A Game-Theoretic Analysis of Link Adaptation in Cellular Radio Networks

Samir V. Ginde, Allen B. Mackenzie, R. Michael Buehrer, and Ramakant S. Komali

Abstract — Game theory is a promising approach for the system-level analysis of power control in wireless networks. This paper extends game-theoretic analysis to the study of link adaptation, which involves the variation of modulation parameters in addition to power control. We use the link adaptation scheme in General Packet Radio Service (GPRS) as an example, although the basic approach is applicable to any centralized wireless system with power and rate control. The action space of a player in our proposed game, called the Link Adaptation Game (LAG), consists of power as usual, and a discrete-valued Adaptable Link Parameter (ALP), e.g., code rate. The utility function is a sigmoid, fitted to the throughput characteristics of a link adaptation scheme, priced by the square of power. We first show the existence of a Nash Equilibrium (NE) in the game. Next, we propose a distributed algorithm to discover the NE. The algorithm is analytically shown to converge to a NE by treating it as a point-to-set map. Simulation results using the GPRS system demonstrate superior throughput and fair system-wide allocation of resources in comparison with other non-game-theoretic methods.

Index Terms — Adaptive modulation, algorithmic map, cellular radio, game theory, power control

I. INTRODUCTION

Non-cooperative game theory [1] has been used in the system-level analysis of power control (PC) in wireless data networks [3]. Specifically, [3] shows how strategic-form [1] game-theoretic techniques may be applied to power control problems. Some notable subsequent research adopting similar game-theoretic approaches is presented in [4][5][6].

It is easy to imagine why game theory is a worthwhile tool in the system-level analysis of radio resource management (RRM) problems. The nodes in a wireless network compete for the use of finite radio resources. If the choice of the radio transmission parameters had no effect on the performance of other users, classical optimization techniques could be applied to obtain optimal parameter assignments. However, the wireless environment is inherently interference-limited, which results in interactions between the nodes.

Consider the classic power control example in CDMA-based systems. Here, the utility of a user may be an increasing function of her signal-to-interference-and-noise ratio (SINR).

If the power levels of all other users are kept constant, this user may simply raise her power level to increase her SINR, and hence her utility. This, however, may have performance-degrading consequences for other users. Increasing one user’s power may necessitate other users to respond by increasing their powers to overcome the interference effects and to maintain their SINRs. This example suggests that RRM problems in wireless networks, in general, often require analyzing an interactive-decision-making process between the players. We adopt the view that the nodes in a wireless network selfishly appropriate as much of the shared radio resources as possible; we also assume that nodes are rational, meaning that they always choose to improve their utilities by selecting actions that are not dominated by other actions in their action set (we discuss the implications of such a best response process in the following sections). From these viewpoints, it is natural to model RRM problems as non-cooperative games [2].

In [3], the power control problem for a code-division multiple access (CDMA) uplink is formulated as a game. The set of players in this game are the mobile terminals. The mobiles choose their powers with the objective of maximizing their utility functions. Each player’s utility varies in direct proportion to the SINR and in inverse proportion to its power. The properties of the utility function are sufficient to show that a Nash Equilibrium (NE) exists in the game. The NE is then shown to be unique (SINRs of all terminals are equal) but not Pareto optimal. In other words, another power vector that improves the utility of some players without decreasing the utilities of the remainder is shown to exist (see Definition 3 in [3]). To bring about a Pareto improvement in the NE the authors introduce linear pricing. This idea has its origins in economics literature (taxation) and has been applied to computer networks [7]. The pricing function penalizes terminals for consuming power. A distributed and asynchronous algorithm that executes at the terminals is shown to generate a sequence of powers that converge to a Pareto-optimal NE. The SINRs of the terminals are no longer equal; they tend to be higher for users in more favorable channel conditions.

In [4], the work in [3] is extended to a multi-cell CDMA network. Assuming that each terminal can only communicate with a single base station - implying that soft handoff is not permitted - the optimal assignment of terminals to base stations is an added dimension to the authors’ earlier problem.
Two solutions are presented. The first reduces the dimension of the problem by assigning each terminal to the closest base station. The second attempts a joint optimization over base station assignments and powers, which results in the assignment of each terminal to the base station at which its SINR is maximized.

The choice of utility function has a significant impact on the properties of the game and the nature of results. It is important for this function to be intuitive, accurate and mathematically tractable. In [5], the uplink of a multi-rate CDMA system is once again studied. The authors use the information-theoretic channel capacity as part of the utility function.

In our work, we employ a sigmoid model [5] for throughput. This model is shown to be the closest approximation (see chapter 4 in [8]) to the typical simulated throughput versus SINR profiles. Unlike the logarithmic utility functions, which work well when a continuous rate/modulation can be selected, the sigmoid model better approximates the throughput performance in the case of discrete code rate selection. Our model is also realistic in the sense that, it doesn’t restrict the action space in any way to ensure minimum SINR guarantees as in the model considered in [12]. This allows users to refrain from transmitting (i.e. select \( p=0 \)), if channel conditions are bad.

The sigmoid model is employed in the utility-based power control (UBPC) scheme presented in [5]. While not strictly employing game theory, the utility-based approach bears several similarities to a more formal game-theoretic approach. The paper also compares the merits of using UBPC to traditional distributed power control (DPC) [9] and justifies that the divergence that occurs in DPC when the system is infeasible is not an issue in UBPC. The reason offered is that the hard SINR target requirement in DPC is “softened” in UBPC. In other words, an unachievable SINR target is reduced, or, in extreme cases, a user is turned off until system feasibility is attained. Detailed analysis is conducted on the choice of sigmoid function parameters and the slope of the linear pricing function. Extensions such as adapting the pricing parameter as either a function of channel conditions, or to mitigate the near-far unfairness that is inherently produced by UBPC, are suggested. Integration of UBPC with admission control, dynamic base-station assignment and link adaptation are further recommendations.

In this paper, the power control game is extended and generalized to incorporate PC schemes with multiple, discrete code rates and/or modulation schemes, also known as link adaptation [10]. Each player chooses a power level and a rate. To date, there has been limited literature on this subject [11][12]. We show that a NE must exist in this Link Adaptation Game (LAG). To find a NE, we propose an iterative and distributed algorithm, termed Algorithm LAG, which follows a best-response dynamic. Algorithm LAG is analytically shown to converge to a NE using the theory of algorithmic maps [13]. The bulk of this paper focuses on the development and convergence of Algorithm LAG.

The second contribution of this paper is to draw system-level performance comparisons between game-theoretical and system-theoretic solutions. In this paper, traditional techniques for the system-level analysis of PC in cellular radio networks involving linear algebra [9][14][15] are collectively referred to as “systems techniques”. Systems techniques have been successfully applied to RRM problems pertaining to third generation (3G) wireless networks [16][17][18]. We conduct a detailed examination of simulation results obtained in a GPRS network using game-theoretic and three other systems approaches, contrasting the merits and demerits of each. The results show that Algorithm LAG achieves a capacity gain over the other competing techniques. Algorithm LAG also results in a relatively fair assignment of rates amongst users.

