The Coulomb Interaction between Pion-Wavepackets: The π^+–π^- Puzzle

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Abstract

The time dependent Schrödinger equation for π^+–π^- pairs, which are emitted from the interaction zone in relativistic nuclear collisions, is solved using wavepacket states. It is shown that the Coulomb enhancement in the momentum correlation function of such pairs is smaller than obtained in earlier calculations based on Coulomb distorted plane waves. These results suggest that the experimentally observed positive correlation signal cannot be caused by the Coulomb interaction between pions emitted from the interaction zone. But other processes which involve long-lived resonances and the related extended source dimensions could provide a possible explanation for the observed signal.

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1 Introduction

Originally, the method of intensity-interferometry ("Hanbury-Brown and Twiss interferometry", HBT) was developed in astronomy and applied to determine the angular radii of stars, which were too small to be resolved in ordinary optical instruments [1]. Shortly afterwards it was recognized that a similar procedure could also be applied to pions emitted in high energy reactions, whereby the size of the pion-production volume becomes a measurable quantity [2]. However, it is clear that the different physical environment in high energy- or heavy ion reactions asks for certain modifications of the original formalism. The following facts have to be taken into account:

1. The wavelength λ of visible light is about 500 nm and the coherence length of a photon $\Lambda \approx \tau\, c$ with a typical emission time of $\tau \approx 10^{-8}$ s is about 7 orders of magnitude larger. Therefore, the photon can be well described as a plane wave. A typical pion, produced via $\Delta(1232)$-decay with a momentum $p_{\rm o} = 150$ MeV/c has the de Broglie wavelength $\lambda_{\rm o} \approx 8$ fm, and the coherence length is defined as ([3], p. 158) $\Lambda = \lambda_{\rm o}^2/\Delta\lambda \approx 6$ fm. Here, $\Delta\lambda$ is the uncertainty in the wavelength, related to the energy width $\Delta E \approx 120$ MeV of the $\Delta(1232)$ resonance via $\Delta\lambda = 2\pi\,\hbar/\Delta E$. Since $\lambda_{\rm o}$ and Λ are of the same size, the pion cannot be described as a plane wave.

2. The radius R of a star is typically 8 orders of magnitude larger than the coherence length of the emitted light, and therefore coherence effects at emission time can be neglected: The star is a chaotic radiator. The pion source has a typical size of a few fm which is of the same order as the coherence length of the pion. Therefore, interference effects in the source have to be taken into account, as it is consistently done in the wavepacket model [4-6].

3. Photons are neutral, whereas the pion correlations (with exception of $\pi^{\rm o}$) are modified by the Coulomb interaction.

The Coulomb distortion of the correlation function and the corresponding correction has been the topic of many recent publications [7-10], where the pions where described either as plane waves or as classical particles. The several existing theoretical approaches all have in common that they do not consider the zero point energy which emerges as a consequence of the finite size of the pion source. However, in a recent wavepacket calculation of identical and Coulomb interacting pions it was found that the correlation function does not exhibit any significant distortion, a fact which was explained by the strong momentum uncertainty of the wavepackets [11]. This argument, if valid, should also apply to the correlations of π^+–π^- pairs. Here, positive correlations were found experimentally [12], which are interpreted as caused by the Coulomb attraction between both pions and equivalently used to Coulomb-correct the correlation functions for like-charge pions.

It is therefore of crucial importance to verify the conclusions of Ref. [11] also for the π^+–π^- system of non-identical mesons, where the wavefunction does not have to be symmetric with respect to particle exchange. This system therefore displays the consequences of the quantum behaviour, and in particular those of the Heisenberg uncertainty principle, most purely, and it allows to study the modifications when the system approaches the classical limit. To this end, we compute the correlation function of π^+–π^- pairs using wavepackets for the pion-states. We again discuss the important role of the dispersion which determines the size of the Coulomb distortion. We compare the results with experimental data and finally discuss the implications.

