating function (MGF) of the combined signal at the destination [24], [25] and use these statistics for various performance evaluation purposes. However, the MGF method does not permit us to obtain average \( P_d \) expressions for our assumed system model. For this very reason we use the tight upper and lower bounds of \( \gamma_{rel} \) in this work to address the problem of spectrum sensing in cooperative relay networks.

Proceeding forward, having obtained \( f_{\gamma_{rel}^{UB}}(y) \) and \( f_{\gamma_{rel}^{LB}}(y) \), we can now derive the PDFs of upper and lower bounds of \( \gamma_{MRC} \) using the formula:

\[
f_{\gamma_{MRC}}(z) = \int_0^z f_{\gamma_{pd}}(z)f_{\gamma_{rel}^{LB}}(z-y) \, dz.
\]

(17)
Fig. 3. ROC curves comparing analytical bounds of average $P_d$ for the relay path with the exact analytical expression of [16] at different SNR values and $u=2$.

Using (12), we obtain the PDFs of $\gamma_{d,MRC}^{UB}$ and $\gamma_{d,MRC}^{LB}$, namely;

$$f_{\gamma_{d,MRC}^{UB}}(z) = \frac{1}{\gamma_{pd} - \left(\frac{1}{\gamma_{pr} + \frac{1}{\gamma_{rd}}}\right)} \left[ e^{\frac{1}{\gamma_{pd}} - e^{-\left(\frac{1}{\gamma_{pr} + \frac{1}{\gamma_{rd}}}\right)}} \right]$$

$$f_{\gamma_{d,MRC}^{LB}}(z) = \frac{1}{\gamma_{pd} - \left(\frac{1}{\gamma_{pr} + \frac{2}{\gamma_{rd}}}\right)} \left[ e^{\frac{1}{\gamma_{pd}} - e^{-\left(\frac{2}{\gamma_{pr} + \frac{2}{\gamma_{rd}}}\right)}} \right]$$

As shown in the previous subsection, now it is straightforward to obtain the upper and lower bounds on average $P_d$ by using (18), (9) and (10), and solving the integrals in similar manner to get

$$P_{d,MRC}^{UB} = \frac{1}{\gamma_{pd} - \left(\frac{1}{\gamma_{pr} + \frac{1}{\gamma_{rd}}}\right)} \sum_{n=0}^{\infty} \left[ \frac{1}{\left(1 + \frac{1}{\gamma_{pd}}\right)^n} - \frac{1}{\left(1 + \frac{1}{\gamma_{pr} + \frac{1}{\gamma_{rd}}}\right)^n} \right] \frac{\Gamma(n + u, \frac{1}{\gamma_{pd}})}{\Gamma(n + u)}$$

$$P_{d,MRC}^{LB} = \frac{1}{\gamma_{pd} - \left(\frac{2}{\gamma_{pr} + \frac{2}{\gamma_{rd}}}\right)} \sum_{n=0}^{\infty} \left[ \frac{1}{\left(1 + \frac{1}{\gamma_{pd}}\right)^n} - \frac{1}{\left(1 + \frac{2}{\gamma_{pr} + \frac{2}{\gamma_{rd}}}\right)^n} \right] \frac{\Gamma(n + u, \frac{1}{\gamma_{pd}})}{\Gamma(n + u)}$$

Likewise, the truncation error bound when truncating the infinite summation series to finite terms in above expressions can be computed as

$$[E_{MRC}^{UB}] = \frac{1}{\gamma_{pd} - \left(\frac{1}{\gamma_{pr} + \frac{1}{\gamma_{rd}}}\right)} \times \left[ \left\{ \frac{1}{\Gamma(1; \frac{1}{\gamma_{pd}})} - \sum_{n=0}^{N} \left(\frac{1}{\frac{1}{\gamma_{pd}}}\right)^n \right\} \frac{1}{1 + \frac{1}{\gamma_{pd}}} \right]$$

Fig. 4. ROC curves comparing analytical bounds of average $P_d$ of the fixed-gain cooperative relay network with its simulation counterpart at different SNR values and $u=2$.

In the next section, we provide the simulation results to show the accuracy of our analysis.

IV. SIMULATIONS AND ANALYTICAL RESULTS

The receiver operating characteristics (ROC) curves are plotted here to validate the accuracy of the derived analytical expressions. To plot these curves, IID flat Rayleigh faded channels are generated with equal average SNR on each link, i.e., $\tilde{\gamma}_{pd} = \tilde{\gamma}_{pr} = \tilde{\gamma}_{rd} = \tilde{\gamma}$. The value of average SNR on each link is varied between 0dB, 3dB and 6dB to show the effect of increased SNR on the detector performance. Further, the number of samples ($u$ in our notation) is fixed at 2 and $\lambda$ is calculated by varying $P_f$ from 0 to 1, using (7). Then each value of $\lambda$ versus $P_f$ and $u=2$ is used to compute the corresponding average $P_d$.

In Fig. 3, the analytical curves obtained from (13) and (14) are plotted for various SNR values. For comparison, exact average $P_d$ curves are also plotted which were obtained from [16, eq. (15)]. It can be seen that the curves for $P_{d,rel}^{UB}$ lie...
above the exact average $P_{d,rel}$ curve of [16], at various SNR values. Likewise, $\overline{P}_{d,rel}$ curves are just below the exact analytical curves of average $P_{d,rel}$, validating the accuracy of the presented analyses. Also, it can be seen that the performance of the energy detector deteriorates as the $\gamma$ decreases, which indicates the general behavior of the energy detectors.

The analytical curves generated from the asymptotic average $P_d$ expressions in (19) and (20) are shown in Fig. 4. Because there is no analytical study which covers the average $P_d$ in fixed-gain cooperative relay networks with MRC, corresponding simulation curves for average $P_d$ in such a system model are also plotted for comparison. Clearly, the analytical asymptotic curves lie tightly close to the exact simulation curve, hence again confirming the accuracy and tightness of the analyses of this study. Further, it is evident that the detection performance of the energy detector increases due to cooperation. It is understandable because with cooperation, the received SNR at the destination improves which contribute towards the better performance of the energy detector.

In Table I, are tabulated the required number of terms $(N)$, obtained from (15), (16), (21) and (22), respectively, for various values of $\gamma$. Interestingly only a fewer terms are required in all cases to achieve high accuracy.

<table>
<thead>
<tr>
<th>Number of terms $(N)$</th>
<th>$\gamma=0$dB</th>
<th>3dB</th>
<th>6dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>From (13)</td>
<td>05</td>
<td>09</td>
<td>16</td>
</tr>
<tr>
<td>From (16)</td>
<td>02</td>
<td>05</td>
<td>09</td>
</tr>
<tr>
<td>From (21)</td>
<td>10</td>
<td>18</td>
<td>33</td>
</tr>
<tr>
<td>From (22)</td>
<td>10</td>
<td>17</td>
<td>32</td>
</tr>
</tbody>
</table>

V. CONCLUSION

Detailed analyses were presented in this paper to evaluate detection performance of energy detector in fixed-gain cooperative relay networks. Asymptotic expressions for average detection probability were obtained both for the fixed-gain relay communication and for the cooperative system with MRC at the destination. Because the derived expressions were in the form of infinite summation series, their respective truncation error bounds were also computed. In the end, analytical results were validated with the help of simulations. These analyses are anticipated to aid in quantifying the future communication networks such as cognitive radios.

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