II. POWER CONTROL AND LINK ADAPTATION IN RADIO RESOURCE MANAGEMENT

The system model used in this paper employs frequency reuse, a typical example of which is a GPRS network. The system consists of a subset of co-channel cells \( T \), consisting of \( N \) users sharing the same frequency. In the following, we focus on the downlink but the analysis could be applied to the uplink. The link gain from each transmitter (in this case the base station) \( j \) to each receiver (in this case the mobile) in cell \( i \), is denoted by \( G_{ij} \). The gain includes both large-scale and small-scale fading. \( G_{ii} \) is the link gain from the transmitter to receiver in cell \( i \in T \). The base-station (BS) transmitter power allocated to link \( i \in T \) is denoted by \( P_i \). The thermal noise power at the receiving mobile station (MS) \( i \in T \) is denoted by \( n_i \). The SINR at MS \( i \), denoted by \( \gamma_i \), is expressed in logarithmic form as follows.

\[
\gamma_i = 10 \log_{10} \left( \frac{G_{ii} P_i}{\sum_{j \neq i} G_{ij} P_j + n_i} \right)
\] (1)

A similar expression can be derived for the uplink. The vector of SINRs of the \( N \) users is defined as \( \Gamma = [\gamma_1, \gamma_2, ..., \gamma_N] \).

Let \( \mathbf{G} = \{G_{ij}\} \) be the link gain matrix of the system. Each element \((i,j)\) of the matrix represents the link gain from BS \( i \) to MS \( j \). The transmitter powers can be expressed in the form of a vector \( \mathbf{P} = [P_1, P_2, ..., P_N]^T \), where the \( P_i \)'s are chosen from set \( P \), defined in (2).

\[
\mathbf{P} = \{p : p \in [P_{\text{min}}, P_{\text{max}}]\}
\] (2)

\( \mathbf{G} \) is assumed to be static for the duration of convergence of the algorithms that execute on each link \( i \in T \). It is assumed that each link \( i \in T \) can adapt a particular link characteristic or parameter \( r_i \), referred to as the adaptable link parameter (ALP). We require that \( r_i \) be chosen from a finite set \( R = \{r_1, r_2, ..., r_M\} \). The number of different values of this parameter is the cardinality \( |R| \) of set \( R \). The ALP might be the modulation scheme or coding scheme of its link. Note that, since the channel conditions are assumed static for each slot
duration, the ALP is adjusted every time slot. In addition, each link \( i \in T \) is power controlled, i.e., it can adjust its power \( P_i \) within the bounds of its power set (2).

The formulation of our model above is quite generic, and allows for its applicability not just to GPRS systems but to CDMA-, WiMAX-, HSD(U)PA-based systems as well. These systems allow for a scalable physical layer, including the support of power control and link adaptation.

In addition to employing power control, the next generation systems (3G, 4G and beyond) such as WiMAX, based on the 802.16 standard, also employ advanced adaptive modulation and coding techniques to enhance achievable data throughput. By mapping the system’s modulation-coding scheme to the channel conditions during a transmission, these systems allow rates to be adjusted (keeping the power of the transmitted signal constant) based on the channel quality. Typically, mobiles close to the base station use higher order modulation and higher code rates than those farther from the base station. By selecting appropriate system parameters, and mappings between code rates and coding schemes, we believe that results similar to those presented in this paper can be drawn for other aforementioned systems as well.

III. A LINK ADAPTATION GAME

The information that is communicated across each link is best-effort type data. There are no pre-assigned priorities among different classes of traffic. The impact of the upper layers (above the medium access control or MAC layer) of the protocol stack on performance, although critical in practice for estimating true application performance, is neglected for simplicity. The techniques proposed in this paper involve the physical and MAC layers only. This simplification does not obstruct the analysis of lower layer performance that we are attempting. For best-effort data, the effective per-user throughput \( L \) is an appropriate measure of link performance. The expression for net throughput at mobile \( i \), in terms of the data rate \( R_i \) bits/sec, and frame error rate (FER) is

\[
L_i = R_i (1 - \text{FER}(\gamma_i, r_i))
\]

(3)

For best-effort data, the maximization of throughput is the objective of each mobile. However, from (3) and (1), we note that this is at cross purposes with the objectives of other mobiles. Elaborating, the throughput is a monotonically increasing function of the SINR \( \gamma_i \). The SINR at the mobile increases in direct proportion to the power it receives (numerator), but in inverse proportion to the interference (denominator). Thus, increasing the throughput for a particular mobile increases the interference it causes to other mobiles. This degrades the throughput of the other mobiles. It is, therefore, possible to model this situation as a non-cooperative game, which we conveniently name the Link Adaptation Game (LAG). A strategic form game is defined by the players, their actions and for each player, a preference relation or utility function, defined over the action sets of all players [1].

A. Players

Each mobile is a player of the LAG, and the player set is denoted by \( T \). The transmitters (base stations in the case of the downlink) adapt their powers and ALPs. The receivers (mobiles in the case of the downlink) estimate interference. In a distributed scheme, the mobiles solve the game-theoretic algorithms on the downlink and convey their decisions to the base stations. The base stations follow the mobiles decisions. Since the mobiles are the decision making entities in the LAG, they are chosen to be the players. Henceforth, we use the term player and mobile interchangeably.

B. Actions

We define the action selected by any player \( i \in T \) as the pair \((P_i, r_i)\), where \( P_i \in P \) is the power of player \( i \) and \( r_i \in R \) is the ALP of player \( i \). The power vector \( P \) and an ALP vector \( r \) are defined as \( P = (P_i)_{i \in T} \), where \( P_i \in [P_{\text{min}}, P_{\text{max}}] \), and \( r = (r_i)_{i \in T} \). The action space \( A_i \) of player \( i \in T \) is defined by \( A_i = P \times R \). The action space of the game is the cross product of the individual action spaces, i.e. \( A = \times (A_i)_{i \in T} \).

C. Utility function

Based on the preceding discussion, we select the effective per-user throughput as a component of the utility function of a player. However, due to the non-cooperative nature of this game, it is clear that in an attempt to maximize throughput at any cost, each mobile is likely to consume maximum power, since throughput increases monotonically with power. This will also create excessive interference, leading instead to performance degradation, since throughput decreases monotonically with interference. Similar observations\(^1\) have been made in other game-theoretic analyses of power control schemes [3]. The solution to this problem in these instances was to introduce pricing. We adopt the same approach to penalize the use of excessive power. This strategy also induces a degree of co-operation amongst players, and can bring about an improvement in system performance. The utility function of player \( i \in T \) is defined as follows.

\[
U_i(P, r) = L_i(\gamma_i, r_i) - C_i(P_i),
\]

(4)

Note that, our model assumes that the channel states are accurately observed and measured. However, if there are errors in the estimation of channel gains, a more robust game theoretic model based on imperfect monitoring is required. Imperfect monitoring games [25] deal with expected payoffs, since the utilities are now random functions. These games are developed to deal primarily with errors in observing the actions of other players; however, the technique can still be applied in principle, in our case.