2 The multiconfigurational procedure

The method to solve the time dependent Schrödinger equation for π^+–π^- pairs,

$$\hat{H}\Psi(\mathbf{r}_1, \mathbf{r}_2, t) = i\hbar\partial_t\Psi(\mathbf{r}_1, \mathbf{r}_2, t) \ , \tag{1}$$

with the Hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2m}(\Delta_1 + \Delta_2) + \frac{-e^2}{|\mathbf{r}_1 - \mathbf{r}_2|} \tag{2}$$

is similar to the method used in [11]. For the initial 2-particle wavefunction we use the product state

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, t = 0) = \psi_1(\mathbf{r}_1, t = 0) \cdot \psi_2(\mathbf{r}_2, t = 0) \ , \tag{3}$$

where the single particle states are Gaussians, i. e.

$$\psi_j(\mathbf{r}_j, t = 0) = (2\pi\sigma_{\mathrm{o}}^2)^{-3/4} \exp\left(\frac{i}{\hbar}\mathbf{P}_j \cdot \mathbf{r}_j - \frac{(\mathbf{r}_j - \mathbf{R}_j)^2}{4\sigma_{\mathrm{o}}^2}\right) \tag{4}$$

with $j \in \{1, 2\}$. $\mathbf{R}_j$ is the initial position of the j'th wavepacket-centre, $\mathbf{P}_j$ the initial momentum and σ_{o} the initial wavepacket-width. Transforming into center of mass (com) and relative (rel) coordinates, the state is rewritten as

$$\Psi(\mathbf{r}_{\mathrm{c}}, \mathbf{r}, t = 0) = \Psi_{\mathrm{com}}(\mathbf{r}_{\mathrm{c}}) \cdot \Psi_{\mathrm{rel}}(\mathbf{r}) \tag{5}$$

with $\mathbf{r}_{\mathrm{c}} = (\mathbf{r}_1 + \mathbf{r}_2)/2$ and $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$,

$$\Psi_{\mathrm{com}}(\mathbf{r}_{\mathrm{c}}) = (\pi\sigma_{\mathrm{o}}^2)^{-3/4} \exp\left(\frac{i}{\hbar}\mathbf{P}_{\mathrm{c}} \cdot \mathbf{r}_{\mathrm{c}} - \frac{(\mathbf{r}_{\mathrm{c}} - \mathbf{R}_{\mathrm{c}})^2}{2\sigma_{\mathrm{o}}^2}\right) \ , \tag{6}$$

and

$$\Psi_{\mathrm{rel}}(\mathbf{r}) = (4\pi\sigma_{\mathrm{o}}^2)^{-3/4} \exp\left(\frac{i}{\hbar}\mathbf{P} \cdot \mathbf{r} - \frac{(\mathbf{r} - \mathbf{R})^2}{8\sigma_{\mathrm{o}}^2}\right) \ . \tag{7}$$

Here, $\mathbf{R}_{\mathrm{c}} = (\mathbf{R}_1 + \mathbf{R}_2)/2$, $\mathbf{R} = \mathbf{R}_1 - \mathbf{R}_2$, $\mathbf{P}_{\mathrm{c}} = \mathbf{P}_1 + \mathbf{P}_2$ and $\mathbf{P} = (\mathbf{P}_1 - \mathbf{P}_2)/2$. Since there are no external forces, the time dependent solution for Ψ_{com} is the solution for a free wavepacket, i. e.

$$\Psi_{\mathrm{com}}(\mathbf{r}_{\mathrm{c}}, t) = (\pi s_{\mathrm{c}}(t)^2)^{-3/4} \exp\left(\frac{i}{\hbar}\left(\mathbf{P}_{\mathrm{c}} \cdot \mathbf{r}_{\mathrm{c}} - E_{\mathrm{c}}t\right) - \frac{(\mathbf{r}_{\mathrm{c}} - \mathbf{R}_{\mathrm{c}})^2}{2s_{\mathrm{c}}(t)\sigma_{\mathrm{o}}}\right) \ . \tag{8}$$

Here, $E_{\mathrm{c}} = P_{\mathrm{c}}^2/(2M)$ with $M = 2m$ and the pion mass $m = 140$ MeV, finally

$$s_{\mathrm{c}}(t) = \sigma_{\mathrm{o}}\left(1 + \frac{i\hbar t}{M\sigma_{\mathrm{o}}^2}\right) \ . \tag{9}$$

In order to evaluate the time dependent solution for $\Psi_{\mathrm{rel}}(\mathbf{r},t)$, an expansion into a set of basis functions is required. As basic components for our basis we use the same time dependent wavefunctions as in [11], i. e.

$$\psi_{l,m,n}(\mathbf{r},t) = \sqrt{\mathcal{N}_{l,m,n}}\, x^l\, y^m\, z^n \exp\left(\frac{i}{\hbar}\left(\mathbf{P}(t)\cdot\mathbf{r} - \theta_{l,m,n}(t)\right) - \frac{(\mathbf{r}-\mathbf{R}(t))^2}{8s(t)\sigma_{\mathrm{o}}}\right) \tag{10}$$

with the phase

$$\theta_{l,m,n}(t) = \int_0^t E(\tau)\, d\tau + \hbar\,(l+m+n+3/2)\, \arctan\left(\frac{\hbar t}{2\mu\sigma_o^2}\right) \tag{11}$$

and $x^l = (\mathbf{r}_x - \mathbf{R}_x)^l$, and similar for the y- and z coordinates. Further, $\mathbf{P} = (\mathbf{P}_1 - \mathbf{P}_2)/2$, μ is the reduced mass, l, m and n are positive integers,

$$s(t) = \sigma_{\mathrm{o}}\left(1 + \frac{i\hbar t}{4\mu\sigma_{\mathrm{o}}^2}\right) \tag{12}$$

and the norm is

$$\mathcal{N}_{l,m,n} = \frac{(4\pi\sigma^2)^{-3/2}\,(\sqrt{2}\,\sigma)^{-2(l+m+n)}}{(2l-1)!!\,(2m-1)!!\,(2n-1)!!} \tag{13}$$

with $\sigma = |s|$ and $j!! = j\,(j-2)\,(j-4)\cdots$. We may call a state with $k = l+m+n = 0$ "s-state", similar "p-state" for $k=1$, "d-state" for $k=2$ and "f-state" for $k=3$. The basis functions $\psi_{l,m,n}(\mathbf{r},t)$ are not orthogonal for different sets $\{l,m,n\}$, but linear combinations of them can be found which are orthogonal [13]. We use the following orthogonal basis:

$$\begin{aligned}
s:&\quad \psi_{000}\\
p:&\quad \psi_{100},\ \psi_{010},\ \psi_{001}\\
d:&\quad \psi_{200}-\psi_{020},\ \psi_{011},\ 2\psi_{002}-\psi_{200}-\psi_{020},\ \psi_{101},\ \psi_{110}\\
f:&\quad \psi_{300}-3\psi_{120},\ \psi_{201}-\psi_{021},\ 4\psi_{102}-\psi_{300}-\psi_{120},\ \psi_{030}-3\psi_{210},\\
&\quad 2\psi_{003}-3\psi_{201}-3\psi_{021},\ 4\psi_{012}-\psi_{210}-\psi_{030},\ \psi_{111}.
\end{aligned} \tag{14}$$

The expansion of Ψ_{rel} now reads

$$\Psi_{\mathrm{rel}}(\mathbf{r},t) = \sum c_k(t)\, \psi_{\vec{l}_k}(\mathbf{r},t) \tag{15}$$

with the c-valued coefficients

$$c_k(t) = \langle \psi_{\vec{l}_k}(\mathbf{r},t) | \Psi_{\mathrm{rel}}(\mathbf{r},t) \rangle \tag{16}$$

and the index-vectors $\vec{l}_k = (l_k, m_k, n_k)$. The equation of motion for the expansion coefficients is

$$i\hbar\, \dot{\mathbf{c}} = \left(\tilde{H} - \tilde{D}\right)\, \mathbf{c} \tag{17}$$

with the time dependent coefficient vector $\mathbf{c}$, the Hamilton matrix $\tilde{H}_{ij} = \langle \psi_{\vec{l}_i} | \hat{H} | \psi_{\vec{l}_j} \rangle$, and the time evolution matrix $\tilde{D}_{ij} = \langle \psi_{\vec{l}_i} | i\hbar\partial_t | \psi_{\vec{l}_j} \rangle$.