The throughput function \( L \) is described by (3) and \( C_i(P_i) \) is called the penalty or pricing or cost function. It is a function of

\(^1\) The analogy with the famous “Tragedy of the Commons” essay [Garrett Hardin, Science, Dec 1968] is illuminating. In this instance, the “commons” refers to the shared wireless medium. Overuse of wireless resources can be discouraged by introducing a usage cost.
only the power of player \( i \in T \) and is defined in (5).

\[
C_i(P_t) = K P_t^q
\]  

The parameters of the cost function, \( K \) and \( q \), are positive constants. In our work we set the weighting or scaling parameter \( K \) to 1, and the index or exponent, \( q \) to 2. We shape the penalty function by using \( q > 1 \), such that players are penalized more severely for consuming power than they would be under linear pricing.

The sigmoid function of SINR in (6) has been used [6][12][19] to accurately model the throughput versus SINR profile of a wireless link.

\[
L_i(\gamma', r) = \frac{\alpha(r)}{1 + e^{\alpha(r)/\gamma'}}
\]  

\( \alpha \) is the peak value of the sigmoid function. \( \delta \) is the abscissa of the point where the sigmoid attains maximum slope. When \( \gamma' = \delta \), \( L(\delta, r) = \alpha(r)/2 \). \( \lambda \) is the steepness factor of the sigmoid function. Since the throughput is a function of the ALP, the parameters of the sigmoid function, namely \( \alpha, \lambda, \) and \( \delta \), must be functions of the ALP, as indicated in (6).

In all our simulations and results, we have exemplified our work through a GPRS network, since GPRS supports link adaptation [20]. In GPRS, the modulation scheme is Gaussian minimum shift keying (GMSK), but four options for code rate, referred to as coding schemes (CS), are specified. Thus, the ALP of a GPRS network is its code rate. The parameters of the GPRS coding schemes are summarized in TABLE 1[21] for the TU-50 channel, with ideal frequency hopping. The sets of parameters of the sigmoid model that best approximate simulation results are selected by computer-aided search [8] and summarized in TABLE 2. A sample utility function profile is shown in Fig. 1.

IV. PROPERTIES OF THE LAG

We state some useful properties of the LAG, which will subsequently be used in establishing important results. Wherever the proof is omitted for brevity, the reader is referred to [7]. We now define the key concept of the Nash Equilibrium of a game [1].

The following notation is used in the definition. \( N \) is the set of players; \( i \) is a player in set \( N \). An action-tuple or vector is denoted by \( (a_i)_{i \in N} \). It is the vector (or profile) of actions (or strategies) chosen by the players, each element corresponding to an action of a player. The notation \( a_i \) is used to indicate the profile of actions taken by all players except player \( i \). Thus, it is possible to express an action-tuple using the shorthand \( a = (a_i, a_{-i}) \).

A. Definition 1 – Nash Equilibrium

The Nash Equilibrium (NE) of a strategic game \( G = <N,A,U> \) is an action-tuple \( a' \) which satisfies the following property for all \( i \in N \) [1]

\[
U_i(a'_i, a_{-i}) \geq U_i(a_i, a_{-i}) \quad \forall a_i \in A_i
\]  

Here \( U_i \) is the utility function of player \( i \).

B. Lemma 1

The action space \( A_i \) of player \( i \in T \) is non-empty and compact [13]. This property is proved in [7].

C. Lemma 2

\( A_i \) is not a convex set [13]). However, under the condition that \( r_i \) is fixed, \( A_i \) is a convex set. This property is proved in [7].

D. Lemma 3

\( U_i \) is continuously differentiable on \( A \), when \( r \) is fixed. Hence, \( U_i \) is continuous on \( A \) [7]. This property is proved in [7].

E. Proposition 1

\( U_i \) has a unique global maximum in \( P_0 \), assuming \( r_i \) is fixed. \( U_i \) is also strictly quasi-concave [13] over \( P_i \).

Proof: Fig. 2 shows the first-derivative of \( U_i \) versus \( P_i \) for all \( r_i \). A number of contours are plotted, each for a different value of interference. The lowest interference yields a maximum SINR (ratio of maximum power to interference) of 30 dB. The highest interference yields a maximum SINR of -6 dB. It is apparent that the first-derivative curves all intersect the \( P_i \)-axis only once, at \( \overline{P}_i \), say. At \( \overline{P}_i \), the derivatives have negative slope. Thus, \( \overline{P}_i \) is the unique global maximum of \( U_i \) with respect to \( P_i \). \( U_i \) is non-decreasing for all \( P_i \geq \overline{P}_i \). \( U_i \) is non-increasing for all values of \( P_i > \overline{P}_i \). To prove \( U_i \) is strictly quasi-concave, consider two arbitrarily-selected points \( p_1 \) and \( p_2 \) in \( P \), \( p_1 < p_2 \).

Case 1: Suppose \( p_1 < \overline{P} \) and \( p_2 < \overline{P} \). Then, due to the monotonic nature of \( U_i \), \( \forall P_i \in (p_1, p_2) \),

\[
U_i(P_l) > \min(U_i(p_1), U_i(p_2)) \in this interval of \( P \).
\]

The case when \( p_1 > \overline{P} \) and \( p_2 > \overline{P} \) is proved similarly.

Case 2: Suppose \( p_1 < \overline{P} < p_2 \). Then, \( \forall P_i \in (p_1, \overline{P}) \),

\[
U_i(P_l) > U_i(p_1) \in this interval of \( P \).
\]

From Case 1 and Case 2, for any \( p_1 \) and \( p_2 \) in \( P, U_i(P_l) > \min(U_i(p_1), U_i(p_2)), \forall P_i \in P \). Therefore, \( U_i \) is strictly quasi-concave. \( \Box \)

F. Proposition 2

A NE exists in game LAG.

Proof: Consider the game with set of players \( T \), who choose their actions from \( P \), the set of powers. The utility function of player \( i \) is

\[
V_i(P) = \max_{a_i \in A_i} \{ U_i(P, r) \}
\]  

It follows from Lemma 1 that the action space \( P_i \) of player...
i ∈ T is non-empty and compact. \( P_i \) is also convex (Lemma 2). We note that \( V_i(P) \) is continuous over \( P \) from Lemma 3. To apply the Glicksberg-Fan Theorem \([22][23]\) for the existence of NE, we must show that \( V_i(P) \) is quasi-concave in \( P_i \). This function is equal to \( \bar{L}_i(P) - KP_i^0 \), where

\[
\bar{L}_i(P) = \begin{cases}
L_i(P_i, r^*) & 0 < P_i \leq \bar{P}_i \\
L_i(P_i, r^*) & \bar{P}_i < P_i \leq \bar{P}_2 \\
\vdots & \vdots \\
L_i(P_i, r^*) & \bar{P}_{|\bar{P}|} < P_i \leq \bar{P}_{\max}
\end{cases}
\]  

(9)

Here we define \( \bar{P}_i \) as the abscissa of the intersection of the throughput versus power profiles for two consecutive values of the ALP. Thus, \( L_i(P_i, r^*) = L_i(P_i, r^{n+1}) \) at \( P_i = \bar{P}_i \). The first derivative of \( V_i(P) \) versus \( P_i \) is shown in Fig. 3. An identical number of contours to those in Fig. 2 are plotted, each for an identical value of interference as their counterpart in Fig. 2. Each contour crosses the \( P_i \)-axis at one point only with negative slope. Thus \( V_i(P) \) has a unique global maximum.