Equation (17) can be regarded as the quantum mechanical part of the time evolution, which is necessary to account for the deformation of the wavepacket. A second, classical contribution is given by the motion of the wavepacket-centre. The corresponding equation of motion is defined as

$$\dot{\mathbf{P}} = -\nabla_{\mathbf{R}} \langle \Psi_{\rm rel}(\mathbf{r}) | V(\mathbf{r}) | \Psi_{\rm rel}(\mathbf{r}) \rangle \tag{18}$$

with $V(\mathbf{r}) = -e^2/|\mathbf{r}|$. The evaluation of the matrix-elements is described in [13]. The numerical solution of the equation of motion was done the same way as described in [11], i. e. a second order integrator was used for the integration, the simulation was stopped after the Coulomb-potential has dropped to 2% of its original value, and the final state was Fourier-transformed into momentum space. In order to evaluate the 2-particle correlation function of the relative momentum $q = |\mathbf{p}_1 - \mathbf{p}_2|$,

$$\mathcal{C}_2(q) = \frac{\mathcal{P}_2(\mathbf{p}_1, \mathbf{p}_2)}{\mathcal{P}(\mathbf{p}_1)\,\mathcal{P}(\mathbf{p}_2)} \ , \tag{19}$$

a Monte-Carlo procedure [4] was used to sample the 2-particle probability density $\mathcal{P}_2(\mathbf{p}_1, \mathbf{p}_2)$, and event-mixing was used to obtain the denominator of Eq. (19). As described in [11], it is possible to generate a large number N_s of pairs from one single momentum state. After a number of N_e independent events are calculated, the total number of pairs equals $N_p = N_e{\cdot}N_s$.

3 Results

As source function, i. e. the initial phase space distribution of the wavepackets, we applied the following simple parametrization:

$$\rho(\mathbf{R}) = (\pi R_s^2)^{-3/2} \exp\left(\frac{-\mathbf{R}^2}{R_s^2}\right) \ , \tag{20}$$

$$f(\mathbf{P}) = (2\pi m T)^{-3/2} \exp\left(\frac{-\mathbf{P}^2}{2mT}\right) \ , \tag{21}$$

i. e. the source was a static Gaussian with rms- radius $\sqrt{3/2}R_s$ and temperature T. The initial width σ_o of the wavepackets is to some extent arbitrary, but there exist certain upper and lower limits. Since any localization of a quantum state is accompanied with the corresponding zero point energy

$$E_o = \frac{3\hbar^2}{8m\sigma_o^2} \ , \tag{22}$$

the observed pion energies [14, 15] do not allow the localization to be well below ≈ 1 fm, which already yields $E_o(\sigma_o = 1\text{fm}) \approx 100$ MeV. On the other hand, the correlation radii which are observed in correlations of identical pions also depend on σ_o, because the effective radius of the source is given in 1. order by

$$R_{\rm eff} = \sqrt{R_s^2 + 2\sigma_o^2} \ . \tag{23}$$

Since the measured radii have a typical size $R_{\rm corr} \approx 6$ fm [12], the upper limit for σ_o may not significantly exceed ≈ 4 fm.

In Fig. 1 the correlation functions are shown for 3 different initial wavepacket widths. In one of the cases ($\sigma_o = 4$ fm, $R_s = 0$ fm) the width of the wavepacket determines the source size. In a second case, $\sigma_o = 1.8$ fm and $R_s = 5$ fm yield the same effective source size $R_{\rm eff} \approx 5.7$ fm as in the first case. The simulation with $\sigma_o = 20$ fm corresponds to the effective source size $R_{\rm eff} = 28$ fm, it is therefore not compatible with the measured radii in like-charge pion interferometry and should be considered as a pure exercise to study the dependence of the Coulomb effect on σ_o. In all cases the source temperature was set to $T = 50$ MeV. The states were evaluated using the *spd*-basis, in test simulations it was checked that the increase to the larger *spdf*-basis did not change the results in any observable way. The following facts are important:

1. The enhancement in the correlation function for small relative momenta q is much weaker than obtained in the classical trajectory calculation (solid curve).
2. The enhancement becomes more pronounced with an increasing value for the initial width σ_o of the states.

These results confirm the observations made in simulations of identical pions [11]. There, the factual disappearance of the Coulomb signal was explained by the dominance of the zero point energy over the Coulomb energy. The maximum Coulomb energy in a system of 2 Gaussian states is reached when both completely overlap, yielding (without interference, i. e. for nonidentical particles)

$$E_{\rm coul}^{\rm max} = \frac{Z^2 e^2}{\sqrt{\pi}\sigma_o} \, . \tag{24}$$

We define the ratio

$$\xi \equiv \frac{E_{\rm coul}^{\rm max}}{E_o} = \frac{8e^2}{3\sqrt{\pi}\hbar^2} \cdot Z^2 m \sigma_o \tag{25}$$

as the *relative Coulomb strength* of the system. We obtain $\xi = 1.4 \cdot 10^{-2}$ for the $\sigma_o = 1.8$ fm case, $\xi = 3.1 \cdot 10^{-2}$ for $\sigma_o = 4$ fm and $\xi = 0.16$ for $\sigma_o = 20$ fm. Due to the smallness of ξ, in all cases the system is dominated by the quantum dispersion. As a crosscheck, the fundamental constant $\hbar$ was reduced to 1/10 of its real value. This implies a reduction of E_o by a factor of 100 and, for $\sigma_o = 1.8$ fm, a relative Coulomb strength $\xi = 1.4$, which is now in the classical domain. The simulation results (with source size $R_s = 5$ fm, stars) are already in good agreement with the corresponding classical trajectory-calculation for pointlike particles ($R_s = 5.7$ fm, solid curve).

Similary, an increase in particle mass and/or charge can shift the ratio ξ towards the classical domain. For example, protons with $\sigma_o = 4$ fm correspond to $\xi = 0.2$, α-particles with $\sigma_o = 4$ fm correspond to $\xi = 3.4$. In simulations, a strong Coulomb signal was observed for these particles. Nevertheless, for small relative momenta, a deviation from the classical results remains, since the momentum uncertainty $\sigma_p = \hbar/(2\sigma_o) = 25$ MeV/c does not depend on the particle mass, but reduces the momentum resolution which becomes significant for the rapid rise of the Coulomb signal when q approaches zero.

4 Discussion

In proton- [16] or nucleus- [12] induced reactions with energies above 10 AGeV, rather strong positive correlations in π^+–π^- pairs have been observed. These are interpreted as a Coulomb effect — a conclusion which is supported by classical trajectory and/or plane-wave scattering calculations. In this work and in [11] we have tried to argue that both assumptions are not applicable for particles which are emitted from a source, the size of which is 2 orders of magnitude smaller than the Bohr-radius of π^+–π^- pairs. Because of Heisenberg's uncertainty principle a spatial uncertainty of size σ_o transforms into a momentum uncertainty which allows the mutual particle interactions to become discernible in momentum correlations only when the parameter ξ in Eq. (25) is of order 1. The value of ξ can increase by either increasing the interaction strength, the delocalization of the states or the reduced system mass. For the π^+–π^- system, the corresponding parameters under normal conditions are too small. Therefore, it is highly desirable to find an interpretation of the π^+–π^- signal. Obviously, there are two ways out of the dilemma:

1. The observed correlations *are* of Coulomb origin, and there is a yet unknown inconsistency in the wavepacket model and/or its interpretation. For example, if the pion-states could be initiated with an arbitrary large wavepacket width σ_o, then the ratio ξ in Eq. (25) easily could reach values above 1 and hence the classical (plane-wave) assumption would be justified.

2. The observed correlations have another origin, for example strong interactions, or residual dynamical correlations which could not be eliminated in the event mixing procedure during data analysis.

With respect to the first point, i. e. the wavepacket size, one has to bear in mind that the pion wavefunction is interpreted as the probability amplitude to detect the pion at position $\mathbf{r}$ in case of a measurement, and there is no physical reason to assume that the pion can be found far out of the source already at emission time. Additionally, the short lifetime of the $\Delta(1232)$-resonance, the scattering and reabsorption by the surrounding nuclear matter and, particulary, the experimentally observed small values for the effective source sizes do not suggest the description of the pion state by a widely extented coherent wavefunction. Consequently, σ_o cannot exceed the size of the source, especially since that size is to be determined in the interferometry of like-charge pions. In order to measure an object with a given spatial extent R_s, one unavoidably has to cope with the corresponding momentum uncertainty.

But there exists a third and to some extent speculative possibility, which has also been discussed in [16]: With increasing energies, more and more long-living resonances are formed, which do not decay within a small volume. Pions which are produced under such circumstances do not necessarily fall under the above considerations, i. e. they may approximately be described as plane waves. These pions could contribute to the observed π^+–π^- peak, but this process would not work for π^+–π^+ and π^-–π^- pairs emitted from the short lived and smaller interaction zone formed in heavy ion collisions. Consequently, the measured correlation function of like-charge pions should be corrected by a Coulomb factor which is weighted with the fraction of pions produced by long-living resonances. This hypothesis does not explain why the observed π^+–π^- correlations are of the observed strength, since evidently not all pions could have been produced far outside the interaction zone — otherwise, there wouldn't be any visible Bose-Einstein signal in the like-charge data. It seems that low energy data, which exclude the influence of long-living resonances, could help to solve the π^+–π^- puzzle.