\( V_i(P) \) is also non-decreasing to the left of its maximum and non-increasing to the right. Therefore, the arguments put forth in Proposition 1 hold for \( V_i(P) \). Thus, \( V_i(P) \) is quasi-concave in \( P_i \).

The Glicksberg-Fan Theorem can now be applied to show that the game \( \{T, P, V_i(P)\} \) has a NE. This theorem states that a strategic game \( G = <N,A,U> \) has at least one NE if, for each player in \( N \), the following conditions hold:

- the set \( A_i \) of actions is a non-empty, compact and convex subset of a Euclidean space;
- the utility function \( U_i \) is continuous on \( A \) and quasi-concave on \( A_i \).

Each of these conditions has been verified in this proof. Let the NE of the game \( \{T, P, V_i(P)\} \) be denoted by \( p^* \). The NE of game LAG is \( \left(p_i^*, \arg\max_{r \in \Phi} \left[U_i(p_i^*, r)\right]\right)_{i \in N} \). By definition, this action tuple always exists. Thus, a NE exists in game LAG. □

V. A DISTRIBUTED ALGORITHM FOR SOLVING THE LAG

In this section, we describe an algorithm that discovers a NE in the LAG. It is not possible for a player to know the complete channel gain matrix \( G \) and the utility functions of all other players. Hence, a set of mobiles in a realistic distributed environment cannot discover a NE immediately. However, suppose each player \( i \) makes a guess, denoted by the ordered pair \((P_i, r_i)\) regarding its equilibrium power and choice of ALP, denoted by \((\bar{P}, \bar{r})\). Then, assuming that the interference, i.e. \( P_{\bar{r}i} \), remains fixed while it makes a decision, player \( i \) improves its guess by selecting a new combination \((P_i, r_i)\) that maximizes \( U_i \). This results in a new approximation to \((\bar{P}, \bar{r})\). Each player repeats this process of utility-maximizing deviations from the previously selected action, so as to obtain further refinements of the approximation to \((\bar{P}, \bar{r})\). In game-theoretic vocabulary, the player is said to be playing a best-response. The algorithm terminates when all players do not deviate from their previously-selected action. In practice, the algorithm may be assumed to have converged to a NE, when the deviations in all players’ actions become negligibly small. In the following formal description of Algorithm LAG, we use the index variable \( k \) to represent an iteration of the algorithm.

A. Definition 2a – Algorithm LAG

Step 1: Initialize \((P(0), r(0))\). \( k = 0 \). Algorithm LAG consists of the following steps.

Step 2: At each iteration, for all \( i \in T \),

\[
r_i(k+1) = \arg\max_{r \in \Phi} \left[U_i(p_i(k), r)\right]
\]

Equation (10) is now briefly explained. The penalty function \( C_i \) does not depend on the ALP. Therefore, ALP \( r' \) is selected if and only if its throughput for SINR \( \gamma \) (which does not depend on ALP) is higher, i.e. \( L(\gamma, r') > L(\gamma, r'') \). If \( L(\gamma, r') = L(\gamma, r'') \), the lower of the two ALP values is arbitrarily chosen.

Step 3: At iteration \( k \), for each \( i \in T \),

\[
P_i(k+1) = \arg\max_{r \in \Phi} \left[U_i(p_i(k), r_i(k+1))\right]
\]

(11)

From Proposition 1, \( U_i \) has a single global maximum in its argument \( P_i \), provided \( r_i \) is fixed. In this step, this is indeed the case. This ensures that the left-hand side of (11) is single-valued.

Step 4: If \( r_i(k+1) = r_i(k) \) AND \( P_i(k+1) = P_i(k) \), \( \forall i \in T \), the algorithm has converged to a NE. \( \bar{P} = P(k+1) \) and \( \bar{r} = r(k+1) \).

Step 5: Else, \( k = k+1 \). Return to step 2. □

In subsequent proofs, we assume that the numerical algorithm used to solve (11) always converges to a solution in \( A \). Several non-linear programming techniques are available to solve this sub-problem. Detailed descriptions of these techniques are available in [13]. In all examples presented in this paper, we have allowed our simulation package MATLAB\textsuperscript{™} to select an appropriate approach. In all cases, a Successive Quadratic Programming algorithm with quasi-Newton approximations was selected.

VI. CONVERGENCE OF ALGORITHM LAG

An important property of a distributed, iterative algorithm is its convergence. From the preceding development of Algorithm LAG, it becomes apparent that proof of its
convergence is equivalent to proof that it will always result in
the discovery of a Nash Equilibrium. To elaborate, when
Algorithm LAG converges, no player can profitably deviate
from its present action. Any deviation by a player would lead
to a reduction in utility for that player. By Definition 1, this
choice of action must be a NE of the LAG.

To demonstrate convergence of Algorithm LAG, we follow
the approach outlined in Chapter 7 of [13]. In this approach,
an algorithm is regarded as being a point-to-set map \( A \) that
assigns a subset of the domain \( X \) to each point \( x_i \in X \). An
iteration of \( \text{A} \) is represented by the equation \( x_{i+1} \in A(x_i) \). Let
\( M \) denote the algorithmic map of Algorithm LAG. The
description of algorithm LAG (Definition 2a) in terms of its
map is as follows.

\[ A: X \rightarrow Y \]

**A. Definition 2b – Mapping LAG to Algorithmic Map M**

Step 1: Initialization - Let \( a_i \in A \) be the starting point of
Algorithm LAG. Set iteration count \( k=1 \).

Step 2: Iterate - \( a_{k+1} = M(a_k) \).

Step 3: Stop, if \( a_{k+1} \in \Omega \), where \( \Omega \) is the solution set. \( \Omega \) is
shown to exist in Proposition 2.

Step 4: Else, set \( k = k+1 \) and return to step 2.

The idea of a closed map is central to proving the
convergence of algorithms using the algorithmic map
approach.

**B. Definition 3 – Closed Map [13]**

Let \( X \) and \( Y \) be non-empty closed sets in Euclidean spaces.

\[ A: X \rightarrow Y \]

is a point-to-set map. The map \( A \) is said to be closed at \( x \in X \) if
\[ A(x) = \{ y \in Y : x \rightarrow y \} \]
implies that \( y \in A(x) \). The map \( A \) is said to be closed on
\( Z \subseteq X \) if it is closed at each point in \( Z \).

In [7], we show that the algorithmic map \( M \) is not closed.

We shall eventually demonstrate that this property is not
strong enough to preclude convergence of Algorithm LAG. If
the map of an algorithm can be decomposed into two maps,
either of which is closed, then provided certain other
conditions are met, the algorithm may be shown to converge.

The convergence theorem for algorithms with composite maps
[13] is presented in Theorem 1 in a slightly modified form, in
Appendix A. The condition in the theorem, which requires that
the sequence produced by the algorithm be contained in a
compact set, is from [24] and replaces a more stringent
condition in [13]. However, the proof of the theorem remains
unchanged after this substitution.

\( M \) can be decomposed into two maps: \( C \) and \( B \),
corresponding to steps 2 and 3 in Definition 2a, respectively.

The maps \( B \) and \( C \) are defined in Definition 4 and Definition
5, respectively. The composite map can be expressed as \( M = CB \).

This composite mapping involves a mapping under \( B \),
followed by a mapping under \( C \).

**C. Definition 4 – Algorithmic Map B**

\[ B: A \rightarrow A \]

Let \( (p, r) \) be the input to \( B \) and \( (p', r') \) be its
output. Then, for each \( i \in T \), \( r_i = r_i, P_i = \arg \max_{r_i \in \Omega} U_i(P_i, r_i, r_i) \)

From Proposition 1, \( U_i \) has a single global maximum in its
argument \( P_i \) since \( r_i \) is fixed. This ensures that \( P_i \) is single-valued.

**D. Definition 5 – Algorithmic Map C**

\[ C: A \rightarrow A \]

Let \( (p, r) \) be the input to \( C \) and \( (p', r') \) be its
output. Then, for each \( i \in T \), \( P'_i = P_i, r'_i \) is selected as in (10).

In the following lemma, we show that the utility function \( U_i \)
is continuous over \( A_i \). The proof follows from the application of
the continuity definition, and is presented in Appendix B.

This result is needed in the convergence proof of Algorithm
LAG.

**E. Lemma 5**

The utility function \( U_i \) is continuous over \( A_i \).

As an intermediate step in proving the convergence of
Algorithm LAG, we show in Proposition 3 that the
algorithmic map \( B \) is closed. We present the proof in
Appendix C.

**F. Proposition 3**

The algorithmic map \( B \) is closed.

We are now equipped with the results we need to prove that
Algorithm LAG converges to a NE.

**G. Proposition 4**

The Algorithm LAG converges to a Nash Equilibrium.

Proof: The proof follows from a systematic application of
Theorem 1 (see Appendix A). Note that by the definition
\( M = CB \), the mappings \( B \) and \( C \) are applied in an order that is
conducive to the direct application of this theorem. We first
define the descent function \( \beta \), at iteration \( k \), as follows.

\[ \beta(x, k) = -\sum_{i=1}^{N} U_i(x, I_i(k)), \quad x \in A, \text{ and } x_i = (P_i, r_i) \] (12)

Here, \( I_i(k) = \sum_{j=1}^{\infty} G_i P_j(k) + n_i \) is the interference at \( i \) at
iteration \( k \).

1. \( \beta \) is the sum of \( N \) continuous functions (using Lemma
5). Hence, \( \beta \) is continuous.

2. We first show that if \( y = C(x), x \in A \), then
\( \beta(y, k) \leq \beta(x, k) \), at iteration \( k \). Let \( x = (P_s, r_s) \) and
\( y = (P_r, r_r) \) By definition of \( C \) (Definition 5), \( P_s = P_r \).

If \( r_s = r_r \), as would occur when the ALPs do converge,
then from (12), we must have \( \beta(y, k) = \beta(x, k) \). It is important to
realize that this does not necessarily mean that the algorithm
terminates here, since the powers might not have converged.

If the ALPs have not converged either, there is such that
\( U_i(P_s, r_s, r_s) > U_i(P_r, r_r, r_r) \). Thus, in this
case, \( \beta(y, k) < \beta(x, k) \). Note that it is not possible for
\[ \beta(y,k) > \beta(x,k) \] since this would be inconsistent with the definition of the Algorithm LAG and becomes apparent by studying (10).

(3) The map \( B \) is closed, by Proposition 3.

(4) We now show that, given \( y = B(x), x \in A \), then \( \beta(y,k) < \beta(x,k) \), if \( x \notin \Omega \).

By contradiction, suppose that at iteration \( k \), \( \beta(y,k) = \beta(x,k) \) and the algorithm has not converged, i.e. \( y \notin \Omega \), where \( x = (P_i, r_i) \) and \( y = B(x) = (P_i, r_i) \). By definition of the map \( B \) (Definition 4), \( r_i = r_i \). Then, we must have \( U_i(P_i, P_i, r_i, r_i) = U_i(P_i, P_i, r_i, r_i) \) for any \( i \). This implies that \( P_i, r_i \) is contained in \( \Omega \). The convergence implies that our original assumption, \( \beta(y,k) = \beta(x,k) \), must be false. Hence, we must strictly have \( \beta(y,k) < \beta(x,k) \). Also note that it is not possible for \( \beta(y,k) > \beta(x,k) \) since this would be inconsistent with the definition of the Algorithm LAG as is evident from (11).

(5) Any sequence produced by the mapping \( M \) is contained in the set \( A \). The set \( A \) is compact by Lemma 1, since \( A_i \) is compact \( \forall i \in T \).

Since all the conditions required by Theorem 1 are satisfied, the algorithm described by the map \( M \) converges. Suppose that the accumulation point is denoted by \( \bar{a} = (\bar{P}, \bar{r}) \). Then, at the point of convergence, the following property is satisfied for all \( i \in T \):

\[ U_i(\bar{a}, \bar{r}) \geq U_i(a_i, r_i), \forall a_i \in A \]  

We observe that (13) is simply the definition of a NE (Definition 1). Thus, we have shown that the convergence point of algorithm LAG is a NE. \( \Box \)

VII. RESULTS

The basis for our results is the seven-cell GPRS network configuration illustrated in Fig. 4. A cutaway consisting of seven co-channel cells is depicted. The intervening area is assigned to adjacent channels, which are not considered in the system model. For computational efficiency, only the first-tier of co-channel interferers relative to the central cell is considered in the subsequent analysis.

The base-stations of the cells numbered 1 to 7 are the co-channel interferers, and constitute the set \( T \). In computing the path loss coefficients, the distance from each interferer to the receiver of interest is approximated as the distance between the centers of their cells. The frequency re-use factor is set to 3. With this setting, the first-tier of interfering base stations lie on a circle of radius 3R, where R is the cell radius. The path loss model is log-distance with attenuation factor 3. Noise is referenced to the maximum possible received power of the weakest user, as shown in (14), and is identical for all \( i \in T \).

\[ n_i = \min_{\text{est}} \left( \frac{G_{e_i} P_{\text{max}}}{\text{SNR}} \right) \quad \forall i \in T \]  

The maximum transmitter power per link \( P_{\text{max}} \) is 10 W, while minimum transmitter power \( P_{\text{min}} \) is 0 W. Permitting mobiles to move is inconsistent with our assumption that the link gain matrix does not change for the convergence period of the techniques presented.