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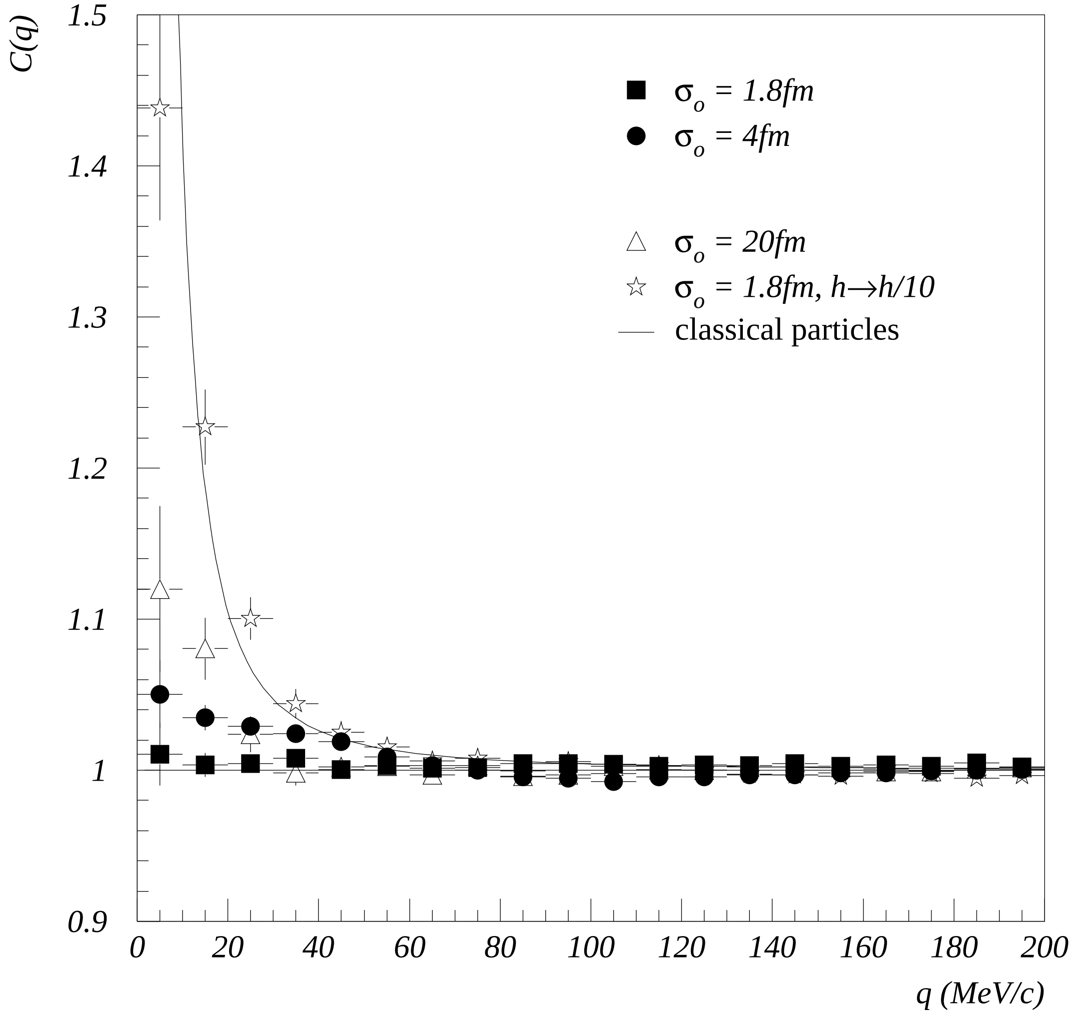

Figure 1: The correlation function for π^+–π^- pairs with different initial localizations σ_o. Systems with $\sigma_o \leq 4$ fm (solid symbols) are compatible with the results from like-charge pion interferometry. The simulation with reduced Planck-constant (stars) displays the approach to the classical limit (solid curve). Number of generated pairs: $5 \cdot 10^7$ (squares), $3 \cdot 10^7$ (circles), $5 \cdot 10^6$ (triangles), $5 \cdot 10^6$ (stars).