The system-theoretic algorithms that are compared with Algorithm LAG are now presented. The detailed descriptions of all these methods appear in [7]. They are:

- Optimal Target Assignment (OTA) and Stepwise Rate Removals (SRR)
- Generalized Selective Power Control with Gradual Rate Removals (GSPC-GRR)
- Greedy Rate Packing (GRP)

A. Optimal Target Assignment (OTA) and Stepwise Rate Removals (SRR)

OTA is a technique of optimally allocating rates in a multi-rate cellular radio network [7]. This technique exploits the relationship between the desired targets, the dominant eigenvalue of the channel gain matrix, and the “row-sum” property of the same matrix in the optimization formulation. The optimization can then be framed as a linear programming problem (LPP), which is easier to solve than a non-linear program. This optimal formulation forms the basis of a systematic algorithm called SRR that achieves the same purpose and performance through an iterative process. Both algorithms are described in detail in [7].

B. Generalized Selective Power Control with Gradual Rate Removals (GSPC-GRR)

In GSPC-GRR, a two-part procedure is followed. The first step involves finding the maximum possible rate (or SINR target) from a finite set of rates that can be assigned to each user within the maximum power, along with the power that achieves these targets. In the next step, the actual SINR values are calculated. If the SINR of any one user is found to be less than its target, the user with the lowest path gain to its serving base station is penalized by a rate decrement. This step is known as GRR. The two-step algorithm is iterated until the powers of all links converge. These algorithms are described in [16].

C. Greedy Rate Packing (modified for non-CDMA networks)

In Greedy Rate Packing (GRP), users are scheduled in descending order of their path gains to their serving base stations. For each user, the maximum achievable target assuming worst-case interference from all other higher priority co-channel links is calculated. As each user is added, the
impact of their interference is accounted for in the resource assignments made to the preceding users in the scheduling order. This modification [7] to the original GRP algorithm [16] is necessary to adapt it to non-CDMA networks. The algorithm is greedy since it assigns SINR to the users with best link quality first, and assigns the remaining capacity to the weaker users.

D. Comparison between Game-Theoretic and System-Theoretic Techniques

We consider an interference-limited scenario, in which the SNR (14) is set to 100 dB. It is worth emphasizing that this value is specifically chosen to model the interference-limited cases as opposed to noise-limited cases. CS selection in all methods is accomplished by mapping SINR to CS according to Table 3. The CS that results in the best throughput for the given SINR is selected.

SRR is implemented using a logarithmically increasing sequence of discrete SINRs, belonging to the set \{1, 2, 3, ..., 50\} dB. GSPC uses the same step-size, but with a maximum value of 30 dB. In GRP, we limit the SINR to occupy the interval [0, 30] dB. The GPRS downlink throughput achieved at 40 dB using the most spectrally-efficient CS-4 coding scheme is 2.5% greater than that at 30 dB. However a 10 dB power increase is required to achieve this increase in throughput. Simulations in [7] showed that GRP and GSPC tended to assign maximum rates to a user in the most favorable conditions at the cost of other mobiles. For the sake of fairness, we use a maximum SINR target of 30 dB for these algorithms.

The penalty function parameters used by Algorithm LAG are \(q = 2\), \(K = 1\). Algorithm LAG is assumed to converge when the deviation in power is less than 1 mW. The smaller this value, the longer it takes for convergence to occur. All players are initialized with power of 20% of \(P_{\text{max}}\) (2W, for \(P_{\text{max}} = 10W\)) and CS-1.

We refer to the sum of per-user throughputs as the **system throughput**. To obtain an unbiased performance comparison between all the techniques considered here, we conducted several simulations for each of them with random mobile locations within a seven-cell configuration. In addition, the path gain on each link also included a log-normal shadowing component with an 8 dB standard deviation. In Fig. 5, we plot the empirical cumulative distribution function (CDF) of downlink system throughput for GSPC-GRR, GRP, SRR, OTA and Algorithm LAG.

The improvement in system throughput achieved by Algorithm LAG is clearly evident in Fig. 5. For example, if we consider the fraction of the time system throughput exceeds 60 kbps, we notice that this works out to 39% for Algorithm LAG, which is clearly greater than the 35% for OTA and SRR, 27% for GRP and 16% for GSPC-GRR. The mean percentage system capacity improvement that Algorithm LAG achieves over its competitors is calculated in Table 4, using average system throughput as a measure of capacity. The nearest competitors are OTA and SRR. Algorithm LAG obtains over 4% improvement over these methods. However, note that Algorithm LAG is distributed, while OTA and SRR are centralized. This observation favors Algorithm LAG. Furthermore, the improvement over GRP, which is centralized, is nearly 10%, while that over GSPC-GRR, which is distributed, approaches 20%. Establishing the efficiency of the convergent NE states analytically, for any utility function, is a non-trivial problem in game theory. Two ways of analytically establishing the efficiency of a NE that we are aware of is by means of S-modularity and potential game theory (see [3],[26]). Even pure power control games, though, are not guaranteed to yield efficient steady states (see [4]). We have shown through simulation that, in fact, the NE of our game is not efficient in all cases studied. Given this, we conducted an exhaustive simulation analysis, which indicates that in a probabilistic sense, LAG is more efficient than the system-theoretic approaches.

Table 5 shows the probability (relative-frequency) of occurrence of the four coding schemes for all solution methods. The following observations can be made. First, we note that the rate assignment frequencies for OTA and SRR are nearly identical. Second, a user is most likely to be assigned either CS-1 or CS-4 when using GSPC-GRR and GRP. These methods result in the highest likelihood of a user being assigned CS-1 (56% for GSPC-GRR and 38% for GRP) or CS-4 (31% for GSPC-GRR and 35% for GRP). Algorithm LAG results in the second lowest probability (after OTA and SRR) of being assigned CS-1 (31%). There is a 62% chance that Algorithm LAG will assign either CS-3 or CS-4. The corresponding value of this statistic for OTA is 58%, but only 52% for GRP and 39% for GSPC-GRR. Based on these observations, it may be stated that Algorithm LAG, OTA and SRR result in the fairest rate assignment amongst the methods considered here.

E. Performance analysis of LAG

To evaluate the performance of LAG more comprehensively we propose a set of Figures of Merit (FOM) that a network designer can use as a rule of thumb to compare the performance of different penalty functions for several starting conditions. We suggest three options and use them to evaluate the performance of some cases.

\[
FOM_1 = \frac{\sum_i L_i}{\sum_i \frac{P_i}{P_{\text{max}}}}
\]

\[
FOM_2 = \sum_i L_i - A_{\text{max}} \sum_i \frac{P_i}{P_{\text{max}}}
\]

\[
FOM_3 = \sum_i L_i
\]

The proposed FOMs are based on the belief that the sum of throughput is to be maximized. However the first two add the condition that the power consumption is to be minimized. In all three cases, a higher FOM is preferable. The effect of penalty exponent, \(q\), on FOM and power consumption

respectively, are illustrated in Fig. 6 and Fig. 7. FOM1 and FOM2 are low for lower values of \( q \) due to excessive power consumption. However, system throughput does not change appreciably with \( q \); hence, \( q > 2 \) is desirable. At low SNR, there is a greater variability in throughput (Fig. 8). Here it is recommended to select \( q > 1 \) to optimize the tradeoff between throughput and power consumption.

We next perform a sensitivity analysis in terms of the variation of system throughput performance when the standard deviation of log-normal shadowing is varied. We varied the standard deviation from 6dB to 12dB. With greater variability, the probability of observing poor channel conditions is greater, and therefore, results in lower throughput. These results are shown in Fig. 9 and Fig. 10. We also observe that, unlike the GRP algorithm, LAG achieves a fair SINR assignment among all the users; this is shown in Fig. 11 where we plot the CDF’s of throughput for all users in the system.

The above model included only log-normal shadowing, but can be easily extended to include multipath fading. We evaluated the system throughput performance by including both Rayleigh fading and log-normal shadowing components in the path gain on each link. As expected, adding multipath fading on top of log-normal shadowing lowers the throughput achieved, as shown in Fig. 12.

VIII. CONCLUSIONS

Systems theory continues to be applied to RRM problems pertaining to cellular data networks. Game-theoretic techniques provide an alternative approach to system-theoretic methods (those using linear algebra) for solving RRM problems. Power control problems have proved to be a popular area in RRM for application of game-theoretic analysis ([3]-[6]).

Our main contribution here is the generalization of power control games to incorporate link adaptation. This resulted in a modification of the typical action set of a power control game to include a discrete-valued Adaptive Link Parameter (ALP), in addition to power. We chose a sigmoid function to model throughput since it permits an accurate approximation to simulated throughput results. Pricing is used from the outset based on previous results [3] and the knowledge that all users would transmit at maximum power in its absence. This would create unnecessary interference and would tend to limit the achievable throughput. Pricing helps avoid this situation. We show that a NE exists in the resulting game - called the Link Adaptation Game (LAG) - using well-known NE existence results [22][23]. We construct the heuristic Algorithm LAG that discovers a NE. The convergence of the algorithm is an important property since proving the existence of a NE does not immediately present the solution to the game. We develop a convergence proof based on the theory of point-to-set algorithmic maps.

In the final section, we show that Algorithm LAG performs favorably compared to analogous system-theoretic schemes in terms of capacity and fairness. In an interference-limited scenario, it achieves a downlink system throughput improvement of 4 – 20% over competing schemes. Our simulations show that Algorithm LAG results in the second lowest probability of a terminal being assigned the lowest coding scheme (almost equal to OTA and SRR) and the highest probability of one being assigned either CS-3 or CS-4. This aspect of performance benchmarking is absent from other studied game-theoretic works. However, we consider this to be important in establishing useful utility models for a class of games, in this case power control games.

It is not clear how our scheme, distributed though it may be, can find its way into an actual implementation. An immediate impediment is estimating the parameters of the utility model. It might help to simplify the utility function, but this might be accompanied by a loss of accuracy. The incorporation of the dynamic aspects of fading is lacking in our model and the subsequent analysis. While this is consistent with the “one-shot event” philosophy that is pervasive in power control literature [14], we expect the nature of a dynamic fading channel to have a significant impact on algorithm convergence and model parameter estimation.

In the future, the RRM of networks of cognitive radios will be complicated by the degree of heterogeneity introduced by having several such radios in a network. It might not be too far-fetched to suggest that software radios might be involved in the formation of ad-hoc networks. Established techniques for studying comparatively simple fixed infrastructure systems, such as cellular networks, will not work without modification. At the same time, this is an excellent reason to consider game-theoretic techniques for this purpose. These techniques are inherently suited to modeling systems of interacting entities that make their decisions autonomously. Finding the right tools for the problems at hand is a challenge confronting researchers adopting the game-theoretic approach.

APPENDIX A: THEOREM 1 – CONVERGENCE OF ALGORITHMS WITH COMPOSITE MAPS [12][24]

Let \( X \) be a non-empty closed set in \( E_n \), and let \( \Omega \subset X \) be a non-empty solution set. Let \( \beta : E_n \rightarrow E_i \) be a continuous function, and consider the point-to-set map \( C : X \rightarrow X \) satisfying the following property: Given \( x \in X \), then \( \beta(y) \leq \beta(x) \) for \( y \in C(x) \). Let \( B : X \rightarrow X \) be a point-to-set map that is closed over the complement of \( \Omega \), and satisfies \( \beta(y) < \beta(x) \) for each \( y \in B(x) \), if \( x \notin \Omega \). Now, consider the algorithm defined by the composite map \( A = CB \). Given \( x_1 \in X \), suppose that the sequence \( \{x_k\} \) is generated as follows:

If \( x_k \notin \Omega \), stop; otherwise, let \( x_{k+1} = A(x_k) \), replace \( k \) by \( k+1 \), and repeat.

Suppose the sequence \( \{x_k\} \) is contained in a compact subset \( \Lambda \) of \( X \). Then, either the algorithm stops in a finite number of steps with a point in \( \Omega \), or all accumulation points of \( \{x_k\} \) belong to \( \Omega \). □
APPENDIX B: PROOF OF LEMMA 5

A real-valued function \( f : S \rightarrow E_1 \), where \( S \) is a convex set in \( E_\alpha \), is said to be continuous at \( x \in S \) if, for any given \( \varepsilon > 0 \), there exists \( \delta > 0 \) such that the conditions
\[
\| x - x \| < \delta \quad \text{implies that} \quad | f(x) - f(x) | < \varepsilon \quad [13].
\]

Consider any two arbitrarily selected points \( A^0 = (P_i, r_i) \) and \( A^1 = (P_i, r_i') \), in \( A \). Since both \( P \) and \( R \) are bounded, it is possible to find \( \delta > 0 \), such that \( | A^0 - A^1 | < \delta \). Using (3) and (6), clearly \( 0 \leq L \leq \max \alpha(r_i) \) and \(-K \nu_{\max} \leq -C \leq 0 \). Hence,

\[
\text{Then, given some } \varepsilon > 0, \text{we must have } |U^0(A^0) - U^1(A^1)| < \varepsilon. \text{Since } U^0 \text{ is bounded, it is possible to make } \varepsilon \text{ as large as necessary so that } | A^0 - A^1 | < \delta, \text{ for any } \delta > 0. \text{Alternatively, let us suppose we shrink } \varepsilon, \text{ such that } \varepsilon \rightarrow 0. \text{Then, as we correspondingly shrink } \varepsilon \text{ and } r_i' = r_i, \text{since the set of ALP } R \text{ is discrete, and the } \delta - \text{neighborhood of any } r_i \text{ contains only a single point, i.e. } r_i \text{ itself.}
\]

Hence,\( |U^0(A^0) - U^0(A^1)| = |\Delta U^0| = |\nabla U^0/\nabla P^0| \Delta P^0 \). Since we now have \( \Delta U^0 = \varepsilon, \Delta P^0 = \delta \) and \( \nabla U^0/\nabla P^0 \) is continuous at any \( A^0 = (P_i, r_i) \), choosing \( \delta < \varepsilon \) results in \( |U^0(A^0) - U^1(A^1)| < \varepsilon \). Hence, the proof is complete. \( \square \)

APPENDIX C: PROOF OF PROPOSITION 3

The proof is an application of Definition 3. We first define an arbitrary converging sequence \( \{X(k) = (P(k), r(k))\} \) in \( A \), Let the limiting point of this sequence be denoted by the vector \( X = (P, r) \). Since the set of ALPs is discrete, a converging sequence of ALPs implies that there exists \( K \) such that, for all \( k \geq K \),
\[
r(k) = r(K) = r.
\]

The image produced by mapping \( \{X(k)\} \) under \( B \) is the sequence \( \{Y(k) = B(X(k)) = (P(k), r(k))\} \). The input sequence \( \{X(k) = (P(k), r(k))\} \) produces a sequence of powers that converge to \( P \) and ALP vectors that converge to \( r \). By the definition of \( B \) (Definition 4), the ALPs remain unchanged. Hence, we must have \( r(k) = r(\kappa) \). Using (18),

\[
\text{clearly } \{r(\kappa)\} \rightarrow r.
\]

From Lemma 5, since \( U_i \) is continuous over \( A_i \), \( \{X(k)\} \) will generate a converging sequence of utility function profiles.
\[
\{U_i(P(k), r(k))\} \rightarrow U_i(P, r), \forall i
\]

Thus, we must have,

\[
\left\{ \max_{r \in R} U_i(P_i, P, r_i) \right\} \rightarrow \max_{r \in R} U_i(P_i, P, r_i)
\]

We conclude that map \( B \) yields a converging sequence of powers \( \{P(k)\} \rightarrow \arg \max_{r \in R} U_i(P, P, r) \). Let the accumulation point for power be denoted by \( P \). Therefore, \( \{Y(k)\} \) converges to a point \( Y = (P, r) \). Further let \( Y' = B(X) \).

REFERENCES


Samir V. Ginde received the Bachelor of Engineering degree in Electronics Engineering from Mumbai University, Mumbai, India in 2001 and Masters degree in Electrical Engineering from Virginia Tech, Blacksburg, VA in 2004. He currently works at Qualcomm Inc, in San Diego, CA. His interests include multimedia application design for wireless environments.

Allen B. MacKenzie [M] has been an Assistant Professor in Virginia Tech’s Bradley Department of Electrical and Computer Engineering since 2003. He joined Virginia Tech after receiving his Ph.D. from Cornell University and (in 1999) his B.Eng. from Vanderbilt University, both in Electrical Engineering. Dr. MacKenzie's research focuses on wireless communications systems and networks. His current research interests include cognitive radio and cognitive network algorithms, architectures, and protocols and the analysis of cooperation in such systems and networks. In addition to the IEEE, Dr. MacKenzie is a member of the ASEE and the ACM. In 2006, he received the Dean's Award for Outstanding New Assistant Professor in the College of Engineering at Virginia Tech.

R. Michael Buehrer joined Virginia Tech as an Assistant Professor with the Bradley Department of Electrical Engineering in 2001. He is currently an Associate Professor and is part of Wireless @ Virginia Tech, a comprehensive research group focusing on wireless communications. His current research interests include dynamic spectrum sharing, Multiple Input Multiple Output (MIMO) communications, intelligent antenna techniques, position location networks, Ultra Wideband, spread spectrum, interference avoidance, and propagation modeling. In 2003 he was named Outstanding New Assistant Professor by the Virginia Tech College of Engineering.

From 1996-2001 Dr. Buehrer was with Bell Laboratories in Murray Hill, NJ and Whippany, NJ. While at Bell Labs his research focused on CDMA systems, intelligent antenna systems, and multiuser detection. He was named a Distinguished Member of Technical Staff in 2000 and was a co-winner of the Bell Labs President’s Silver Award for research into intelligent antenna systems. Dr. Buehrer received the BSEE and MSEE degrees from the University of Toledo in 1991 and 1993 respectively. He received a Ph.D. from Virginia Tech in 1996 where he studied under the Bradley Fellowship.

Dr. Buehrer has co-authored 30 journal papers and holds 11 patents in the area of wireless communications. He is currently a Senior Member of IEEE, and an Associate Editor for IEEE Transactions on Wireless Communications, IEEE Transactions on Vehicular Technologies and IEEE Transactions on Signal Processing.

Ramakant S. Komali is a Ph.D. candidate at Virginia Tech’s Bradley Department of Electrical and Computer Engineering since 2004. He received a Master of Science degree in Physics from Indian Institute of Technology, Kharagpur, India in 1996. He also received an M.S. in Mathematics from Syracuse University in 1998, and an MSEE from University of Texas at Dallas in 2004. His interests include wireless network design and optimization, topology control in ad-hoc networks, spectrum access and management, cognitive networks, and game theory.
Fig. 1. A set of sample utility functions versus power for GPRS CS-4. The penalty function parameters are $K=1$ and $q=2$. Each profile is for a different value of interference; the collection of profiles in each plot is obtained by varying the interference to cover a range of SINR from -6 dB to 30 dB. The profiles corresponding to lower values of interference yield higher utility.

Fig. 2. First-derivative of the utility-function of player $i$ with respect to the power of player $i$. The power-axis (x-axis) uses a logarithmic scale. Each plot corresponds to a particular GPRS coding scheme (the ALP). The collection of profiles in each plot is obtained by varying the interference to cover a range of SINR from -6 dB to 30 dB. All plots show a single zero-crossing of the first derivative.

Fig. 3. First-derivative of the maximum (over all ALP) utility function of player $i$ with respect to the power of player $i$. The power-axis (x-axis) uses a logarithmic scale. The collection of profiles in each plot is obtained by varying the interference to cover a range of SINR from -6 dB to 30 dB. All plots show a single zero-crossing of the first derivative.

Fig. 4. Seven-cell configuration, re-use factor = 3. A set of co-channel cells is shown. The center of each cell is occupied by the base station. In cell 1, the mobile is shown by the large dot at the cell edge.

Fig. 5. Performance comparison of GSPC-GRR, GRP, SRR, OTA and Algorithm LAG using CDF of system throughput (kbps) for the interference-
limited scenario (SNR = 100 dB). Note that the performances of SRR and OTA are nearly identical.

Fig. 6. Effect of penalty exponent on power consumption

Fig. 7. Effect of penalty exponent on FOM's, SNR = 100 dB

Fig. 8. Effect of penalty exponent on FOM's, SNR = 5 dB

Fig. 9. Variation in system throughput CDF with varying levels of shadowing. The collection of profiles is obtained by varying the standard deviation of the log-normal shadowing from 6 dB to 12 dB.

Fig. 10. Effect of channel quality on system throughput performance
Fig. 11. CDF of each of the seven users’ SINR in NE state under LAG

Fig. 12. CDF of system throughput (kbps) obtained for LAG for the interference-limited scenario (SNR = 100 dB), with and without channel fading.